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LUCAS ANTIMAGIC LABELING OF SOME MIRROR GRAPHS PAPER-1

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ABSTRACT
A (p, q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \rightarrow \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^*: V(G) \rightarrow \{1,2, \dots \sum L_q\}$ given by $f^*(u) = \sum_{e \in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).In this paper the Lucas Antimagic Labeling of some Mirror graphs are found.

KEYWORDS: Bistar graph , Complete bipartite graph, Lucas Antimagic graph, Mirror graph.

1.INTRODUCTION

In this paper, graph $G(V, E)$ is considered as finite, simple and undirected with p vertices and q edges. A graph labeling is a fundamental concept in graph theory, where integers are assigned to vertices or edges. Its enormous applications in astronomy, theory of coding and other fields has propelled it to the frontend of research. After referring, the seminal work of Gallian, as showcased in his comprehensive survey [1], we have embarked on this research endeavor. Furthermore, the innovative concept of Antimagic labeling, introduced by N.Hartsfield and G.Ringel in the year 1990, has opened up new avenues of exploration. Inspired by these groundbreaking contributions, we introduced Lucas Antimagic labeling and further Lucas Antimagic labeling has been investigated on $B(m,n), M\big(B(m,n)\big), K_{m,n}$, $M\big(K_{m,n}\big)$.

2.DEFINITIONS

Definition 2.1: Lucas number is defined by the linear recurrence relation

$L_1 = 2,\ \ L_2 = 1\ and\ \ L_n = L_{n-1} + L_{n-2}\ , n > 2$

The first few Lucas numbers are 2,1,3,4,7,11,18,29,47,…

Definition 2.2:[2] A (p,q) graph G is said to be a Lucas antimagic graph if there exists a bijection $f: E(G) \to \{L_1, L_2, \cdots L_q\}$ such that the induced injective function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ given by $f^*(u) = \sum_{e\in E(u)} f(e)$ are all distinct (where E(u) is the set of edges incident to u).

Definition 2.3:[1] A complete bipartite graph is a special type of bipartite graph where every vertex of one set is connected to every vertex of the other set.

Definition 2.4:[5] The Bistar graph $B(m,n)$ is the graph obtained from K_2 by joining m pendant edges to one end of K_2 and n pendant edges to the other end of K_2 .

Definition 2.5:[6] Let G be bipartite graph with partite sets V_1 and V_2 and let G' be a copy of G and $V_1'\ and\ V_2'$ be the copies of V_1 and V_2.The mirror graph denoted by M(G) is obtained from G and G' by joining each vertex of V_2 to the corresponding vertex in V_2' by an edge.

3.MAIN RESULTS

Theorem 3.1:

The Bistar graph $B(m,n), m \geq 2, n \geq 2$ is Lucas antimagic graph.

Proof:

Let G be $B(m,n)$

Let $V(G) = \{u, v, u_i, v_j : 1 \leq i \leq m\ ,1 \leq j \leq n\}$

$E(G) = \{uv, uu_i,\ vv_j\ : 1 \leq i \leq m\ ,1 \leq j \leq n\}$

Define a function $f: E(G) \to \{L_1, L_2, \dots L_q\}$ by

$f(uv) = L_1$

$f(uu_i) = L_{i+1}$,$1 \leq i \leq m$

$f(vv_j) = L_{m+1+j}$,$1 \leq j \leq n$

The induced function $f^* : V(G) \to \{1,2, \ldots \sum L_q\}$ is given by

$$f^*(\mathrm{u}) = L_1 + \sum_{i=1}^{m} L_{i+1}$$

$f^*(u_i) = L_{i+1}, 1 \leq i \leq m$

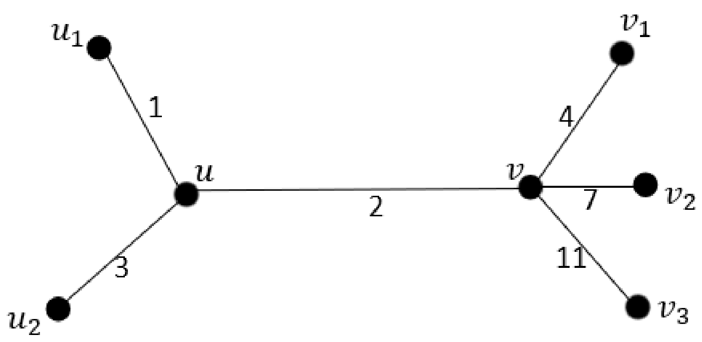
$f^*(v) = L_1 + \sum_{j=1}^{n} L_{m+1+j}$

$f^*(v_j) = L_{m+1+j}, 1 \leq j \leq n$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.1.1: The Bistar graph $B(2,3)$ and its Lucas antimagic labeling.



Theorem 3.2:

The Mirror graph $M(B(m,n))$ of a Bistar graph $B(m,n), m \geq 2, n \geq 2$ is Lucas antimagic.

Proof:

Let G be $M(B(m,n))$

Let $V(G) = \{u, u', v, v', u_i: 1 \leq i \leq m, v_j: 1 \leq j \leq n, u_i': 1 \leq i \leq m, v_j': 1 \leq j \leq n\}$

$E(G) = \{uv, u'v', uu_i: 1 \leq i \leq m, vv_j: 1 \leq j \leq n, u'u_i': 1 \leq i \leq m, v'v_j': 1 \leq j \leq n, u_iu_i': 1 \leq i \leq m, v_jv_j': 1 \leq j \leq n\}$

Define a function $f: E(G) \to \{L_1, L_2, \ldots L_q\}$ by

$f(uv) = L_1$

$f(u'v') = L_2$

$f(uu_i) = L_{2+i}, 1 \leq i \leq m$

$f(u'u_i') = L_{m+2+i}, 1 \leq i \leq m$

$f(u_i u_i') = L_{2m+2+i}, 1 \leq i \leq m$

$f(vv_j) = L_{3m+2+j}, 1 \leq j \leq n$

$f(v'v_j') = L_{3m+n+2+j,} 1 \leq j \leq n$

$f(v_j v_j') = L_{3m+2n+2+j}, 1 \leq j \leq n$

The induced function $f^* : V(G) \to \{1,2, \dots \sum L_q\}$ is given by

$$f^*(\mathrm{u}) = L_1 + \sum_{i=1}^{m} L_{2+i}$$

$f^*(u_i) = L_{2+i} + L_{2m+2+i}, 1 \leq i \leq m$

$f^*(u_i') = L_{m+2+i} + L_{2m+2+i}, 1 \leq i \leq m$

$$f^*(u') = L_2 + \sum_{i=1}^{m} L_{m+2+i}$$

$f^*(v) = L_1 + \sum_{j=1}^{n} L_{3m+2+j}$

$f^*(v_j) = L_{3m+2+j} + L_{3m+2n+2+j}, 1 \leq j \leq n$

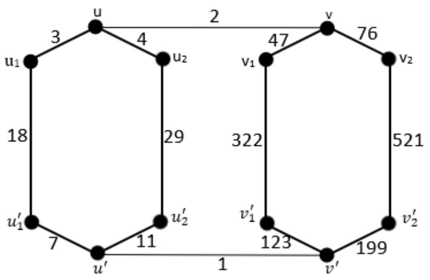
$f^*(v_j') = L_{3m+2n+2+j} + L_{3m+n+2+j}, 1 \leq j \leq n$

$$f^*(v') = L_2 + \sum_{j=1}^{n} L_{3m+n+2+j}$$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.2.1: The Mirror graph $M(B(2,2))$ and its Lucas antimagic labeling.



Theorem 3.3:

The Complete bipartite graph $K_{m,n}, m \geq 2, n \geq 2$ is Lucas antimagic graph.

Proof:

Let G be $K_{m,n}$.

Let $V(G) = \{u_i : 1 \leq i \leq m \,, v_j : 1 \leq j \leq n\}$

$E(G) = \{u_i v_j \;: 1 \leq i \leq m \,, 1 \leq j \leq n\}$

Define a function $f: E(G) \rightarrow \{L_1, L_2, \dots L_q\}$ by

$f(u_1 v_1) = L_2$

$f(u_1 v_2) = L_1$

$f(u_1 v_j) = L_j \,, 3 \leq j \leq n$

$f(u_i v_j) = L_{(i-1)n+j} \,, 2 \leq i \leq m, 1 \leq j \leq n$

The induced function $f^* : V(G) \rightarrow \{1,2, \dots \sum L_q\}$ is given by

$$f^*(u_1) = L_1 + L_2 + \sum_{j=3}^{n} L_j$$

$f^*(u_i) = \sum_{j=1}^{n} L_{(i-1)n+j} \,, 2 \leq i \leq m$

$f^*(v_1) = L_2 + \sum_{i=2}^{m} L_{(i-1)n+1}$

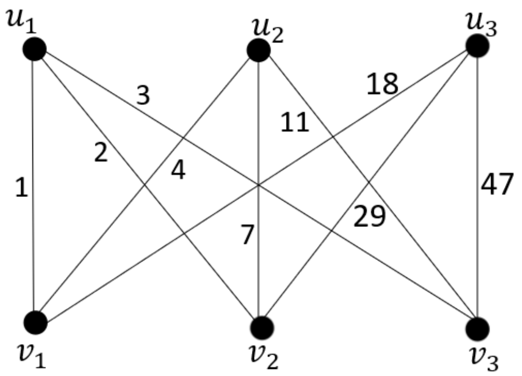
$f^*(v_2) = L_1 + \sum_{i=2}^{m} L_{(i-1)n+2}$

$$f^*(v_j) = \sum_{i=1}^{m} L_{(i-1)n+j} \,, 3 \leq j \leq n$$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.3.1: The Complete bipartite graph $K_{3,3}$ and its Lucas antimagic labeling.



Theorem 3.4:

The Mirror graph $M(K_{m,n}) m \geq 2, n \geq 2$ of a Complete bipartite graph $K_{m,n}$ is Lucas antimagic.

Proof:

Let G be $M(K_{m,n})$

Let $V(G) = \{u_i : 1 \leq i \leq m, v_j : 1 \leq j \leq n, u_i' : 1 \leq i \leq m, v_j' : 1 \leq j \leq n\}$

$E(G) = \{u_i v_j : 1 \leq i \leq m, 1 \leq j \leq n, u_i' v_j' : 1 \leq i \leq m, 1 \leq j \leq n, v_j v_j' : 1 \leq j \leq n\}$

Define a function $f : E(G) \to \{L_1, L_2, \ldots L_q\}$ by

$f(u_i v_j) = L_{(i-1)n+j}$,$1 \leq i \leq m, 1 \leq j \leq n$

$f(u_i' v_j') = L_{mn+(i-1)n+j}, 1 \leq i \leq m$, $1 \leq j \leq n$

$f(v_j v_j') = L_{2mn+j}, 1 \leq j \leq n$

The induced function $f^* : V(G) \to \{1,2, \ldots \sum L_q\}$ is given by

$$f^*(u_i) = \sum_{j=1}^{n} L_{(i-1)n+j}, 1 \leq i \leq m$$

$f^*(u_i') = \sum_{j=1}^{n} L_{mn+(i-1)n+j}, 1 \leq i \leq m$

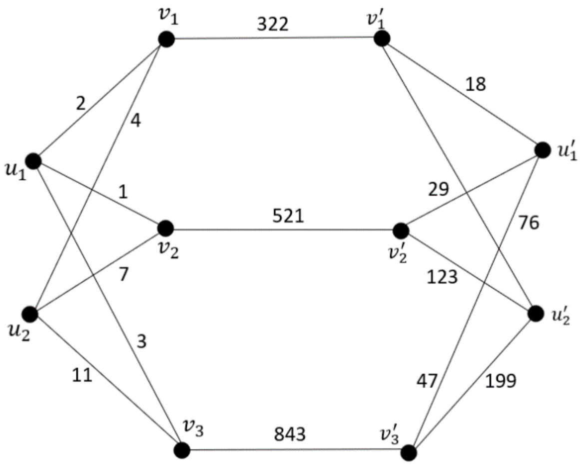
$$f^*(v_j) = L_j + \sum_{i=2}^{m} L_{(i-1)n+j} + L_{2mn+j}, 1 \leq j \leq n$$

$$f^*(v_j') = L_{mn+j} + \sum_{i=2}^{m} L_{mn+(i-1)n+j} + L_{2mn+j}, 1 \leq j \leq n$$

We observe that the vertices are all distinct.

Hence G is Lucas antimagic graph.

Example 3.4.1: The Mirror graph $M(K_{2,3})$ and its Lucas antimagic labeling.



4.CONCLUSION

In this paper, We have successfully demonstrated that various Mirror graphs are Lucas antimagic and through this mathematical analysis, we have established the profound relation between mirror graphs and Lucas antimagic labeling. Our future research endeavour is to determine various other mirror graphs are also Lucas antimagic and hence similar investigations are in process.

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