https://doi.org/10.48047/AFJBS.6.Si3.2024.2816-2829

AFJBS
African Journal of Biological Sciences
Journal homepage: http://www.afjbs.com

Research Paper Open Access

A Review on Mathematical Growth Models Related to Different Wings of Biological Field

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Article Info

Volume 6, Issue Si3, 2024
Received: 01 May 2024
Accepted: 15 June 2024
Published: 04 July2024
doi: 10.48047/AFJBS.6.Si3.2024.2816-2829

ABSTRACT:

Everything has a Mathematical component, but the natural sciences-including biology has the strongest connections. The relationship between Mathematical and Biological sciences has grown at a faster pace in recent years. The mathematical modeling of the growth of forestry, micro-organisms, fisheries, tumor cells, and agricultural crops are the important part in the fields of Biology. The main objective of this paper is to review different types of Mathematical growth and the convergence of Mathematical and biological sciences which are mainly related to biological growth. This paper is empathized by providing 80 growth equations which describes different types of growth.

Keywords: Mathematical model, Growth model, Biological Growth

1. Introduction

Mathematical growth models play a central role in understanding and quantifying the intricate process of biological growth. Growth or alternation in organism size with ageing is a fundamental but a very crucial biological process that combinesvarious processes and effects [20]. Growth and development occur across the life stages and focuses on the physical, intellectual, emotional and social changes that human go through. Growth predominately concentrates on the physical changes that take place across various parts of the body.Growth model is a representation of the growth mechanics and growth plan of the product; a growth model can help in forecast how much growth to expect over a certain period assumption. It can also help to simulate growth outcomes for different initiatives. Growth models shows specific and predictive pattern of growth of the respective field with time.

The process of creating a mathematical representation of a real world scenario for the purpose of making a prediction or provide insight is referred to as mathematical modeling. The main purpose of this paper to review different kinds of growth models in different biological fields. Various kinds of growth model were studied in the paper which are specially related to biological growth such as bacterial growth, fishery growth, Plant growth, Tumor growth, Agricultural growth and all other growth about certain body parts of animal and various types of bird.How this growth model works in various biological fields: Mathematical growth models are widely used in various biological fields to explain and understand the growth and development of organism,populations and ecosystems.These models are based on Mathematical equations that capture the underlying processes driving growth.By increasing the size of simulated tree in the forest on an annual or more frequent basis, tree growth models imitate the growth and development of forest ecosystems [15]. Several methods have been put out in fisheries to calculate the average growth of individual. For fish population analysis, a mathematical formulation of the mean individual body growth of fish is required that links the species size to its age [37]. There are mathematical models that can be used to represent the relatively simple equations that govern tumor development kinetics. Tumor growth model describe the physiological processes and measurements as well [3]. Agricultural development is beneficial to planting crops, harvesting and processing for safer food conditions. Enhancing agricultural growth has been widely acknowledged as a significant means to alleviate poverty [31].

2. Bacterial Growth

Life is fundamentally linked with growth [64].The study of bacterial growth can be performed in situ in environmental matrices like soil, water, or organic wastes, or in a controlled laboratory conditions using pure cultures of microorganisms[41]. It has been considered that lack of carbon is most frequent limiting factor for bacterial growth in soil, also other nutrients like nitrogen, phosphorus are also considered as the limiting factors for bacterial growth in soil[13]. Bacteria grow in water when it is stagnant or when it is not treated with enough water treatment chemicals, such as chlorine[16]. The organic waste provides the bacteria with nourishment for growth and reproduction[22]. Bacterial growth can be described in 4 phases: Lag phase, Exponential phase, Stationary phase, and death phase.In lag phase, bacteria take time to adjust to their new environment in the bioreactor. When this process take place there is no growth.In exponential phase, the growth is rapid and bacteria present in the bioreactor doubles at high rate.In stationary phase, there is no growth phase. But still bacteria continue to multiple while some continues to die.In death phase, nutrients required for bacterial growth are completely used up [42].The paper is reviewing thirty growth models which are used to describe the specific growth rate of bacteria.

3. Tumor Growth

The intricate process of tumor growth ultimately depends on tumor cells multiplying and spreading across the host tissues [5]. Tumour growth modelling using mathematics and technologies is not new- in fact it goes back over 50 years. Due to necessity, most cancer models were general phenomenological and type specific, as a result, they struggled due to lack of experimental data for modelling and verification[57]. This paper is specifically review about the growth models which describes the growth of brain tumour. The phrase “brain tumour” describes a group of neoplasms, each of which has its own biology, prognosis, and course of therapy, these tumours are accurately known as “intracranial neoplasm”, as some of them do not develop from brain tissues [17]. As brain tumors are frequently seen, it is important for general practitioners to understand how to diagnose and treat them[56]. In recent decades, numerous models have been proposed. But this paper is reviewing 5 models: Hossfield, Korf, Levakovic III, Levakovic I, Sloboda [67].

4. Plant Growth

Physiology, community dynamics, and environmental characteristics are all integrated into th e fundamental ecological process of plant growth [61].Plant growth and associated changes depend on secondary metabolic products. The daily biological cycles control these metabolic activities. Plant growth involves both elastic(reversible) and plastic/viscoelastic(irreversible)deformations, making plant growth a highly mechanical process.This paper review about the few growth models which describes the growth of cotton tree, growth of babul tree (acacia nilotica), height growth of different Eucalyptus species.
Cotton is also known as the ‘white gold’ of India[43]. Cotton is an important fiber crop of India being cultivated over an area of about 9.5 million hectares (mha) representing approximately one quarter of the global area of 35 million hectares under this crop[34]. The major goal of this paper is to review appropriate nonlinear growth models analytical models(Logistic, Gompertz, Monomolecular, Richards, Reciprocal and Quadratic) to concentrate on past and future patterns of cotton area output and productivity in India[35].
The Indian leguminous tree Acacia nilotica, also known as babul, is a true multipurpose nitrogen-ixing tree and the source of Indian gum Arabic [46]. It is a complex species with nine subspecies,six of which are indigenous to the tropics of Africa and three of which are indigenous to the Indian subcontinent. This paper review about the growth models(Chapmen Richards, Von Bertalanffy, Logistic, Gompertz, Monomolecular, Negative Exponential) for the Babul growth in India[10, 29,30,53].
Eucalyptus is a genus of over seven hundred species of flowering trees, shrubs or mallees in myrtle family.Most species of Eucalyptus are native to Australia, and every state and territory has representative species[6]. The growth models which are considered in this paper are used to study about the five different types Eucalyptus species, namely, *E. tereticornis*, *E. hybrid, E. grandis, E. palleta* and *E. dandeli*[44]. This paper review about the growth models which are used to study the height growth of different Eucalyptus species in India using seven nonlinear growth models(Logistic, Gompertz, Monomolecular, Chapman- Richards (with three and four parameters) and Von-Bertalanffy(with three and four parameters)[44,45,46].

5. Fishery Growth

Fisheries are one of the four major systems that constitute the foundation of global economic system [25]. The management of fisheries and aquaculture is a very complicated issue. It stands to reason to review the most successful cases after more than 40 years of applying models or applications to the management of fisheries in order to assess past performance as well as to identify present issues and potential research areas[8].Recent years have witnessed an increase in the number of growth models available and the comparison of many growth models for a certain species or study in the field of fisheries science. Growth models estimate life history parameters that are employed in the management of fishery stocks, such as growth rates and asymptotic size[20]. There are several uses for being able to precisely simulate fish growth in population dynamics[60]. It was usual practice in fisheries science to apply a single growth model to size-at-age data.However, many model types have been proposed and evaluated to estimate the growth of fishes[52]. This paper is reviewing seventeen growth models which describes the growth pattern of fishes[20, 38, 54].

6. Agricultural Growth

This paper mainly reviews about the nonlinear regression models in agricultural research. Many crop and soil processes are better captured by nonlinear than linear models, nonlinear regression models are crucial tools[1].Parsimony, interpretability,and prediction are the major benefits of nonlinear models[4]. The growth Equations which are mentioned in this paper are based on five groups (exponential, sigmoid functions, temperature dependence, peak or bell shaved curves). Several fields of soil and plant sciences use the exponential decay and exponential gives rise to maximum functions.They are frequently employed to characterize the vertical distributions of light in plant canopies [50].Another significant class of nonlinear models are sigmoid curves, which are mathematical functions with a S shape. These models are frequently used to characterize plant characteristics as a function of time, herbicide dose, or seed germination[26,47]. The most crucial biological process for plant growth is photosynthesis, and factors affecting its rate include irradiance, temperature, N supply, the vapour pressure deficit, and CO_2 concentration[32,27]. Temperature dependence models describes monotonic increase.The temperature dependency of numerous soil and plant processes has been modeled using a wide range of nonlinear regression models that have been devised and evaluated [39,59]. In agricultural research, the bell-shaped or peak functions have been used to explain the relationship between temperature and phenological growth rates, leaf size and rank in a plant, and the impact of soil moisture on N_2O emissions [28, 55].

7. Others

The paper reviews other growth models which are related to biological fields such as growth of microalgae[18], Baluchi sheep[3], growth curves of Broiler chicken fed on different levels of corn bran[49], growth in the Andalusian turkey breed[2], growth curves in Mule duck[63], growth of scrotal circumference in Awassi male lambs[12], artificial neural networks and nonlinear regression analysis in Cherry Valley ducks[33], growth in body weight at different ages in Toy poodles[48], growth in Japanese Quail[51], growth in Creole chickens in Maxico[40], Biomass Growth Prediction in Batch Fermentation[21].

Table 1: Integral forms of Growth model

Sl. No.	Model Name	Integral Form	Parameters	Name	Category
1.	Exponential model	$y = ae^{bt}\ , b > 0$	a, b, c	Gogoi et al 2022	Bacteria
2.	Logistic	$y = a/(1 + be\^(-ct))$	a, b, c	Gogoi et al 2022	Bacteria

	Model	$X_t = X_0.\exp(\mu t)\left[1-\left(\frac{X_0}{X_m}\right)\right][1 - \exp(\mu t)]$	X_0, μ, X_m	Dhanasekar et al 2003A	Bacteria
		$w_t = \beta_0/(1+\beta_1\exp(-\beta_2)$	w_t, β_0, β_1 β_2	Narinc et al 2010	Japanese quail
		$y = y_{asym}/\{1+\exp[-k(t-t_m)]$	y, y_{asym} t_m, t, k	Archontoulis 2015	Agriculture
3.	Monomolecular model	$y = a(1 - be^{(-ct)})$	a, b, c	Gogoi et al 2022	Bacteria
		$y = A(1 - Be^{-kt})$	y, A, B, k	Mahanta et al 2018	Plant growth
4.	Gompertzmodel	$y = ae^{-(b-ct)}$	a, b, c	Gogoi et al 2022	Bacteria
		$y = Ae^{-Be^{-kt}}$	A, B, k	Mahanta et al 2018	Plant growth
		$w_t = \beta_0\exp(-\beta_1\exp(-\beta_2 t)$	w_t, β_0, β_1 β_2	Narinc et al 2010	Japanese quail
		$y = y_{asym}\exp\{-\exp[-k(t-t_m)]\}$	y, y_{asym} t_m, t, k	Archontoulis 2015	Agriculture
5.	Richards model	$y = a.\{1 + v.e^{[k(\tau - t)]}\}^{(-\frac{1}{v})}$	a, k, τ, v	Gogoi et al 2022	Bacteria
		$L_t = L_\infty\left(1 - \delta e^{-k_6(t-t_4)}\right)^{\frac{1}{\delta}}$	L_α, k_6, t L_t, t_4, δ	Flinn and Midway 2021	Fisheries
		$w_t = \beta_0/\left(1+\beta_1\exp\left(-\beta_2 t\right)\right)^{\frac{1}{\beta_3}}$	w_t, β_0, β_1 β_2, β_3	Narinc et al 2010	Japanese quail
		$y = y_{asym}/\{1 + v\exp[-k(t-t_m)]\}^{\frac{1}{v}}$	y, y_{asym} t_m, t, k, v	Archontoulis 2015	Agriculture
6.	Von Bertanalffy model	$y = \left\{a^{1-m} - be^{[-k(t-t_0)]}\right\}^{\frac{1}{1-m}}$	a, b, k, m	Gogoi et al 2022	Bacteria
		$L_t = L_\alpha - L_\alpha e^{-k(t-T_0)}$	L_t, L_α, T	Pawlak and hanumara 1990	Fisheries
		$w_t = \beta_0^{1-\beta_3} - \beta_1\exp\left[(-\beta_2 t)^{\frac{1}{1-\beta_3}}\right]$	k	Narinc et al 2010	Japanese quail
7.	The schnute model	$y = \left[y_1^b + \frac{(y_2^b - y_1^b)(1 - e^{(-a)(t-\tau_1)}}{1 - e^{(-a)(\tau_2 - \tau_1)}}\right]^{\frac{1}{b}}$	$a, b, y_1,$ y_2	Gogoi et al 2022	Bacteria
8.	The stannard model	$y = a\left\{1 + e^{\left[-\frac{(1+kt)}{p}\right]}\right\}^{(-p)}$	a, k, p	Gogoi et al 2022	Bacteria
9.	Ratkowsky Non linear Square model	$\sqrt{r} = b(T - T_0)(1 - e^{c(T - T_{max})})$	b, c	Gogoi et al 2022	Bacteria
10.	Weibull Model	$y = a - be^{-ct^m}$	a, b, c, m	Gogoi et al 2022	Bacteria

11.	Monod model	$\mu = \frac{\mu_{max} \cdot S}{k_s + s}$; $\mu = \frac{\mu_{max} x_s}{k_s + s}$; $\mu = \mu_{max} \frac{s^n}{s^n + k_s} (using parameter n$ (n>1))	μ_{max}, k_s	Gogoi et al 2022, Mulyani et al 2018, Vijaylaxmi et al 2018, Gunawan et al 2018,Farred et al 2019,Muloiwa et al 2020	Bacteria
13.	Moser model	$\mu = \frac{\mu_{max} S^n}{k_s + s^n}$ $\mu = \frac{\mu_m S^n}{k_s + s^n}$	μ_{max}, k_s, n	Gogoi et al 2022, Gunwan et al 2020, Farred et al 2019, Muloiwa et al 2020, Zaffer et al 2021.	Bacteria
14.	Powell Model	$\mu = \frac{(\mu_{max} + m)S}{k_s + s} - m$;	μ_{max}, k_s, m	Gogoi et al 2022, Gunwan et al 2020, Farred et al 2019, Muloiwa et al 2020, Zaffer et al 2021.	Bacteria
15.	Haldane model	$\mu = (\mu\_(max\ ) S)/(k\_s + S + s\^2/k\_i\ )$	μ_{max}, k_s, k_i	Gogoi et al 2022, Mohdzahri et al 2017	Bacteria
16.	Aiba model	$\mu = \mu_{max} \frac{S}{k_s + S} \exp\left(-\frac{s}{k_i}\right)$ $\mu = \mu_{max} \frac{S}{k_s + S} \exp\left(-\frac{S}{k_I}\right)$	$\mu_{max}, k_s, k_{i,} k_I$	Zahri et al 2017, Muloiwa et al 2020, ShabnamSharifyazd et al 2019	Bacteria
17.	Teisser Edward model	$\mu\_max\ [1 - \exp(-s/k\_i\ ) - \exp(s/k\_s\ )]$	$\mu_{max}, , k_i, k_s$	Mohdzahri et al 2017	Bacteria
18.	Yano model	$\mu = \frac{\mu_{max} S}{S + k_s + \left(\frac{S^2}{k_1}\right)\left(1 + \frac{S}{k}\right)}$	$\mu_{max}, , k_s, k_1, S$	Mohdzahri et al 2017, Muloiwa et al 2020	Bacteria
19.	Tesier model	$\mu = \mu_{max}(1 - e^{ki^s})$	μ_{max}, k_i	Vijaylaxmi et al 2018, Muloiwa et al 2022	Bacteria
20.	Product inhibitor model	$\mu = \mu_{max} \left(\frac{S}{k_s + S}\right)\left(1 - \frac{C_P}{C_{Pm}}\right)^n$	μ_{max}, k_s, C_p, S	Gunwan et al 2020	Bacteria
21.	Contais Model	$\mu = \frac{\mu_{max} S}{k_s X + S}$	μ_{max}, k_s, X, S	Muloiwa et al 2022	Bacteria
22.	Logarithm model	$\mu = a + b\ln(s)$	a, b, s	Muloiwa et al 2022	Bacteria
23.	Han and levenspid model	$\mu = \mu_{max} \left[1 - \left(\frac{S}{S_m}\right)\right]^n \left[\frac{S}{S + k_s \left(1 - \frac{S}{S_m}\right)^m}\right]$	μ_{max}, S_m, k_s, S	Muloiwa et al 2022	Bacteria

24.	Verhulst model	$\mu = \mu_{max}\left[1 - \frac{X}{X_m}\right]$	μ_{max}, X_m, , X	Muloiwa et al 2022	Bacteria
25.	Luong model	$\mu = \mu_{max}\frac{K}{K_s + S}\left(1 - \frac{S}{S_m}\right)^m$	μ_{max}, k_s, S_m, S	Muloiwa et al 2022	Bacteria
26.	Webb model	$\mu = \frac{\mu_{max}S\left(1 + \frac{S}{K_i}\right)}{S + K_s + \left(\frac{S^2}{k_i}\right)}$	μ_{max}, k_s, K_i, S	Muloiwa et al 2022	Bacteria
27.	Formal kinetic (unstructured and low structured) model	$\mu = \mu_{max}\frac{S}{S + K_s}$	μ_{max}, k_s, S	Shabnam sharifyazd 2019	Bacteria
28.	Microbial growth model	$\mu = \mu_{max}\frac{S}{k_n} if S \leq K_n$ $\mu = \mu_{max} if S \geq K_n$	μ_{max}, $k_s, k_n S$	Shabnam sharifyazd 2019	Bacteria
29.	Andrew model	$\mu = \frac{\mu_{max}}{\left(1 + \frac{k_s}{S}\right)\left(1 + \frac{S}{k_{I,s}}\right)}$	μ_{max}, k_s, k_I, S	Shabnam sharifyazd2019, Mulyani et al 2018	Bacteria
30.	Monod incorporate LeudkingPiret(MLP)	X_t $P_t = P_0 + \alpha X_0\left[\exp\left(\frac{\mu_{max}St}{K_s + S}\right) - 1\right]$ $+ \frac{\beta X_0}{\left(\frac{\mu_{max}S}{K_s+S}\right)\left[\exp\left(\frac{\mu_{max}St}{K_s+S}\right) - 1\right]}$	μ_{max}, k_s, S, X_0	Shabnam sharifyazd 2019	Bacteria
31.	Flexible non linear model	$y_i = ae^{-b\left(\frac{t_i - c}{t_i}\right)}$	a, b, c	Lugert et al 2014	Fisheries
32.	Pitcher and Macdonald Model (PM model)	$L_t = L_\alpha - L_\alpha e^X$ where $X = -K(T - T_0) - Csin\left(2\pi\frac{(T-T_2)}{52}\right)$	L_α, K, T_0 C, T_2	Pawlak and hanumara 1990	Fisheries
33.	Cloerm and Nichols (CN)model	$L_t = L_\alpha - (L_\alpha - L_0)e^x$ Where $X = -A(t - T_0)$ $-\frac{180\,A_1}{\pi}[\cos(\frac{\pi(T_0+\theta)}{180})]$	L_α, K, T_0 C, T_2	Pawlak and hanumara 1990	Fisheries
34.	Pauly and Gaschutz mode	$L_t = L_\alpha - L_\alpha e^X$ Where $X = -K(T - T_0) - \frac{CK}{2\pi}\sin 2\pi(T - T_2)$	L_α, K, T_0 C, T_2	Pawlak and hanumara 1990	Fisheries
35.	Hoenig and Hanumara (HH)model	$L_t = L_\alpha - L_\alpha e^X$ Where $X = -K(T - T_0) - \frac{KC}{2\pi}\sin 2\pi(T - T_2)$ $+ \frac{KC}{2\pi}\sin 2\pi(T - T_c)$	L_α, K, T_0 C, T_2	Pawlak and hanumara 1990	Fisheries
36.	Two parameter VBGM	$L_t = L_\infty$(1-$e^{[-k_1(t)]}$)	L_α, K_1, t L_t	Flinn and Midway 2021	Fisheries

37.	Three parameter VBGM	$L_{(t)} = L_\infty(1\text{-}e^{[-k_1(t-t_0)]})$	L_α, K_1, t L_t	Flinn and Midway 2021	Fisheries
38.	Two parameter Gompertz	$L_t = L_0 e^{G(1-e^{-k_2 t})}, G = \ln\frac{L_\infty}{L_0}$	L_α, K_1, t L_t	Flinn and Midway 2021	Fisheries
39.	Three parameter Gompertz model	L_t = L_∞ e^(−(1 e^(−k_2 (t − α)))/k_2)	L_α, K_2, t L_t, α	Flinn and Midway 2021	Fisheries
40.	Three parameter Logistic model	L_t = L_α/(1 + e^(−k_3 (t − α)))	L_α, K_1, t L_t	Flinn and Midway 2021	Fisheries
41.	Linear VBGM	$L_t = (b_0 + b_1 t)(1 - e^{-k_1(t-t_0)}$	b_0, b_1, k_1 t, t_0	Flinn and Midway 2021	Fisheries
42.	Generalized VBGM	$L_t = L_\infty\left(1 - e^{-k_1(t-t_0)}\right)^p$	L_α, K_1, t L_t, t, t_0	Flinn and Midway 2021	Fisheries
43.	Seasonal VBGM	$L_t = L_\infty\left(1 - e^{-k_1(t-t_0)}\right) - \left(\frac{ck}{2\pi}\right)[sin2\pi(t - t_s) - sin2\pi(t_0 - t_s)$	L_α, K_1, t L_t, t_s	Flinn and Midway 2021	Fisheries
44.	Power model	$L_t = a_0 + a_1 t^b$	L_t, a_0, a_1 b	Flinn and Midway 2021	Fisheries
45.	Hossfeld model	$Y = \frac{t^c}{b + \frac{t^c}{a}}$	$Y, t^c, a,$ b	Boris zeide 1993	Brain tumor
46	Korf model	$Y = ae^{-bt^{-c}}$	$Y, a,$ b, c	Boris zeide 1993	Brain tumor
47.	Levakovic 1	Y = a(t^d/(b + t^d))^c	Y, a, t^d b	Boris zeide 1993	Brain tumor
48.	Levakovic 3	Y = a(t^2/(b + t^2))^c	Y, a, b, t	Boris zeide 1993	Brain tumor
49.	Sloboda model	$Y = ae^{-be^{-ct^d}}$	$Y, a, b,$ c, d, t	Zeide, 1993	Brain tumor
50.	Chapman Richards(3 parameter)	$y = A\{1 - e^{-kt}\}^d$	A, K, d	Mahanta et al 2018	Plant growth
51.	Chapman Richards(4 parameter)	$y = A\{1 - Be^{kt}\}^d$	$A, B, K,$ d	Mahanta et.al., 2018	Plant growth
52.	Exponential decay	$y = y_0 \exp(-kt)$	y, y_0, k, t	Archontoulis 2015	Agricultu re
53.	Exponential give rise to maximum	$y = y_0[1 - \exp(-kt)]$	y, y_0, k, t	Archontoulis 2015	Agricultu re
54.	Weibull model	$y = y_{asym}[1 - \exp(-at^b)$	y, y_{asym} a, t	Archontoulis 2015	Agricultu re
55.	Beta model	$Y = Y_{max}(1 + \frac{t_e - t}{t_e - t_m})(\frac{t}{t_e})^{\frac{t_e}{(t_e - t_m)}}$	y, y_{max} t_m, t, t_e	Archontoulis 2015	Agricultu re
56.	Hill model	$Y = t_c^n/(t_c^n + t^n)$	y, t_c^n t^n	Archontoulis 2015	Agricultu re

57.	Asymptotic exponential	$y = y_{asym}\left[1-\exp\left(-\frac{al}{y_{asym}}\right)\right] - R_d$	y, y_{asym} a, l, R_d	Archontoulis 2015	Agriculture
58.	Rectangular hyperbola	$y = {aly_{asym}}/{(y_{asym}+al)} - R_d$	y, y_{asym} a, l, R_d	Archontoulis 2015	Agriculture
59.	Nonrectangular hyperbola	$Y = \frac{Y_{asym} + aI - \sqrt{(Y_{asym}+aI)^2 - 4\theta aI}}{2\theta} - R_d$	y, y_{asym} a, l, R_d	Archontoulis 2015	Agriculture
60.	Van't Hoff	$y = Q_{10}^{(T-T_{ref})/10}$	$y, Q_{10},$ T, T_{ref}	Archontoulis 2015	Agriculture
61.	Arrhenius	$y = \exp\left\{\frac{E}{R\left[\frac{1}{T_{ref}+273} - \frac{1}{T+273}\right]}\right\}$	y, E T, T_{ref}	Archontoulis 2015	Agriculture
62.	Lloyd and Taylor	$y = \exp\left\{E_0\left[\frac{1}{T_{ref}+273-T_x} - \frac{1}{T+273+T_x}\right]\right\}$	$y, E_0,$ T_{ref}, T_x $, T$	Archontoulis 2015	Agriculture
63.	Bell curve model	$y = y_{asym}\exp[a(x-x_0)^2 + b(x-x_0)^3]$	$y, y_{asym},$ a, x, x_0, b	Archontoulis 2015	Agriculture
64.	Gaussian function	$y = y_{asym}\exp\{-0.5[(x-x_0)/b^2$	$y, y_{asym},$ $b, x, x_0,$	Archontoulis 2015	Agriculture
65.	Power	$y = aX^b$	$y, a, X,$ b	Archontoulis 2015	Agriculture
66.	Modified hyperbola	$y = aX/(1+bX)$	y, a, X $, b$	Archontoulis 2015	Agriculture
67.	Michaelis-Menten	$y = \mu X/(X + C_{sat})$	$y, \mu, X,$ C_{sat}	Archontoulis 2015	Agriculture
68.	Rational function	$y = a_1 X^{a_2}/(1 + a_3 X^{a_4})$	$y, a_1, X,$ a_3, a_4	Archontoulis 2015	Agriculture
69.	Ricker curve	$y = a_1 X exp(-a_2 X)$	y, a_1, X a_2	Archontoulis 2015	Agriculture
70.	Morgan Mercer Flodin model	$w_t = (\beta_0\beta_1 + \beta_2 t^{\beta_3})(\beta_1 + t^{\beta_3})$	$w_t, \beta_0,$ $\beta_1, \beta_3,$ β_2, t	Narinc et al 2010	Japanese Quail
71.	Negative exponential model	$w_t = \beta_0(1 - \beta_1\exp(-\beta_2 t))$	$w_t, \beta_0,$ $\beta_1,$ β_2, t	Narinc et al 2010	Japanese quail
72.	Hyperbolastic 1	$w_t = \beta_0(1 + \beta_1\exp(-\beta_0\beta_2 t - \beta_4 arc\sin h(t))$	$w_t, \beta_0,$ $\beta_1,$ β_2, t	Narinc et al 2010	Japanese quail

73.	Hyperbolastic 2	$w_t = \beta_0/(1 + \beta_1 \operatorname{arc}\sin h\left(\exp(-\beta_0\beta_2 t^{\beta_3})\right))$	$w_t, \beta_0, \beta_1, \beta_2, t$	Narinc et al 2010	Japanese quail
74.	Hyperbolastic 3	$w_t = \beta_0 - \beta_1(\exp(-\beta_2 t^{\beta_3} - arc \sin h(\beta_4 t)))$	$w_t, \beta_0, \beta_1, \beta_3, \beta_2, t$	Narinc et al 2010	Japanese quail
75.	Tanka model	$S_{c(t)} = \left(\frac{1}{\sqrt{\beta_1}}\right) \ln \lvert 2\beta_1(t-\beta_3) + 2\sqrt{\beta_1^2(t-\beta_3^2)+\alpha\beta_1} \rvert + \beta_2$	$\beta_1, \beta_2, \alpha, \beta_3$	Bilgin et al 2004	Awassi male lamb
76.	Gompertz laired model	$w_t = w_o \exp[\left(\frac{L}{K}\right)(1-\exp(kt)]$	w_t, w_o L, k	Estrade et al 2019	Creole chicken of Mexico
77.	Verhust model	$y = a/(1 + bexp(-kt)$	y, a, b k, t	Arando et al 2021	Andalssian Turkey breed
78.	Sinusoidal Model	$y = a_1 + a_2(\sin(ct-\theta))^n$	a, b a_1, a_2 k, θ	Arando et al 2021	Andalssian Turkey breed
79.	Hinshelwood model	$\mu = \mu_{max}\frac{s(t)}{s(t)+k_s}(1-k_i q)$	μ, μ_{max} $k_i,$	Islam et al 2021	Biomass
80.	Proposed model	$\mu = \mu_{max}\frac{s(t)}{s(t)+k_s+\frac{s^2}{k_i}}(1-k_i q)$	μ, μ_{max} $k_i,$	Islam et al 2021	Biomass

8. Discussion

In this work, we attempt to provide an overview of the use and effectiveness of mathematical growth models in many branches of biology. These models can be used by scientist in almost all aspects of scientific inquiry, development, and production. Models are beneficial instruments for describing concepts and explanations and scientist use them frequently to describe, analyze and forecast phenomena that take place in the natural world. The relationship between Mathematical and Biological sciences is a quiet new development into present era. Various Mathematical models are being used to develop deeper into different biological phenomena. In our review paper, we are putting efforts to show how mathematical growth model are being applied to analyses different areas of biology such as bacterial growth, environment and plant growth, production of crops in fishery,tumor growth and so on. In all these biological fields, Mathematical growth models provide a quantitative framework to test hypothesis,make predictions and guide decision-making.The prime objective of this review paper is to foster a comprehensive understanding of the wide application of the Mathematical growth model in Biology. It has now become a proven fact that the use of Mathematics and Biology paves the way for breaking new grounds in the fields of agriculture, fishery and so on. Apart from this,the study we have embarked on will surely help the policy makers of a country developed suitable policy regarding health care, agriculture,fishery, plants for the sake of better society.

9. Conclusion

It is evident from the above discussion that a large number of growth models are studies in the last decades, but this paper is reviewing a few growth models which are specifically related to the growth phenomena. This paper clearly shows that relationship and also analyses how the Mathematical models study the biological growth process and these methods are essential tools for researchers, biologist and ecologists as they enable the analyses of complex biological phenomena in a systematic and quantifiable manner.

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