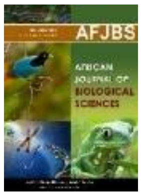
Sunita Kuppayya Poojari / Afr. J. Bio. Sc. 6(6) (2024) ISSN: 2663-2187
https://doi.org/10.33472/AFJBS.6.6.2024.2403-2410

African Journal of Biological Sciences



Translations In Bipolar Valued I-Fuzzy Subsemirings Of A Semiring

Sunita Kuppayya Poojari[1]*, M. Muthusamy[2], K. Arjunan[3]

[1]*Department of Mathematics, Gurukul College of Commerce, Mumbai - 400077, Maharashtra, India, Email:sunita_1005@rediffmail.com

[2]Department of Mathematics, Dr.Zakir Husain College, Ilayangudi-630702, Tamilnadu, India,
Email: msamy0207@yahoo.com

[3]Department of Mathematics, Alagappa Government Arts college, Karaikudi - 630003, Tamilnadu, India,
Email: arjunan.karmegam@gmail.com

***Corresponding Author:** Sunita Kuppayya Poojari
*Department of Mathematics, Gurukul College of Commerce, Mumbai - 400077, Maharashtra, India, Email:sunita_1005@rediffmail.com

Volume 6, Issue 6, June 2024
Received: 06 April 2024
Accepted: 11 May 2024
doi: 10.33472/AFJBS.6.6.2024.2403-2410

This work defines and studies several sorts of translations in bipolar valued I-fuzzy subsemiring of a semiring; translations are applied and certain translation theorems are presented.

Key Words: Interval valued fuzzy subset, bipolar valued fuzzy subset, bipolar valued I-fuzzy subset, bipolar valued I-fuzzy subsemiring and translations.

INTRODUCTION:

The concept of a fuzzy subset of a set was first presented by Zadeh [14] in 1965. Fuzzy sets are a helpful mathematical structure that may be used to describe a group of objects whose boundaries are not clearly defined. Since then, there have been many generalisations of this basic idea, including intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets, etc. It has also become a burgeoning field of study in other disciplines. The concept of bipolar valued fuzzy sets was suggested by Lee [6]. Fuzzy sets that have their membership degree range expanded from [0, 1] to [-1, 1] are called bipolar valued fuzzy sets. When elements in a bipolar valued fuzzy set have a membership degree of 0, it signifies that they are not relevant to the corresponding property. When an element has a membership degree of (0, 1] it means that they partially satisfy the property, and when an element has a membership degree of [-1, 0), it means that they partially satisfy the implicit counter property. Intuitionistic fuzzy sets and bipolar valued fuzzy sets have a similar appearance. They are distinct from one another, [6, 7]. Azriel Rosenfeld introduced the fuzzy subgroup[4]. Bipolar valued fuzzy subgroups of a group, as well as homomorphism and anti-homomorphism, are defined by Anitha M. S. et al. [1, 2]. Subsequently, K. Murugalingam and K. Arjunan[8] talked about interval valued fuzzy subsemirings of a semiring, while Yasodara.B and KE. Sathappan[12] presented bipolar valued multi fuzzy subsemirings of a semiring. The many kinds of translations in bipolar valued multifuzzy subnearrings of a nearing were defined by Muthukumaran, S. & B. Anandh

[9]. The bipolar valued I-fuzzy subsemirings of a semiring were defined by Sunita Kuppayya Poojari et al. [11]. Here the concept of translations in bipolar valued I-fuzzy subsemiring of a semiring are given.

1.PRELIMINARIES.

Definition 1.1. [14] A fuzzy subset Ɗ of the set Γ is a function Ɗ : Γ →[0, 1].

Definition 1.2. [14] An interval valued fuzzy subset Ɗ of the set Γ is a function Ɗ : Γ →D[0, 1]. Here D[0, 1] denotes the family of all closed subintervals of [0, 1].

Definition 1.3. [6] *The ordered structure* $\mathfrak{T} = \{(\mathfrak{z}, \mathfrak{T}^{+}(\mathfrak{z}), \mathfrak{T}^{-}(\mathfrak{z})) : \mathfrak{z} \in \mathbb{W}\}$ *is called* a bipolar valued *fuzzy subset*($\mathbb{BVFS}$) *of* $\mathbb{W}$, *where* $\mathfrak{T}^{+}: \mathbb{W} \rightarrow [0,1]$ *is a positive* membership map and $\mathfrak{T}^{-}: \mathbb{W} \rightarrow [-1,0]$ is a negative membership map.

Example 1.4. Let Γ = {ϖ, ω, υ} be a set. Then 𝔗 = {⟨ϖ, 0.7, –0.6⟩, ⟨ω, 0.4, –0.5⟩, ⟨υ, 0.2, –0.3⟩} is a bipolar valued fuzzy subset of Γ.

Definition 1.5. [11] *The ordered structure* $\mathfrak{T} = \{(\mathfrak{z}, \mathfrak{T}^{+}(\mathfrak{z}), \mathfrak{T}^{-}(\mathfrak{z})) : \mathfrak{z} \in \mathbb{W}\}$ *is called* a bipolar valued I-fuzzy subset or bipolar interval valued fuzzy subset ($\mathbb{BV}I\mathbb{FS}$) *of* $\mathbb{W}$, *where* $\mathfrak{T}^{+}: \mathbb{W} \rightarrow D[0,1]$ *is a positive* membership map and $\mathfrak{T}^{-}: \mathbb{W} \rightarrow D[-1,0]$ is a negative membership map. Here D[0, 1] denotes the family of all closed subintervals of [0, 1] and $D[-1,0]$ denotes the family of all closed subintervals of [−1, 0].

Example 1.6. Let Γ = {ϖ, ω, υ} be a set. Then 𝔗 = {⟨ϖ, [0.7, 0.8], [–0.6, –0.5]⟩, ⟨ω, [0.4, 0.7], [–0.5, –0.3]⟩, ⟨υ, [0.2, 0.7], [–0.3, –0.1]⟩} is a bipolar valued I-fuzzy subset of Γ.

Definition 1.7. [9] *Let* $\mathfrak{P} = \langle \mathfrak{P}^{+}, \mathfrak{P}^{-} \rangle$ be $\mathbb{BV}I\mathbb{FS}$ of the set $\mathbb{N}_1$. Then the following transformation are defined as,

(i) $\between(\mathfrak{P}) = \langle \between(\mathfrak{P}^{+}), \between(\mathfrak{P}^{-}) \rangle$, where $\between(\mathfrak{P}^{+})(\varrho) = rmin\,\{[½, ½], \mathfrak{P}^{+}(\varrho)\}$ and $\between(\mathfrak{P}^{-})(\varrho) = rmax\,\{[-½, -½], \mathfrak{P}^{-}(\varrho)\}$, for all $\varrho \in \mathbb{N}_1$.

(ii) $\bowtie(\mathfrak{P}) = \langle \bowtie(\mathfrak{P}^{+}), \bowtie(\mathfrak{P}^{-}) \rangle$, where $\bowtie(\mathfrak{P}^{+})(\varrho) = rmax\,\{[½, ½], \mathfrak{P}^{+}(\varrho)\}$ and $\bowtie(\mathfrak{P}^{-})(\varrho) = rmax\,\{[-½, -½], \mathfrak{P}^{-}(\varrho)\}$, for all $\varrho \in \mathbb{N}_1$.

(iii) $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^{+}), \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^{-}) \rangle$, where $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^{+})(\varrho) = rmin\,\{\varpi, \mathfrak{P}^{+}(\varrho)\}$ and $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^{-})(\varrho) = rmax\,\{\varsigma, \mathfrak{P}^{-}(\varrho)\}$, for all $\varrho \in \mathbb{R}_1$, $\varpi \in D[0,1]$ *and* $\varsigma \in D[-1,0]$.

(iv) $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^{+}), \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^{-}) \rangle$, where $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^{+})(\varrho) = rmax\,\{\varpi, \mathfrak{P}^{+}(\varrho)\}$ and $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^{-})(\varrho) = rmin\,\{\varsigma, \mathfrak{P}^{-}(\varrho)\}$, for all $\varrho \in \mathbb{R}_1$, $\varpi \in D[0,1]$ *and* $\varsigma \in D[-1,0]$.

(v) $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^{+}), \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^{-}) \rangle$, where $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^{+})(\varrho) = \varpi\mathfrak{P}^{+}(\varrho)$ and $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^{-})(\varrho) = -\varsigma\mathfrak{P}^{-}(\varrho)$, for all $\varrho \in \mathbb{R}_1$, $\varpi \in D[0,1]$ *and* $\varsigma \in D[-1,0]$.

Definition 1.8. [11] *A* $\mathbb{BV}I\mathbb{FS}$ $\mathfrak{M} = \langle \mathfrak{M}^{+}, \mathfrak{M}^{-} \rangle$ of a semi-ring S *is said* to *be a* bipolar valued I − fuzzy subsemi − ring of S ($\mathbb{BV}I\mathbb{FSSR}$) *if* $\mathfrak{M}$ *has the following condition* ,

(i) $\mathfrak{M}^{+}(\mathfrak{y} + \mathfrak{w}) \geq rmin\{\mathfrak{M}^{+}(\mathfrak{y}), \mathfrak{M}^{+}(\mathfrak{w})\}$,

(ii) $\mathfrak{M}^{+}(\mathfrak{y}\mathfrak{w}) \geq rmin\{\mathfrak{M}^{+}(\mathfrak{y}), \mathfrak{M}^{+}(\mathfrak{w})\}$,

(iii) $\mathfrak{M}^{-}(\mathfrak{y} + \mathfrak{w}) \leq rmax\{\mathfrak{M}^{-}(\mathfrak{y}), \mathfrak{M}^{-}(\mathfrak{w})\}$,

(iv) $\mathfrak{M}^{-}(\mathfrak{y}\mathfrak{w}) \leq rmax\{\mathfrak{M}^{-}(\mathfrak{y}), \mathfrak{M}^{-}(\mathfrak{w})\}$, *for all* $\mathfrak{y}, \mathfrak{w} \in S$.

Example 1.9. Let $\mathbb{N} = \mathbb{Z}_3 = \{0, 1, 2\}$ *be a semi* − *ring with* $\oplus_3$ *and* $\otimes_3$. Then $\mathfrak{M}$ is defined as $\mathfrak{M} = \{(0, [0.072, 0.81], [-0.91, -0.081]), (1, [0.051, 0.61], [-0.61, -0.051]), (2, [0.051, 0.61], [-0.61, -0.051])\}$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $\mathbb{N}$.

2. SOME THEOREMS.

Theorem 2.1. [11] *If* $\mathfrak{K} = \langle \mathfrak{K}^{+}, \mathfrak{K}^{-} \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^{+}, \mathfrak{W}^{-} \rangle$ *are two* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then their intersection $\mathfrak{K} \cap \mathfrak{W}$ *is also a* $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Theorem 2.2. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … and $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then their intersection $\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots \cap \mathfrak{P}_m$ is also *a* $\mathbb{BVMFSNR}$ of $\mathbb{N}_1$.

Proof. The Proof follows from the Theorem 2.1.

Theorem 2.3. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then their intersection $\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots$ *is also a* $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. The Proof follows from the Theorem 2.2.

Theorem 2.4. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of the semi − ring* S_1, then $\eth(\mathfrak{P}) = \langle \eth(\mathfrak{P}^+), \eth(\mathfrak{P}^-) \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. Let ϱ, υ be in S_1. Then

$\eth(\mathfrak{P}^+)(\varrho+\upsilon)$ = rmin{[½,½], $\mathfrak{P}^+(\varrho+\upsilon)$}

≥ rmin{[½,½], rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}

= rmin{rmin{[½,½], $\mathfrak{P}^+(\varrho)$}, rmin{[½,½], $\mathfrak{P}^+(\upsilon)$}}

= rmin{$\eth(\mathfrak{P}^+)(\varrho)$, $\eth(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.

And $\eth(\mathfrak{P}^+)(\varrho\upsilon)$ = rmin{[½,½], $(\mathfrak{P}^+)(\varrho\upsilon)$}

≥ rmin{[½,½], rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}

= rmin{rmin{[½,½], $\mathfrak{P}^+(\varrho)$}, rmin{[½,½], $\mathfrak{P}^+(\upsilon)$}}

= rmin{$\eth(\mathfrak{P}^+)(\varrho)$, $\eth(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.

Also $\eth(\mathfrak{P}^-)(\varrho + \upsilon)$ = rmax{[−½,−½], $\mathfrak{P}^-(\varrho + \upsilon)$}

≤ rmax{[−½,−½], rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}

= rmax{rmax{[−½,−½], $\mathfrak{P}^-(\varrho)$}, rmax{[−½,−½], $\mathfrak{P}^-(\upsilon)$}}

= rmax{$\eth(\mathfrak{P}^-)(\varrho)$, $\eth(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.

And $\eth(\mathfrak{P}^-)(\varrho\upsilon)$ = rmax{[−½,−½], $\mathfrak{P}^-(\varrho\upsilon)$}}

≤ rmax{[−½,−½], rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}

= rmax{rmax{[−½,−½], $\mathfrak{P}^-(\varrho)$}, rmax{[−½,−½], $\mathfrak{P}^-(\upsilon)$}}

= rmax{$\eth(\mathfrak{P}^-)(\varrho)$, $\eth(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.

Hence $\eth(\mathfrak{P})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Corollary 2.5. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi − ring* S_1, then $\eth(\mathfrak{P} \cap \mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Theorem 2.1 and 2.4, it is trivial.

Corollary 2.6. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi − rings* S_1 and S_2, then $\eth\, \mathfrak{P} \cap \eth\, \mathfrak{W}$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $S_1 \cap S_2$.

Proof. From the Theorem 2.1 and 2.4, it is trivial.

Corollary 2.7. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi − rings* S_1, then $\eth\, \mathfrak{P} \cap \eth\, \mathfrak{W}$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Corollary 2.6, it is trivial.

Theorem 2.8. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − rings $S_1, S_2, \ldots, S_m$ respectively, then $\eth\, (\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring $S_1 \cap S_2 \cap \ldots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.4, *the proof is trivial.*

Corollary 2.9. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − rings $S_1, S_2, \ldots, S_m$ respectively, then $\eth\, \mathfrak{P}_1 \cap \eth\, \mathfrak{P}_2 \cap \ldots \cap \eth\, \mathfrak{P}_m$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring $S_1 \cap S_2 \cap \ldots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.4, *the proof is trivial.*

Corollary 2.10. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then $\eth\, (\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of* the semi − ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.4, *the proof is trivial.*

Corollary 2.11. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then $\eth\, \mathfrak{P}_1 \cap \eth\, \mathfrak{P}_2 \cap \ldots \cap \eth\, \mathfrak{P}_m$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.4, *the proof is trivial.*

Theorem 2.12. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of the semi – ring* S_1, then $\bowtie(\mathfrak{P}) = \langle \bowtie(\mathfrak{P}^+), \bowtie(\mathfrak{P}^-) \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. Let ϱ, υ be in S_1. Then,

$\bowtie(\mathfrak{P}^+)(\varrho+\upsilon)$ = rmax{[½,½], $\mathfrak{P}^+(\varrho+\upsilon)$}
≥ rmax{[½,½], rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmax{[½,½], $\mathfrak{P}^+(\varrho)$}, rmax{[½,½], $\mathfrak{P}^+(\upsilon)$}}
= rmin{$\bowtie(\mathfrak{P}^+)(\varrho)$, $\bowtie(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.

And $\bowtie(\mathfrak{P}^+)(\varrho\upsilon)$ = rmax{[½,½], $(\mathfrak{P}^+)(\varrho\upsilon)$}
≥ rmax{[½,½], rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmax{[½,½], $\mathfrak{P}^+(\varrho)$}, rmax{[½,½], $\mathfrak{P}^+(\upsilon)$}}
= rmin{$\bowtie(\mathfrak{P}^+)(\varrho)$, $\bowtie(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.

Also $\bowtie(\mathfrak{P}^-)(\varrho + \upsilon)$ = rmin{[–½,–½], $\mathfrak{P}^-(\varrho + \upsilon)$}
≤ rmin{[–½,–½], rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}
= rmax{rmin{[–½,–½], $\mathfrak{P}^-(\varrho)$}, rmin{[–½,–½], $\mathfrak{P}^-(\upsilon)$}}
= rmax{$\bowtie(\mathfrak{P}^-)(\varrho)$, $\bowtie(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.

And $\bowtie(\mathfrak{P}^-)(\varrho\upsilon)$ = rmin{[–½,–½], $\mathfrak{P}^-(\varrho\upsilon)$}}
≤ rmin{[–½,–½], rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}
= rmax{rmin{[–½,–½], $\mathfrak{P}^-(\varrho)$}, rmin{[–½,–½], $\mathfrak{P}^-(\upsilon)$}}
= rmax{$\bowtie(\mathfrak{P}^-)(\varrho)$, $\bowtie(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.

Hence $\bowtie(\mathfrak{P})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Corollary 2.13. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi – ring* S_1, then $\bowtie(\mathfrak{P} \cap \mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Theorem 2.1 and 2.12, it is trivial.

Corollary 2.14. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi – rings* S_1 and S_2, then $\bowtie \mathfrak{P} \cap \bowtie \mathfrak{W}$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $S_1 \cap S_2$.

Proof. From the Theorem 2.1 and 2.12, it is trivial.

Corollary 2.15. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$*s of the semi – rings* S_1, then $\bowtie \mathfrak{P} \cap \bowtie \mathfrak{W}$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Corollary 2.14, it is trivial.

Theorem 2.16. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – rings $S_1, S_2, \dots, S_m$ respectively, then $\bowtie(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.12, *the proof is trivial.*

Corollary 2.17. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – rings $S_1, S_2, \dots, S_m$ respectively, then $\bowtie \mathfrak{P}_1 \cap \bowtie \mathfrak{P}_2 \cap \dots \cap \bowtie \mathfrak{P}_m$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.12, *the proof is trivial.*

Corollary 2.18. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}$s of the semi – ring S_1, then $\bowtie(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of* the semi – ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.12, *the proof is trivial.*

Corollary 2.19. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – ring S_1, then $\bowtie \mathfrak{P}_1 \cap \bowtie \mathfrak{P}_2 \cap \dots \cap \bowtie \mathfrak{P}_m$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.12, *the proof is trivial.*

Theorem 2.20. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of the semi – ring* S_1, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+), \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-) \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1, where $\varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$.

Proof. Let ϱ, υ be in S_1, $\varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$. Then,

$\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho+\upsilon)$ = rmin{ϖ, $\mathfrak{P}^+(\varrho+\upsilon)$}
≥ rmin{ϖ, rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmin{ϖ, $\mathfrak{P}^+(\varrho)$}, rmin{ϖ, $\mathfrak{P}^+(\upsilon)$}}

= rmin{$\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho)$, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.
And $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho\upsilon)$ = rmin{ϖ, $(\mathfrak{P}^+)(\varrho\upsilon)$}
≥ rmin{ϖ, rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmin{ϖ, $\mathfrak{P}^+(\varrho)$}, rmin{ϖ, $\mathfrak{P}^+(\upsilon)$}}
= rmin{$\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho)$, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.
Also $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho+\upsilon)$ = rmax{ς, $\mathfrak{P}^-(\varrho+\upsilon)$}
≤ rmax{ς, rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}
= rmax{rmax{ς, $\mathfrak{P}^-(\varrho)$}, rmax{ς, $\mathfrak{P}^-(\upsilon)$}}
= rmax{$\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho)$, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.
And $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho\upsilon)$ = rmax{ς, $\mathfrak{P}^-(\varrho\upsilon)$}}
≤ rmax{ς, rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}}
= rmax{rmax{ς, $\mathfrak{P}^-(\varrho)$}, rmax{ς, $\mathfrak{P}^-(\upsilon)$}}
= rmax{$\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho)$, $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.
Hence $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Corollary 2.21. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s\ of\ the\ semi-ring\ S_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Theorem 2.1 and 2.20, it is trivial.

Corollary 2.22. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s\ of\ the\ semi-rings\ S_1$ and S_2, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $S_1 \cap S_2$.

Proof. From the Theorem 2.1 and 2.20, it is trivial.

Corollary 2.23. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s\ of\ the\ semi-rings\ S_1$, then $\mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Corollary 2.22, it is trivial.

Theorem 2.24. $If\ \mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle,\ \mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle,\ \ldots,\ \mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle\ are\ \mathbb{BV}I\mathbb{FSSR}\mathrm{s\ of\ the\ semi-rings}\ S_1, S_2, \ldots, S_m\ \mathrm{respectively, then}\ \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}\ \mathrm{of\ the\ semi-ring}\ S_1 \cap S_2 \cap \ldots \cap S_m$.

Proof. $From\ the\ Theorem\ 2.2\ and\ 2.20, the\ proof\ is\ trivial.$

Corollary 2.25. $If\ \mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle,\ \mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle,\ \ldots,\ \mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle\ are\ \mathbb{BV}I\mathbb{FSSR}\mathrm{s\ of\ the\ semi-rings}\ S_1, S_2, \ldots, S_m\ \mathrm{respectively, then}\ \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \ldots \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}\ \mathrm{of\ the\ semi-ring}\ S_1 \cap S_2 \cap \ldots \cap S_m$.

Proof. $From\ the\ Theorem\ 2.2\ and\ 2.20, the\ proof\ is\ trivial.$

Corollary 2.26. $If\ \mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle,\ \mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle,\ \ldots,\ \mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle\ are\ \mathbb{BV}I\mathbb{FSSR}\mathrm{s\ of\ the\ semi-ring}\ S_1, \mathrm{then}\ \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \ldots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}\ of\ \mathrm{the\ semi-ring}\ S_1$.

Proof. $From\ the\ Theorem\ 2.2\ and\ 2.20, the\ proof\ is\ trivial.$

Corollary 2.27. $If\ \mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle,\ \mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle,\ \ldots,\ \mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle\ are\ \mathbb{BV}I\mathbb{FSSR}\mathrm{s\ of\ the\ semi-ring}\ S_1, \mathrm{then}\ \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \ldots \cap \mathfrak{Q}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}\ \mathrm{of\ the\ semi-ring}\ S_1$.

Proof. $From\ the\ Theorem\ 2.2\ and\ 2.20, the\ proof\ is\ trivial.$

Theorem 2.28 If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}\ of\ the\ semi-ring\ S_1$, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+), \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-) \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1, where $\varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$.

Proof. Let ϱ, υ be in $S_1, \varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$. Then,
$\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho+\upsilon)$ = rmax{ϖ, $\mathfrak{P}^+(\varrho+\upsilon)$}
≥ rmax{ϖ, rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmax{ϖ, $\mathfrak{P}^+(\varrho)$}, rmax{ϖ, $\mathfrak{P}^+(\upsilon)$}}
= rmin{$\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho)$, $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)$}, for all ϱ, υ in S_1.
And $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho\upsilon)$ = rmax{ϖ, $(\mathfrak{P}^+)(\varrho\upsilon)$}
≥ rmax{ϖ, rmin{$\mathfrak{P}^+(\varrho)$, $\mathfrak{P}^+(\upsilon)$}}
= rmin{rmax{ϖ, $\mathfrak{P}^+(\varrho)$}, rmax{ϖ, $\mathfrak{P}^+(\upsilon)$}}

$= \text{rmin}\{\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho), \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)\}$, for all ϱ, υ in S_1.
Also $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho+\upsilon) = \text{rmin}\{\varsigma, \mathfrak{P}^-(\varrho+\upsilon)\}$
$\leq \text{rmin}\{\varsigma, \text{rmax}\{\mathfrak{P}^-(\varrho), \mathfrak{P}^-(\upsilon)\}\}$
$= \text{rmax}\{\text{rmin}\{\varsigma, \mathfrak{P}^-(\varrho)\}, \text{rmin}\{\varsigma, \mathfrak{P}^-(\upsilon)\}\}$
$= \text{rmax}\{\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho), \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)\}$, for all ϱ, υ in S_1.
And $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho\upsilon) = \text{rmin}\{\varsigma, \mathfrak{P}^-(\varrho\upsilon)\}\}$
$\leq \text{rmin}\{\varsigma, \text{rmax}\{\mathfrak{P}^-(\varrho), \mathfrak{P}^-(\upsilon)\}\}$
$= \text{rmax}\{\text{rmin}\{\varsigma, \mathfrak{P}^-(\varrho)\}, \text{rmin}\{\varsigma, \mathfrak{P}^-(\upsilon)\}\}$
$= \text{rmax}\{\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho), \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)\}$, for all ϱ, υ in S_1.
Hence $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Corollary 2.29. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^-\rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^-\rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi – ring* S_1, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Theorem 2.1 and 2.28, it is trivial.

Corollary 2.30. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^-\rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^-\rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi – rings* S_1 and S_2, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $S_1 \cap S_2$.

Proof. From the Theorem 2.1 and 2.28, it is trivial.

Corollary 2.31. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^-\rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^-\rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi – rings* S_1, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Corollary 2.30, it is trivial.

Theorem 2.32. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^-\rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^-\rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^-\rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – rings $S_1, S_2, \dots, S_m$ respectively, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.1.11 *and* 3.1.36, *the proof is trivial.*

Corollary 2.33. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^-\rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^-\rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^-\rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – rings $S_1, S_2, \dots, S_m$ respectively, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \dots \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.28, *the proof is trivial.*

Corollary 2.34. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^-\rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^-\rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^-\rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – ring S_1, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of* the semi – ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.28, *the proof is trivial.*

Corollary 2.35. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^-\rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^-\rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^-\rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi – ring S_1, then $\mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \dots \cap \mathfrak{R}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi – ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.28, *the proof is trivial.*

Theorem 2.36. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^-\rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of the semi – ring* S_1, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}) = \langle \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+), \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-) \rangle$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1, where $\varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$.

Proof. Let ϱ, υ be in S_1, $\varpi \in D[0,1]$ and $\varsigma \in D[-1,0]$. Then,
$\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho+\upsilon) = \varpi\ \mathfrak{P}^+(\varrho+\upsilon)$
$\geq \varpi\ \text{rmin}\{\mathfrak{P}^+(\varrho), \mathfrak{P}^+(\upsilon)\}$
$= \text{rmin}\{\varpi\ \mathfrak{P}^+(\varrho), \varpi\ \mathfrak{P}^+(\upsilon)\}$
$= \text{rmin}\{\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho), \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)\}$, for all ϱ, υ in S_1.
And $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho\upsilon) = \varpi\ (\mathfrak{P}^+)(\varrho\upsilon)$
$\geq \varpi\ \text{rmin}\{\mathfrak{P}^+(\varrho), \mathfrak{P}^+(\upsilon)\}$
$= \text{rmin}\{\varpi\ \mathfrak{P}^+(\varrho), \varpi\ \mathfrak{P}^+(\upsilon)\}$
$= \text{rmin}\{\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\varrho), \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^+)(\upsilon)\}$, for all ϱ, υ in S_1.
Also $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho+\upsilon) = (-\varsigma)\ \mathfrak{P}^-(\varrho+\upsilon)$
$\leq (-\varsigma)\ \text{rmax}\{\mathfrak{P}^-(\varrho), \mathfrak{P}^-(\upsilon)\}$
$= \text{rmax}\{(-\varsigma)\mathfrak{P}^-(\varrho), (-\varsigma)\mathfrak{P}^-(\upsilon)\}$

= rmax{$\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho)$, $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.
And $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho\upsilon) = (-\varsigma)\ \mathfrak{P}^-(\varrho\upsilon)$
≤ $(-\varsigma)$ rmax{$\mathfrak{P}^-(\varrho)$, $\mathfrak{P}^-(\upsilon)$}
= rmax{$(-\varsigma)\mathfrak{P}^-(\varrho)$, $(-\varsigma)\mathfrak{P}^-(\upsilon)$}
= rmax{$\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\varrho)$, $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}^-)(\upsilon)$}, for all ϱ, υ in S_1.
Hence $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Corollary 2.37. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi − ring* S_1, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P} \cap \mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Theorem 2.1 and 2.36, it is trivial.

Corollary 2.38. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi − rings* S_1 and S_2, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of $S_1 \cap S_2$.

Proof. From the Theorem 2.1 and 2.36, it is trivial.

Corollary 2.39. If $\mathfrak{P} = \langle \mathfrak{P}^+, \mathfrak{P}^- \rangle$ and $\mathfrak{W} = \langle \mathfrak{W}^+, \mathfrak{W}^- \rangle$ are $\mathbb{BV}I\mathbb{FSSR}s$ *of the semi − rings* S_1, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}) \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{W})$ is a $\mathbb{BV}I\mathbb{FSSR}$ of S_1.

Proof. From the Corollary 2.38, it is trivial.

Theorem 2.40. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − rings $S_1, S_2, \dots, S_m$ respectively, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.36, *the proof is trivial.*

Corollary 2.41. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − rings $S_1, S_2, \dots, S_m$ respectively, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \dots \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring $S_1 \cap S_2 \cap \dots \cap S_m$.

Proof. *From the Theorem* 2.2 *and* 2.36, *the proof is trivial.*

Corollary 2.42. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1 \cap \mathfrak{P}_2 \cap \dots \cap \mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ *of* the semi − ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.36, *the proof is trivial.*

Corollary 2.43. *If* $\mathfrak{P}_1 = \langle \mathfrak{P}_1^+, \mathfrak{P}_1^- \rangle$, $\mathfrak{P}_2 = \langle \mathfrak{P}_2^+, \mathfrak{P}_2^- \rangle$, … , $\mathfrak{P}_m = \langle \mathfrak{P}_m^+, \mathfrak{P}_m^- \rangle$ *are* $\mathbb{BV}I\mathbb{FSSR}$s of the semi − ring S_1, then $\mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_1) \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_2) \cap \dots \cap \mathfrak{S}_{(\varpi,\varsigma)}(\mathfrak{P}_m)$ is a $\mathbb{BV}I\mathbb{FSSR}$ of the semi − ring S_1.

Proof. *From the Theorem* 2.2 *and* 2.36, *the proof is trivial.*

CONCLUSION

Properties of transformations of $\mathbb{BV}I\mathbb{FSSR}$ of a semi-ring have been discussed. The above concepts can be extended into bipolar interval valued multi fuzzy subsemi-ring of a semi-ring, bipolar interval valued multi fuzzy subspace of a linear space and any other algebraic system.

REFERENCES:

1. Anitha.M.S., Muruganantha Prasad & K.Arjunan, "Notes on bipolar valued fuzzy subgroups of a group", *Bulletin of Society for Mathematical Services and Standards,* Vol. 2 No. 3 (2013), pp. 52-59.
2. Anitha.M.S, K.L.Muruganantha Prasad & K.Arjunan, "Homomorphism and anti-homomorphism of bipolar valued fuzzy subgroups of a group", *International Journal of Mathematical Archive*, 4(12), 2013, 1-4.
3. Arsham Borumand Saeid, "Bipolar valued fuzzy BCK/BCI-algebras", *World Applied Sciences Journal*, 7 (11) (2009), 1404-1411.
4. Azriel Rosenfeld, "Fuzzy groups", *Journal of mathematical analysis and applications*, 35(1971), 512-517.

5. Kyoung Ja Lee, "Bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras", *Bull. Malays.Math. Sci. Soc.,* (2) 32(3) (2009), 361-373.
6. K.M.Lee, "Bipolar valued fuzzy sets and their operations", *Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand,* (2000), 307-312.
7. K.M.Lee, "Comparison of interval valued fuzzy sets, intuitionistic fuzzy sets and bipolar valued fuzzy sets", *J. fuzzy Logic Intelligent Systems,* 14 (2) (2004), 125-129.
8. K.Murugalingam & K.Arjunan, "A study on interval valued fuzzy subsemiring of a semiring", *International Journal of Applied Mathematics Modeling,* Vol.1, No.5 (2013), 1 –6.
9. Muthukumaran.S & B.Anandh, "Some Translation Theorems in bipolar valued multi fuzzy subnearrings of a nearing", *Malaya Journal of Matematik*, Vol.9, No.1, 817- 822 (2021)
10. Samit Kumar Majumder, "Bipolar valued fuzzy Sets in Γ-Semigroups", *Mathematica aieterna,* Vol. 2, no. 3(2012), 203 - 213.
11. Sunita Kuppayya Poojari, M.Muthusamy and K.Arjunan, "Bipolar valued I-fuzzy subsemirings of a semiring", *Adalya Journal,* Vol. 9, Issue 7 (2020), 316 –321.
12. Yasodara.B and KE.Sathappan, "Bipolar-valued multi fuzzy subsemirings of a semiring", *International Journal of Mathematical Archive*, 6(9) (2015), 75 – 80.
13. Young Bae Jun and Seok Zun Song, "Subalgebras and closed ideals of BCH-algebras based on bipolar valued fuzzy sets", *Scientiae Mathematicae Japonicae Online,* (2008), 427-437.
14. L.A.Zadeh, "Fuzzy sets", *Inform. and control,* 8(1965), 338 –353.