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Tribonacci Heinz Quarter Mean Labelingof Graphs

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ABSTRACT:

In this paper, we utilize Tribonacci numbers, which are a generalization of Fibonacci numbers. Let T_n be the n-th Tribonacci numbers defined by$T_{n+3} = T_n + T_{n+1} + T_{n+2}$; $T_0 = 0, T_1 = T_2 = 1$. Here, we introduce a novel concept called Tribonacci Heinz Quarter Mean labeling. An injective function $f: V(G) \to W$ is said to be Tribonacci Heinz Quarter Mean labeling if the induced function f : $E(G) \to \{T_1, T_2, \dots\dots\dots, T_r\}$ definedby$(e=uv)$= $\left\lceil \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$or$\left\lfloor \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rfloor$,then the resulting edge labels are distinct is called Tribonacci Heinz quarter mean labeling and a graph which admits Tribonacci Heinz quarter mean labeling is called Tribonacci Heinz quarter mean graphs.

Keywords: Tribonacci numbers, Heinz quarter mean labeling,.

1. Introduction

Graph theory has been a vital area of research in mathematics due to its extensive applications in various scientific and engineering fields. One of the intriguing aspects of graph theory is the study of graph labeling, where vertices or edges (or both) of a graph are assigned labels (usually numbers) subject to certain rules or conditions. Graph labeling has numerous practical applications in coding theory, X-ray crystallography, communication network design, and many other domains.

Among the many types of graph labeling, the concept of mean labeling has garnered significant attention. Mean labeling involves assigning labels to vertices and edges such that the label of an edge is derived from the labels of its end vertices according to some mean function. This concept has been extended and generalized in various forms, including the Heinz mean, which is a specific type of mean function.

In 2003, Somasundaram and Ponraj [2] introduced the concept of mean labelings for graphs. S.S. Sandhya and S. Latha proposed a labeling scheme known as Heinz Quarter mean labeling. Recently, a novel approach called the Tribonacci Heinz quarter mean labeling has been introduced. This method combines the Tribonacci sequence with the Heinz mean to produce a unique labeling scheme. The Tribonacci sequence, a generalization of the Fibonacci sequence, Tribonacci numbers are defined by $T_1 = 0$, $T_2 = 1$ and $T_3 = 1$ in which$T_n = T_{n-1} + T_{n-2} + T_{n-3}$. The integration of this sequence into mean labeling provides a richer structure and potentially broader applications.

The study of graph labeling has evolved significantly since its inception. In the early works by Rosa (1967), the concept of graceful labeling was introduced, which has since inspired a multitude of labeling techniques. This pioneering work laid the foundation for subsequent developments in various graph labeling methodologies.

In the realm of mean labeling, the Heinz mean, named after the Heinz numbers used in chemistry, provides an interesting framework. The Heinz mean of two positive numbers u and v is given by $f(u,v;\ y) \ = \frac{f(u)^y f(v)^{1-y} + f(u)^{1-y} f(v)^y}{2}$, for $0 \leq y \leq \frac{1}{2}$.

This concept has been adapted in graph theory to create the Heinz mean labeling, where the edge label is derived using the Heinz mean of the vertex labels.The Fibonacci sequence has also been extensively studied in graph labeling, particularly in the context of Fibonacci graceful graphs. The extension to the Tribonacci sequence, which introduces an additional term in the recurrence relation, offers a new dimension to sequence-based labeling techniques. The Tribonacci sequence has found applications in diverse fields, including computer science, combinatorics, and dynamic systems.

In recent studies, researchers have explored the properties and applications of Tribonacci Heinz quarter mean labeling in various types of graphs, including paths, cycles, and complete graphs. These investigations have revealed interesting structural properties and potential applications in network design and analysis, particularly in areas requiring hierarchical and recursive structures.

A **path** is a sequence of vertices in which each vertex is distinct from the others. A **cycle** is a path that starts and ends at the same vertex, forming a closed loop. A **tree** is a special type of undirected graph that contains no cycles. In a rooted tree, one vertex is designated as the root, and all other vertices have a unique parent except for the root, which has no parent. The vertices that are directly connected to a given vertex are known as its **children**.

Definition 1.1. A graph is said to be **Heinz – quarter mean graph** if it is possible to label the vertices$x \in V$with distinct labels $f(x)$ from $1, 2, 3, \ldots \ldots, q + 1$ then each edge is labeled with $f(e = uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$ then the resulting edge labels are distinct. Here, $f$is a called **Heinz – quarter mean labeling** of G.

Definition1.2. Let $f: V(G) \to W$ is said to be Tribonacci Heinz quarter mean labeling if the induced function $(e = uv) = \left\lfloor \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rfloor$ or $\left\lceil \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$ then the resulting edgelabeling are distinct is called Tribonacci Heinz quarter mean labeling. A graph which admit Tribonacci Heinz quarter mean labeling is called Tribonacci Heinz quarter mean graph (T.H.Q.M.G).
Throughout this chapter $|V(G)|$ and $|E(G)|$ are used for cardinality of vertex set and edge set respectively and assume the Tribonacci number be $T_1 = 1, T_2 = 2$ and $T_3 = 3$

Remark: 1.3. If G is a Heinz-quarter mean graph, then one of its vertices must be labeled with '1', as an edge needs to be assigned the label '1'.
Remark: 1.4. For a Heinz-quarter mean graph G , the vertices should be labeled with the integers $1,2, \dots, q + 1$while the edges should be labeled with the integers $1,2, \dots, q$

2. Main results:

Lemma 2.1. The Graph G is a Tribonacci Heinz quarter mean graph where $G = P_n$ for all integers $n \geq 2$.
Proof. Let $\{u_1, u_2, \dots, u_n\}$ be a vertices of a path of length n.
The graph $G = P_n$.
Now the vertex set of $V(G) = \{u_1, u_2, \dots, u_n\}$ and
the edge set $E(G) = \{u_i u_{i+1}/1 \leq i \leq n - 1\}$
The assignment of vertex labels are distinct from the set W

$$f(v_i) = T_i 1 \leq i \leq 3$$

$$f(v_i) = 2T_{i-2} + T_{i-3} 4 \leq i \leq 7$$

$f(v_i) = T_{i-1} + 2T_{i-4} 8 \leq i \leq 12$and so on
Edges are labeled with $f(u_i u_{i+1}) = T_i, 1 \leq i \leq n - 1$
The edge labels are from Tribonacci numbers $\{T_1, T_2, \dots, T_r\}$.
Hence it satisfies Tribonacci Heinz quarter mean labeling.
So the graph G is a Tribonacci Heinz quarter mean graph.

Theorem 2.2. Let G be a graph with having only one cycle
(i) If $G = C_3$ where C_3 is a cycle of length 3 then G admits Tribonacci Heinz quarter mean graph.
(ii) If $G = C_3 \odot nP_q$ iff $n \leq 3$ then G admits Tribonacci Heinz quarter mean graph.
Proof. Let G be a Cycle.
Case (i). Let $G = C_3$
Let $\{v_1, v_2, v_3\}$ be the vertices of a cycle C_3.
Let $V(G) = v_i$; $1 \leq i \leq 3$
Define a function $f : V(G) \to W$ by

$$f(v_i) = T_{i+1} - T_1 i \leq 3.$$

Here we get the edge labels are distinct Tribonacci numbers.
So the given graph is Tribonacci Heinz quarter mean graph.
Case (ii). Let G be a graph with having only one cycle of length 3. Let $\{v_1, v_2, v_3\}$ be the vertices of a cycle C_3.
Here we consider the following sub cases.
SubCase (i). If $n = 1$ then $G = C_3 \odot P_q$

Define the graph G be obtained by joining the vertex of a closed cycle of length 3 can be attached with a path$l_1 l_2 \dots \dots l_x$.
Define a function $f : V(G) \rightarrow W$ by
$f(v_i) = T_{i+1} - T_1$ $1 \leq i \leq 4$.
$f(v_i) = T_{i+1} - 2T_3$ $5 \leq i \leq 6$. And so on
Define the induced function,
$f^*: E(G) \rightarrow \{T_1, T_2, \dots . T_r\}$ by
$f^*(v_i v_{i+1}) = T_i$ $1 \leq i \leq n-1$.
Here the edge labels are distinct tribonacci numbers
Thus G admits Tribonacci Heinz quarter mean labeling.
Hence $G = C_3 \odot P_q$is a Tribonacci quarter mean graph.
SubCase (ii). If $n = 2$ then $G = C_3 \odot 2P_q$
Define the graph G be obtained by joining any two vertices of a closed cycle C_3 can be attached with a path.
Let $l_1 l_2 \dots \dots l_x$be a path of length x and $m_1 m_2 \dots \dots m_y$ is an another path of length y attached with the vertices v_1 and $v_2$respectively.
By Applying H.Q.M.L , We get the distinct Tribonacci numbers
Thus G admits T.H.Q.M.L. Hence $G = C_3 \odot 2P_q$is Tribonacci Heinz quarter mean graph.
SubCase (iii). If $n = 2$ then $G = C_3 \odot 3P_q$
Define the graph G be obtained by joining any two vertices of a closed cycle of length 3 can be attached with a path.
Let $l_1 l_2 \dots \dots l_x$ be a path of length x , $m_1 m_2 \dots \dots m_y$ is a path of length y and $n_1 n_2 \dots \dots n_z$ is a path of length z attached with the vertices v_1 , $v_2$and v_3 respectively.
By Applying Heinz quarter mean labeling of G, different edge labels are found.
Thus G admits Tribonacci Heinz quarter mean labeling. Hence $G = C_3 \odot 3P_q$is a Tribonacci Heinz quarter mean graph.

Remark 2.3. A graph with only one cycle of length 3 has satisfies first three Tribonacci numbers.
Theorem 2.4. Any Tree admits Tribonacci Heinz quarter mean labeling.
Proof: Let G be a Tree graph.
Let $\mathbb{T}$ be a root vertex of tree.
Let $c_1, c_2, c_3, \dots . . c_n$be the children of v. Now if $c_{1i}^1 (1 \leq i \leq t)$are children of p^1
If there are r vertices at level two of $c^1$and out of these r vertices, $r_1$be the children of c_{11}^1
Let there are $r_2$vertices, which are children of c_{12}^1
Let $c^1, c^2, c^3, c^4, \dots . , c^n$ be the vertices of a sub tree.
We denote c_{ij}^n**,** here the level of vertex be i and number of vertices at i^{th}level be j.
If $c_{1i}^n (1 \leq i \leq s)$is children of c^n and $c_{2i}^n (1 \leq i \leq b)$vertices in the level two of$c^n$and $b_1$be the children of c_{11}^n be b_1,and the children of c_{12}^n be b_2.
Let $E(G) = \{u_i u_{i+1} / 1 \leq i \leq n-1\}$
By Applying Heinz quarter mean labeling of G. Here We get the edge labels are distinct and which from$\{T_1, T_2, \dots . T_r\}$
Thus G admits Tribonacci Heinz quarter mean labeling. That is, Trees are Tribobacci Heinz quarter mean graph. Hence all trees admits Tribonacci Heinz quarter mean graph.

3.Tribonacci Heinz Quarter mean Cordial labeling.

A graph which admits Tribonacci Heinz Quarter mean cordial labeling is called Tribonacci Heinz quarter mean cordial graph. Here we investigate some graphs which are Tribonacci Heinz quarter mean cordial or not.

Definition 3.2. A function $f : V(G) \to \{T_1, T_2, \dots\dots\dots\dots, T_q\}$is said to Tribonacci Heinz quarter mean Cordial labeling if the induced function: $f: E(G) \to f\{0,1\}$ defined by $f(e = uv) = \left\lceil \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$(mod 2) satisfies the condition $|e_f(0) - e_f(1)| \leq 1$. A graph which admit Tribonacci Heinz quarter mean Cordial labeling is called Tribonacci Heinz quarter mean cordial graph.

Definition 3.3. Let G be a graph with $V = X_1 \, U \, X_2 \, U \, X_3 \, U \dots X_i U Y$ where each $X_i$is a set of vertices having at least two vertices of the same degree and $Y = V \setminus U\, X_i$. The degree splitting graph of G designated by $DS(G)$ is acquired from G by adding vertices $z_1, z_2, z_3, \dots, z_y$and joining to each vertex of $x_i$for $i \in [1, t]$.

Definition 3.4. A Globe graph is a type of planar graph that represents the connections between points on a sphere. It is symbolized by $Gl_{(n)}$.

Theorem 3.5. The graph P_n is Tribonacci Heinz quarter mean cordial graph.

Proof. Let $G = P_n$be a path $u_1 u_2 \dots u_n$ of length n.

Let $f : V(P_n) \to \{T_1, T_2, \dots, T_r\}$ be a labeling such that $f(v_i) = T_i$ for all $i = 1, 2, \dots., n$.

Applying Tribonacci Heinz quarter mean Cordiallabeling, the value of $|e_f(0) - e_f(1)| \leq 1$.

Therefore $P_n$is Tribonacci Heinz quarter mean Cordial graph.

Theorem 3.6. Petersen graph admits Tribonacci Heinz quarter mean cordial graph.

Proof. The Petersen graph, a 3-regular graph, contains 10 vertices and 15 edges

Let $\{u_1, u_2, \dots, u_{10}\}$ be the vertices and $\{v_1, v_2, \dots, v_{15}\}$ be the edges of Petersen graph.

Let $f : V(P_n) \to \{T_1, T_2, \dots, T_r\}$ such that $f(e = uv) = \left\lceil \frac{\sqrt[4]{f(u)f(v)}(\sqrt{f(u)}+\sqrt{f(v)})}{2} \right\rceil$ (mod 2) then the resulting vertices are $|e_f(1) - e_f(0)| \leq 1$.

HencePetersen graph admits Tribonacci Heinz quarter mean cordial graph.

Theorem 3.7. A graph $G$is obtained by joint sum of two copies of Globe $Gl_{(n)}$is Tribonacci Heinz quarter mean cordial graph.

Proof. Consider G is joint sum of two copies of $Gl_{(n)}$. Let $\{x, x', x_1, x_2, \dots, x_n\}$ be the vertices of first copy and $\{y, y', y_1, y_2, \dots, y_n\}$ be the vertices of the second copy of $Gl_{(n)}$.

Let the function $f: V(G) \to \{T_1, T_2, \dots, T_{2n+4}\}$, as below.

$f(x) = T_1$, f(x') = T_2, $f(x_i) = T_{i+3}$, $1 \leq i \leq n$.

$f(y) = T_3$, $f(y') = T_4$, $f(y_i) = T_{n+i+3}$, $1 \leq i \leq n$.

Tribonacci Heinz quarter mean cordiallabeling, $e_f(0) = n + 1$ and $e_f(1) = n$.

Therefore, $|e_f(1) - e_f(0)| \leq 1$.

Thus, G is Tribonacci Heinz quarter mean cordial graph.

Theorem 3.8. $DS(P_n)$is Tribonacci Heinz quarter mean cordial graph.

Proof . Consider $P_n$with V $(P_n) = \{v_i : i \in [1, n]\}$.

Here V $(P_n) = X_1$U X_2, where $X_1 = \{x_i$: i∈ [2, n-1]} and $X_2 = \{x_1, x_n\}$.

To get $DS(P_n)$ from G we add $w_1$and $w_2$corresponding to $X_1$and X_2.

Then $|V(DS(P_n))|$ = n + 2 and $E(DS(P_n)) = \{X_1 w_2, X_2 w_2\} \, U \, \{w_1 x_i : i \in [2, n-1]\}$.

So, $|E(DS(P_n))| = -1 + 2n$.

Here determinelabeling function $f: V(G) \to \{T_1, T_2, \dots, T_{n+2}\}$ as below

$f(w_1) = T_2$, $f(w_2) = T_{n+2}$, $f(x_1) = T_1$, $f(x_i) = T_i$, $2 \leq i \leq n$.

Therefore, $|e_f(1) - e_f(0)| \leq 1$.

Therefore, $DS(P_n)$ Tribonacci Heinz quarter mean cordial graph.

Theorem 3.9. $K_{m,n}$ is Tribonacci cordial graph for all m, n.

Proof. Let $\{u_1, u_2 \dots.. u_m, v_1, v_2, \dots., v_n\}$ be the vertices of a complete Bipartite graph $K_{m,n}$.

Case 1. Assume either m or n is even. To take m is even

Then $f(u_i) = T_{i-1}$,$f(v_i) = T_{m+i-1}$

Here, $m/2$ even and $m/2$ odd Apply Heinz quarter mean Cordial labels ,$|e_f(1) - e_f(0)|$ = 0, **Case 2.**Assume both m and n are odd.

Let $m = 2K_1 + 1$and $n = 2K_2 + 1$, Apply Heinz quarter mean Cordial labels ,there are either $K_1 + 1$ and K_2 , or K_1 and $K_2 + 1$ labels used. The previous case $|e_f(1) - e_f(0)|$= K_1K_2 + (K_1 + 1)(K_2 + 1) -K_1 (K_2 + 1) -K_2 (K_1 + 1) = 1.

2. Conclusion

The study of Tribonacci Heinz quarter Mean Labeling of graphs is significant because of its various applications. Although cycle graphs of all Heinz quarter Mean Graphs are Tribonacci Heinz quarter Mean cordial, they are not necessarily Tribonacci Heinz quarter Mean graphs. Investigating which graphs admit Tribonacci Heinz quarter Mean Labeling is a fascinating area of research. In this paper, we demonstrate that path and cycle graphs are Tribonacci Heinz quarter Mean Graphs. The results are supported with ample illustrations to enhance understanding. Similar investigations can be conducted for many other types of graphs.

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