https://doi.org/10.48047/AFJBS.6.Si3.2024.2807-2815

African Journal of Biological Sciences

Journal homepage: http://www.afjbs.com

Research Paper Open Access

Arithmetic Sequential Graceful Labeling for Swastik Graph

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Article Info

Volume 6, Issue Si3, 2024
Received: 01 May 2024
Accepted: 15 June 2024
Published: 04 July2024
doi: 10.48047/AFJBS.6.Si3.2024.2807-2815

ABSTRACT:

Let G be a simple, finite, connected, undirected, non-trivial graph with p vertices and q edges. V(G) be the vertex set and E(G) be the edge set of G. Let f:V(G)⟶{a,a+d,a+2d,a+3d,…,2(a+qd)} where a ≥0 and d≥1 is an injective function. If for each edge uv∈E(G) ,f^*:E(G)⟶{d,2d,3d,4d,…,qd} defined by f^* (uv)=|f(u)-f(v)| is a bijective function then the function f is called arithmetic sequential graceful labeling. The graph with arithmetic sequential graceful labeling is called arithmetic sequential graceful graph. Here we proved that swastik graph, path union of swastik graph, cycle of swastik graph is arithmetic sequential graceful graph.

Keywords: Graceful labeling, Arithmetic sequential graceful labeling, swastik graph, path union of swastik graph, cycle of swastik graph.

1. Introduction

A fascinating area of research in graph theory is labeling. Giving values to edges or vertices is the process of labeling. It was Alexander Rosa [2] who first proposed the idea of graceful labeling. Later, a few labeling techniques were presented. See Gallian's dynamic survey [3] for further details. V J Kaneria1 , Meera Meghpara , H M Makadia Pasaribu[4] proved that grid graph is graceful labeling .V. J. Kaneria, H. M. Makadia and M. M. Jariya [5] proved that cycle of graph is graceful labeling.V. J. Kaneria, H. M. Makadia[7] proved that swastik graphs are graceful. In this paper we have proved that swastikgraph [Fig-1] is arithmetic sequential graceful graph. We also proved that $P(t.Swg_\eta)$[Fig-2]and $C(t.Swg_\eta)$[Fig-3] are arithmetic sequential graceful graph. We start with simple, finite, connected, undirected, non-trivial graph with p vertices and q edges. $V(G)$ be the vertex set and $E(G)$ be the edge set of G. Here Swg_η denotes swastik graph.The motivation for studying the swastik graph as an arithmetic sequential graceful graph is to advance the understanding of graph labeling by manipulating its unique symmetry and cultural significance. This research has potential applications in network design, data structure optimization, algorithm development, cryptography, and biological network analysis.

Definitions

Definition 2.1:

A function f is called graceful labeling of graph $G = (V.E)$ if $f: V \to \{0,1,\dots,q\}$ is injective and the induced function $f^*: E \to \{1,2,\dots,q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u,v) \in E$.The graph called graceful graph if it admits gracefullabeling.

Definition 2.2:[4]

let $G_1, G_2, G_3, \dots, G_n, n \geq 2$ be n copies of graph G. Then the graph attained by adding an edge from $G_i$to $G_{i+1} (1 \leq i \leq n-1)$ is called path union of G and it is denoted by $P(G)$.

Definition 2.3:[5]

For a cycle C_n, each vertex of C_n is displaced by connected graphs $G_1, G_2, G_3, \dots, G_n$ and is known as cycle of graphs. We shall mention it by $C(G_1, G_2, G_3, \dots, G_n)$. If we displace each vertex by a graph G, i.e.$G_1 = G, G_2 = G, G_3 = G, \dots, G_n = G$,such cycle of graph G is denoted by $C(n.G)$.

Definition 2.4:[7]

Swastik graph is an unionof 4 copies on $C_{4n.}$If $V_{i,j}(for\ all\ 1 \leq i \leq 4\ \&\ 1 \leq j \leq 4n)$ be vertices of i^{th} copy of $C_{4n}^{(i)}$ then we shall unite$V_{1,4r}\&V_{2,1}$,$V_{2,4r}\&V_{3,1}$, $V_{3,4r}\&V_{4,1}$, $V_{4,4r}\&V_{1,1}$by a single vertex. If we bend arms of graph rightward at the centre then the graph seemsa swastik. It is denoted bySw_n, where $n \in N - \{1\}$. Obviously $|V(Sw_n)| = 16(n) - 4\ \&|E(Sw_n)| = 16(n)$.

Definition 2.5:

Let G be a simple, finite, connected, undirected, non-trivial graph with p vertices and q edges. $V(G)$ be the vertex set and $E(G)$ be the edge set of G. Let $f: V(G) \to \{a, a+d, a+2d, a+3d, \dots, 2(a+qd)\}$ where a ≥ 0 and $d \geq 1$ is an injective function. If for each edge $uv \in E(G)$, $f^*: E(G) \to \{d, 2d, 3d, 4d, \dots, qd\}$ defined by $f^*(uv) = |f(u) - f(v)|$ is a bijective function then the function f is called arithmetic sequential graceful labeling. The graph with arithmetic sequential graceful labeling is called arithmetic sequential graceful graph and it is denoted by ASG_g.

2. Main Results

Theorem 3.1:
A graph $Swg_\eta$is anarithmetic sequential graceful graph,$\eta \geq 2$.
Proof:
Considering $u_{1,4\eta}\&u_{2,1}$, $u_{2,4\eta}\&u_{3,1}$ and $u_{3,4\eta}\&u_{4,1}$ as single vertex $u_{2,1}$, $u_{3,1}$ and $u_{4,1}$ respectively.
Let $V(Swg_\eta) = \{u_{i,j}: 1 \leq i \leq 4; 1 \leq j \leq 4\eta\}$ and
$E(Swg_\eta) = \{u_{i,j}u_{i,j+1}: 1 \leq i \leq 4; 1 \leq j \leq 4\eta - 1\} \cup \{u_{i,1}u_{i+1,1}:\ 1 \leq i \leq 3\} \cup \{u_{1,1}u_{4,1}\}$.
Here $|V| = 16\eta - 4$, $|E| = 16\eta$.
We define a function $f: V(G) \rightarrow \{a, a+d, a+2d, a+3d, \dots, 2(a+qd)\}$.
The vertex labeling are as follows,

$f(u_{1,j}) = a + \left[16\eta - \left(\frac{j-1}{2}\right)\right] d$ if $j = 1,3,\dots,4\eta - 1$
$f(u_{1,j}) = a + \left[\left(\frac{j-2}{2}\right)\right] d$ if $j = 2,4,\dots,2\eta$
$f(u_{1,j}) = a + \left[\left(\frac{j}{2}\right)\right] d$ if $j = 2\eta + 2, 2\eta + 4,\dots,4\eta$
$f(u_{2,j}) = a + \left[2\eta + \left(\frac{j-1}{2}\right)\right] d$ if $j = 1,3,\dots,2\eta + 1$
$f(u_{2,j}) = a + \left[2\eta + \left(\frac{j+1}{2}\right)\right] d$ if $j = 2\eta + 3, 2\eta + 5,\dots,4\eta - 1$
$f(u_{2,j}) = a + \left[14\eta - \left(\frac{j-2}{2}\right)\right] d$ if $j = 2,4,\dots,2\eta$
$f(u_{2,j}) = a + \left[14\eta - \left(\frac{j}{2}\right)\right] d$ if $j = 2\eta + 2, 2\eta + 4,\dots,2\eta$
$f(u_{3,j}) = a + \left[12\eta - \left(\frac{j-1}{2}\right)\right] d$ if $j = 1,3,\dots,4\eta - 1$
$f(u_{3,j}) = a + \left[4\eta + 1 + \left(\frac{j}{2}\right)\right] d$ if $j = 2,4,\dots,2\eta$
$f(u_{3,j}) = a + \left[4\eta + 2 + \left(\frac{j}{2}\right)\right] d$ if $j = 2\eta + 2, 2\eta + 4,\dots,4\eta$
$f(u_{4,j}) = a + \left[6\eta + 2 + \left(\frac{j-1}{2}\right)\right] d$ if $j = 1,3,\dots,4\eta - 1$
$f(u_{4,j}) = a + \left[10\eta - \left(\frac{j-2}{2}\right)\right] d$ if $j = 2,4,\dots,4\eta - 2$

From the function $f^*: E(G) \rightarrow \{d, 2d, 3d, 4d, \dots, qd\}$ we get the edge labels of the swastik graph Swg_nas follows

Table:1 Edge labels of the graphSwg_η

$f^*(u\,v)$	**Edge labels**	**Value**
$f^*(u_{1,i}u_{1,k})$	$\left\lvert\left[16n - \left(\frac{i-1}{2}\right) - \left(\frac{k}{2}\right)\right] d\right\rvert$	$i = 1, k = 4\eta$
$f^*(u_{2,i}u_{2,l})$	$\left\lvert\left[\left(\frac{i-1}{2}\right) - 12\eta + \left(\frac{l}{2}\right)\right] d\right\rvert$	$i = 1, l = 4\eta$
$f^*(u_{3,i}u_{3,k})$	$\left\lvert\left[8\eta - 2 - \left(\frac{i-1}{2}\right) - \left(\frac{k}{2}\right)\right] d\right\rvert$	$i = 1, k = 4\eta$
$f^*(u_{4,i}u_{1,i})$	$\lvert[1 + i - 10\eta]d\rvert$	$i = 1$

It is clear that the function f is injective and also table 1 shows that

$f^*: E \rightarrow \{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph $Swg_n$is arithmetic sequential graceful graph.

Example 3.1.1: Graceful labeling ofSwg_4shown in figure-1.

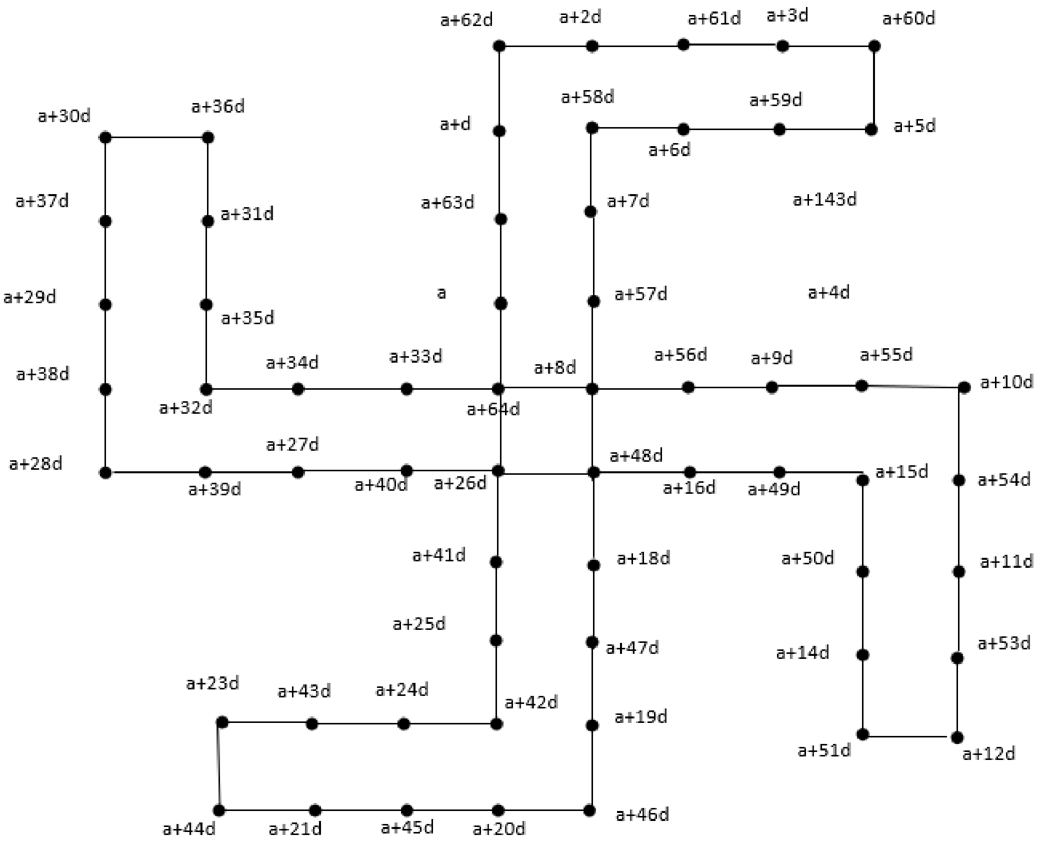

Figure -1: Swastik graph Swg_4.

Theorem 3.2:

Union of finite copies of the path of Swg_η is an arithmetic sequential graceful graph, $\eta \geq 2$.

Proof:

Considering, for $1 \leq k \leq t$, $u_{k,1,4\eta}$&$u_{k,2,1}$, $u_{k,2,4\eta}$&$u_{k,3,1}$ and $u_{k,3,4\eta}$&$u_{k,4,1}$ as single vertex $u_{k,2,1}$, $u_{k,3,1}$ and $u_{k,4,1}$ respectively.

Let $V(Swg_\eta) = \{u_{k,i,j}: 1 \leq k \leq t; 1 \leq i \leq 4; 1 \leq j \leq 4\eta\}$ and

$E(Swg_\eta) = \{u_{k,i,j}u_{k,i,1+j}: 1 \leq k \leq t;\ 1 \leq i \leq 4; 1 \leq j \leq 4\eta - 1\} \cup \{u_{k,i,1}u_{k,1+i,1}: 1 \leq k \leq t;\ 1 \leq i \leq 3\} \cup \{u_{k,1,1}u_{k,4,1}: 1 \leq k \leq t\} \cup \left\{u_{k,1,\left(\frac{4\eta}{2}+1\right)}u_{k+1,1,\left(\frac{4\eta}{2}+1\right)}: 1 \leq k \leq t-1\right\}$.

Let $G = P(t.Swg_\eta)$ be a union of finite copies of the path of Swg_η, $\eta \geq 2$.

Here $|V| = t(16\eta - 4)$&$|E| = (t-1) + t(16\eta)$.

Let $u_{k,i,j}$(for allfor all $1 \leq i \leq 4$ &for all $1 \leq j \leq 4\eta$) be the vertices of k^{th} copy of Swg_η, for all $1 \leq k \leq t$ where the vertices of k^{th} copy of Swg_η is $16\eta - 4$ and the edges of k^{th} copy of Swg_η is 16η.

Join the vertices $u_{k,1,2\eta+1}$ to $u_{k+1,1,2\eta+1}$ for $k = 1,2,\dots,t-1$ by an edge to form the path union of t copies of swastik graph.

We define a function $f: V(G) \to \{a, a+d, a+2d, a+3d, \dots, 2(a+qd)\}$.

The vertex labeling are as follows,

$$f(u_{1,i,j}) = a + [f(u_{i,j}) - a]d, \qquad if \ \ f(u_{i,j}) \le \frac{16\eta}{2} + 1$$

$$f(u_{1,i,j}) = a + [f(u_{i,j}) - a + \{[(t-1) + t.16\eta] - 16\eta\}]d, \qquad if \ \ f(u_{i,j}) > \frac{16\eta}{2} + 1$$

(for all $i = 1,2,3,4$ &for all $j = 1,2,\dots,4\eta$)

$$f(u_{2,i,j}) = a + [f(u_{1,i,j}) + \{[(t-1) + t.16\eta] - 16\eta\}]d, \qquad if \ \ f(u_{1,i,j}) < \frac{(t-1) + (t.16\eta)}{2}$$

$$f(u_{2,i,j}) = a + [f(u_{1,i,j}) - \{[(t-1) + t.16\eta] - 16\eta\}]d, \qquad if \ \ f(u_{1,i,j}) > \frac{(t-1) + (t.16\eta)}{2}$$

(for all $i = 1,2,3,4$ &for all $j = 1,2,\dots,4\eta$)

$$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) + (16\eta + 1)]d, \qquad if \ \ f(u_{k-1,i,j}) < \frac{(t-1) + (t.16\eta)}{2}$$

$$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) - (16\eta + 1)]d, \qquad if \ \ f(u_{k-1,i,j}) > \frac{(t-1) + (t.16\eta)}{2}$$

(for all $i = 1,2,3,4$, for all $j = 1,2,\dots,4\eta$ &for all$k = 3,4,\dots,t$)

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \dots, qd\}$ we get the edge labels of the graph$P(t.Swg_\eta)$as follows

Table:2 Edge labels of the graph $P(t.Swg_\eta)$

$\boldsymbol{f^*(u\,v)}$	**Edge labels**	**Value**
$f^*(u_{1,i,j}u_{2,i,j})$	$\lvert[f(u_{i,j}) - a + 2\{[(t-1) + t.16\eta] - 16\eta\} - f(u_{1,i,j})]d\rvert if\ f(u_{i,j}) > \frac{16\eta}{2} + 1,$ $f(u_{1,i,j}) > \frac{(t-1) + (t.16\eta)}{2}$	$i = 1, j = 2\eta + 1$
$f^*(u_{2,i,j}u_{3,i,j})$	$\lvert[f(u_{1,i,j}) - \{[(t-1) + t.16\eta] - 16\eta\} - f(u_{2,i,j}) + (16\eta + 1)]d\rvert$ $if\ f(u_{1,i,j}) > \frac{(t-1) + (t.16\eta)}{2},$ $f(u_{2,i,j}) > \frac{(t-1) + (t.16\eta)}{2}$	$i = 1, j = 2\eta + 1$

It is clear that the function f is injective and also table 2 shows that $f^*: E \to \{d, 2d, 3d, 4d, \dots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph $P(t.Swg_\eta)$is arithmetic sequential graceful graph.

Example 3.2.1: Union of 3 copies of the path of Swg_3 and its graceful labeling shown in figure-2.

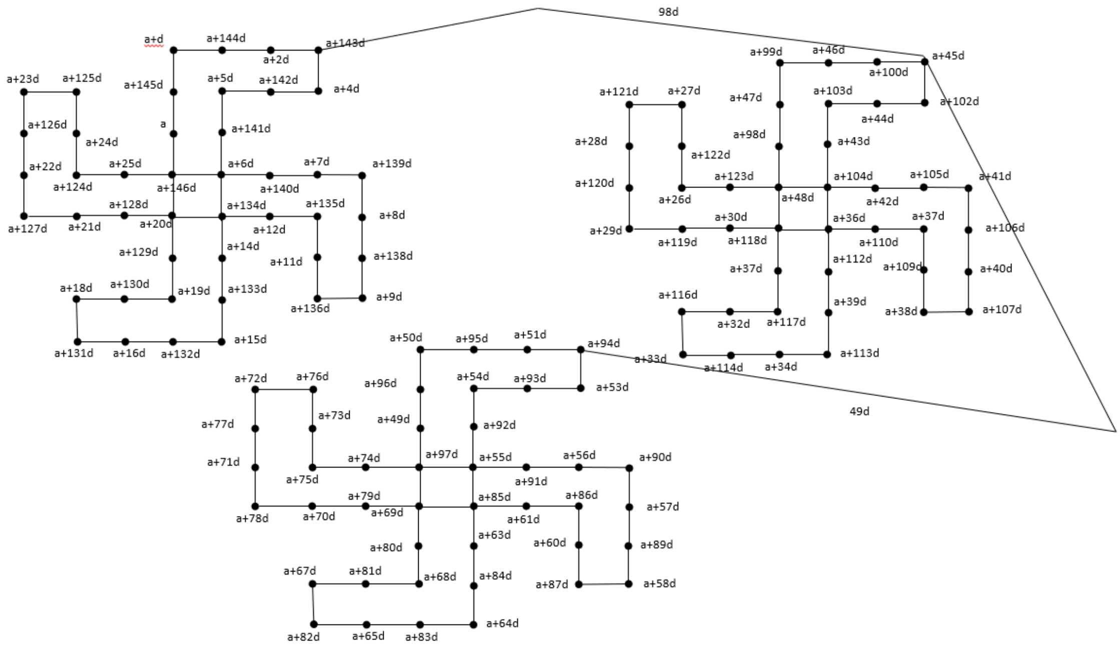

Figure -2: $P(3. Swg_3)$

Theorem 3.3:
Cycle of t copies of$C(t.Swg_\eta)$ is an arithmetic sequential graceful graph,

$$\eta \geq 2 \; and \; r \equiv 0,3(mod\ 4).$$

Proof:
Considering, for $1 \leq ķ \leq t,\ u_{ķ,1,4\eta}\&u_{ķ,2,1},\ u_{ķ,2,4\eta}\&u_{ķ,3,1}$ and $u_{ķ,3,4\eta}\&u_{ķ,4,1}$ as single vertex $u_{ķ,2,1}$, $u_{ķ,3,1}$ and $u_{ķ,4,1}$ respectively.
Let $V(Swg_\eta) = \{u_{ķ,i,j}: 1 \leq ķ \leq t; 1 \leq i \leq 4; 1 \leq j \leq 4\eta\}$ and
$E(Swg_\eta) = \{u_{ķ,i,j}u_{ķ,i,j+1}: 1 \leq ķ \leq t;\ 1 \leq i \leq 4; 1 \leq j \leq 4\eta - 1\} \cup \{u_{ķ,i,1}u_{ķ,i+1,1}: 1 \leq ķ \leq t;\ 1 \leq i \leq 3\} \cup \{u_{ķ,1,1}u_{k,4,1}: 1 \leq ķ \leq t\} \cup \left\{u_{ķ,1,\left(\frac{4\eta}{2}+1\right)}u_{ķ+1,1,\left(\frac{4\eta}{2}+1\right)}: 1 \leq ķ \leq t-1\right\} \cup \left\{u_{ķ,1,\left(\frac{4\eta}{2}+1\right)}\ u_{1,1,\left(\frac{4\eta}{2}+1\right)}\right\}$.
Let $G = \ C(t.Swg_\eta)$ be a cycle of swastik graph,$\eta \geq 2$.
Here $|V| = t[16\eta - 4]$, $|E| = t[16\eta + 1]$.
Let $u_{ķ,i,j}$(for all $1 \leq i \leq 4$ &for all $1 \leq j \leq 4\eta$) be the vertices of $ķ^{th}$ copy of Swg_η, for all $1 \leq ķ \leq t$ where the vertices of $ķ^{th}$ copy of Swg_η is $16\eta - 4$ and the edges of $ķ^{th}$ copy of Swg_η is 16η.
Join the vertices $u_{ķ,1,2\eta+1}$ to $u_{ķ+1,1,2\eta+1}$ for $ķ = 1,2,\ldots,t-1$ and $u_{t,1,2\eta+1}$to $u_{1,1,2\eta+1}$ by an edge to form $C(t.Swg_\eta)$.
We define a function $f: V(G) \rightarrow \{a, a+d, a+2d, a+3d, \ldots, 2(a+qd)\}$.
The vertex labeling are as follows,

$$f(u_{1,i,j}) = a + [f(u_{i,j}) - a]d, \qquad if \quad f(u_{i,j}) \leq \frac{16\eta}{2} + 1$$

$$f(u_{1,i,j}) = a + [f(u_{i,j}) - a + \{t[16(\eta) + 1] - 16\eta\}]d, \qquad if \quad f(u_{i,j}) > \frac{16\eta}{2} + 1$$

(for all $i = 1,2,3,4$ &for all $j = 1,2, \ldots ,4\eta$)

$f(u_{2,i,j}) = a + [f(u_{1,i,j}) + \{t[16(\eta) + 1] - 16\eta\}]d, \quad if \;\; f(u_{1,i,j}) < \frac{t[16(\eta) + 1]}{2}$

$f(u_{2,i,j}) = a + [f(u_{1,i,j}) - \{t[16(\eta) + 1] - 16\eta\}]d, \quad if \;\; f(u_{1,i,j}) > \frac{t[16(\eta) + 1]}{2}$

(for all $i = 1,2,3,4$ &for all $j = 1,2, \dots ,4\eta$)

$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) + (16\eta + 1)]d, \quad if \;\; f(u_{k-1,i,j}) < \frac{t[16(\eta) + 1]}{2}$

$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) - (16\eta + 1)]d, \quad if \;\; f(u_{k-1,i,j}) > \frac{t[16(\eta) + 1]}{2}$

(for all $i = 1,2,3,4$, for all $j = 1,2, \dots ,4\eta$ &for all$k = 3,4, \dots , \frac{t}{2}$)

$f\left(u_{\frac{t}{2}+1\ ,i,j}\right) = a + \left[f\left(u_{\frac{t}{2}-1\ ,i,j}\right) + (16\eta + 2)\right]d, \quad if \;\; f(u_{\frac{t}{2}-1,i,j}) < \frac{t[16(\eta) + 1]}{2}$

$f\left(u_{\frac{t}{2}+1\ ,i,j}\right) = a + \left[f\left(u_{\frac{t}{2}-1\ ,i,j}\right) - (16\eta + 2)\right]d, \quad if \;\; f(u_{\frac{t}{2}-1,i,j}) > \frac{t[16(\eta) + 1]}{2}$

(for all $i = 1,2,3,4$ &for all $j = 1,2, \dots ,4\eta$)

$f\left(u_{\frac{t}{2}+2\ ,i,j}\right) = a + \left[f\left(u_{\frac{t}{2}+1\ ,i,j}\right) + (16\eta + 2)\right]d, \quad if \;\; f(u_{\frac{t}{2}+1,i,j}) < \frac{t[16(\eta) + 1]}{2}$

$f\left(u_{\frac{t}{2}+2\ ,i,j}\right) = a + \left[f\left(u_{\frac{t}{2}+1\ ,i,j}\right) - (16\eta + 2)\right]d, \quad if \;\; f(u_{\frac{t}{2}+1,i,j}) > \frac{t[16(\eta) + 1]}{2}$

(for all $i = 1,2,3,4$ &for all $j = 1,2, \dots ,4\eta$)

$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) + (16\eta + 1)]d, \quad if \;\; f(u_{k-1,i,j}) < \frac{t[16(\eta) + 1]}{2}$

$f(u_{k,i,j}) = a + [f(u_{k-1,i,j}) - (16\eta + 1)]d, \quad if \;\; f(u_{k-1,i,j}) > \frac{t[16(\eta) + 1]}{2}$

(for all $i = 1,2,3,4$, for all $j = 1,2, \dots ,4\eta$ &for all$k = \frac{t}{2} + 3, \frac{t}{2} + 4, \dots , t$)

From the function $f^*: E(G) \to \{d, 2d, 3d, 4d, \dots , qd\}$ we get the edge labels of $C(t.Swg_\eta)$as follows

Table:3 Edge labels of $C(t.Swg_\eta)$

$\boldsymbol{f^*(u\,v)}$	**Edge labels**	**Value**
$f^*(u_{1,i,j}u_{2,i,j})$	$\lvert[f(u_{i,j}) - a - f(u_{1,i,j}) + 2\{t[16(\eta) + 1] - 16\eta\}]d\rvert$ $if\ f(u_{i,j}) > \frac{16\eta}{2} + 1,$ $f(u_{1,i,j}) > \frac{t[16(\eta) + 1]}{2}.$	$i = 1, j = 2\eta + 1$
$f^*(u_{2,i,j}u_{3,i,j})$	$\lvert[f(u_{1,i,j}) - \{t[16(\eta) + 1] - 16\eta\} - f(u_{2,i,j}) + (16\eta + 1)]d\rvert$ $if\ \ f(u_{1,i,j}) > \frac{t[16(\eta) + 1]}{2},$ $f(u_{2,i,j}) > \frac{t[16(\eta) + 1]}{2}$	$i = 1, j = 2\eta + 1$

$f^*\left(u_{3,i,j}u_{t,i,j}\right)$	$\left\|\left[\left(u_{\frac{t}{2}-1\ ,i,j}\right)-f\left(u_{\frac{t}{2}+1\ ,i,j}\right)\right]d\right\|$ $if\quad f(u_{\frac{t}{2}-1,i,j})>\frac{t[16(\eta)+1]}{2},$ $f(u_{\frac{t}{2}+1,i,j})>\frac{t[16(\eta)+1]}{2}$	$i=1, j=2\eta+1$
$f^*\left(u_{t,i,j}u_{1,i,j}\right)$	$\left\|\left[f\left(u_{t-1,i,j}\right)-(16\eta+1)-f\left(u_{i,j}\right)-\{t[16(\eta)+1]-16\eta\}\right]d\right\|$ $if\quad f(u_{t-1,i,j})>\frac{t[16(\eta)+1]}{2},$ $f(u_{i,j})>\frac{16\eta}{2}+1$	$i=1, j=2\eta+1$

It is clear that the function f is injective and also table 3 shows that $f^*: E\rightarrow\{d, 2d, 3d, 4d, \ldots, qd\}$ is bijective. Hence f is arithmetic sequential graceful labeling and the graph is arithmetic sequential graceful graph.

Example 3.3.1: Thecycle of 4 copies of Swg_2 and its graceful labeling shown in figure-3.

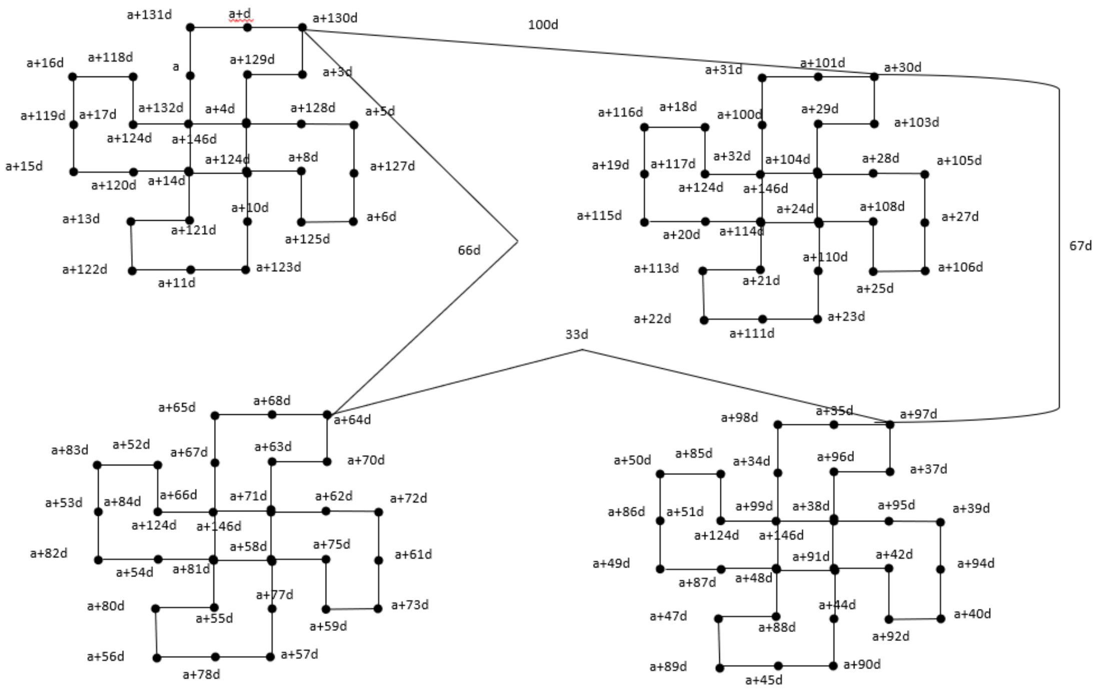

Figure-3: $C(4.Swg_2)$

3. Conclusion

In this article, we have presented an exploration of arithmetic sequential graceful labeling applied to the Swastik graph. Our study has yielded three novel results, namely the graceful labeling of the Swastik graph, the path union of the Swastik graph, and the cycle of the Swastik graph. Through the use of illustrative examples, we have showcased the labeling pattern, enhancing comprehension of the derived outcomes. Our future endeavours will involve extending the analysis of arithmetic sequential graceful labeling to other graph families.

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