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Application of Z-Fuzzy Relationship to Medical Diagnosis

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Abstract

We look at an important application of Z-Fuzzy Relation (ZFR) [5,6] in medical diagnosis. The objective of the paper is, we wish to construct an appropriate mathematical model utilising Z-Fuzzy Subset **(**ZFS) which can be implemented as a computer aided medical diagnosis system. Here additionally, reliability of data has to be taken into account and z-fuzzy relationship between symptoms and diagnosis has been found with ultimate accurate reliability solutions and also, these generalizations are substantiated with numerical example.

Key words:

Fuzzy set, Crisp Relation, Fuzzy Relation, Z-number, Z-Fuzzy Relation, Z-Fuzzy Subset, medical diagnosis, Z-Fuzzy Relationship function, Compositions of Z-Fuzzy Relations.

1 Introduction

In our daily life, fuzzy sets were developed as a generalization of usual sets to take into account the uncertainty which can't be avoid in our vague environment. Zadeh [9] further extended an idea and introduced the concept of z-number which model the 'fuzziness' of information and in addition include the reliability factor in 2011. Thus z-number is simply an ordered pair of fuzzy numbers where the first component gives the information and the second component deals with the reliability of information. So the embedding of ZFR with z-number is efficient application to medical diagnosis [1,2,4,7].

2 Preliminaries

Definition 2.1:

Crisp Relation

Relation from X to Y can be thought of as a subset of X×Y. So it can be represented by a characteristic function χ_R:

$\chi_R(x,y)$ = 1 if $(x,y) \in R \subset X \times Y$
= 0 if $(x,y) \notin R$
ie.$\chi_R(x,y)$ = 1 if xRy
= 0 if $x\not{R}y$

Definition 2.2:

Fuzzy Relation [3,11]

Fuzzy relation from X to Y is represented by fuzzy relationship function R where R: X×Y → [0,1].

Definition 2.3:

Z-number [9]

Z-number has two components Z = (A, B). The first component A, is a restriction on the values which a real-valued uncertain variable X is allowed to take. Second component B, is a measure of reliability of the first component. Typically, A and B are described in a natural language.

Definition 2.4:

Z-Valuation [9]

A Z-valuation is an ordered triple (X,A,B) where X is an uncertain variable, 'A' is a fuzzy set defined on the real line and 'B' is a fuzzy number whose support is contained in [0,1].
The Z-valuation (X,A,B) may also be denoted as ' X isz (A,B) '. A z-valuation is equivalent to an assignment statement "X is (A,B)". It may be viewed as a restriction on X defined by FEP(X is A) is B.
More explicitly

$$\text{Possibility}(FEP(x \in A) = u) = \mu_B(u)$$

The computations with Z-numbers have been studied in both continuous and discrete cases.

Definition 2.5:

Z-Fuzzy Relation [5,6]

Let X and Y be arbitrary sets of z-numbers. A z-fuzzy relation $R(X, Y)$ from X to Y can be described by specifying the z-fuzzy relationship function.

A z-fuzzy relationship function f from X to Y must be in the following form.

For all, $x \in X$, $y \in Y$

$$f\,(x, y) = (A(x, y), B(x, y)) \qquad [9]$$

Where $A(x, y)$ and $B(x, y)$ are normal fuzzy sets with supports in [0, 1].

Here $A(x, y)$ specifies fuzzy estimate of the strength of relationship and $B(x, y)$ refers to the reliability of the estimate of the strength of relationship.

Note: In other words $f(x, y)$ is a z-number with support in unit square.

Example 2.6:

Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_{2,}\}$ where $x_i = (A_i\,, B_i)$; $y_i = (C_i, D_i)$ are z-numbers for i = 1, 2, 3. Then f can be specified in a matrix form where $f(x_i, y_j)$ is given by the element in i-th row, j-th column of the matrix.

$$.\begin{bmatrix} (.6, .7, .8), [.9,1]) & ((.3, .4, .5), [8, .9]) \\ (.7, .8, .9), [.8, .9] & ((.2, .3, .4), [1,1]) \\ (.5, .6, .7), [.9,1]) & ((.5, .6, .7)[.7,8]) \end{bmatrix}$$

Definition 2.7:

Type1 Z-Fuzzy Relation [5,6]

A z-fuzzy relationship function f from X to Y is said to be of Type1 if $\forall x \in X, y \in Y, f(x, y)$ is a type1 z–number. In other words, $A(x, y)$ and $B(x, y)$ are real numbers.

Note: In rest of this paper, z-fuzzy relation ZFR will refer to type1 z-fuzzy relation, unless otherwise specified.

Example 2.8:

Let $X = \{x_1, x_2, x_3\}\ and\ Y = \{y_1, y_2, y_3\}$

Consider a z-fuzzy relationship function f be given in the matrix format

$$\begin{pmatrix} (.8, 1) & (.7, .6) & (.4, .8) \\ (0, 1) & (0, 1) & (.9, .8) \\ (1, .9) & (.9, .9) & (0, .9) \end{pmatrix}$$

where $f(x_i, y_j)$ is an element in i-th row, j-th column of the matrix.

For example, we note that, $f(x_1\,, y_2) = (.7, .6)$

So this means the strength of relationship between x_1 and y_2 is indicated as .7 and the reliability of this information is .6.

3 Compositions of Z-Fuzzy Relations [5,6]

Let us see how we can compose ZFRs

Let X $= \{x_{1,}x_{2,\dots,}x_n\}$; Y $= \{y_{1,}y_{2,\dots,}y_m\}$; W $= \{w_{1,}w_{2,\dots,}w_l\}$

$Let\ P(i,j) = (a(i,j), b(i,j))\ and$

$Q(i,j) = (c(i,j), d(i,j))$

Definition 3.1:

Fuzzy Number [3]

A fuzzy set A of R must have the following properties

1. A must be normal fuzzy set
2. α-cut of A must be a closed interval for every $\alpha \in (0, 1]$
3. The support of A must be bounded.

Definition 3.2:

Composition of Fuzzy Relation [3,11]

Let the two binary relations $P(X,Y)$ and $Q(Y,Z)$ with a common set Y. The standard composition of these relations which is denoted by $P(X,Y) \circ Q(Y,Z)$, produces a binary relation $R(X,Z)$ on $X \times Z$ defined by

For all, $x \in X$, $y \in Y$ and all $z \in Z$.

$R(x,z) = P \circ Q(x,z) = max_{y\in Y}$ min { $P(X,Y), Q(Y,Z)$}

Definition 3.3:

Ranking Function

Let F be a set of fuzzy numbers. A ranking function $r_k$on F is a real valued function defined on F. Given a ranking function r_k on F, we can order the elements in F as follows

$$A_1 \leq A_2\ if\ and\ only\ if\ \ r_k(A_1) \leq r_k(A_2)$$

Definition 3.4:

Momentum Ranking Function

Let $r_{1,}r_2$ be any two ranking functions for fuzzy numbers. Then for the z-number Z = (A, B) define the momentum ranking function (MRF) by

MRF (Z) = M (r_1, r_2)(Z) = $r_k(A)$. r_k(B)

Definition 3.5:

Z-Fuzzy Subset

Let X be a set. Then

$$\text{A: X} \rightarrow [0,1] \times [0,1]$$

$$A(x) = \left(A_1(x), A_2(x)\right)$$

is said to define a z-fuzzy subset A of X. Here $A_1(x)$ specifies the membership of x in Z-FSA and $A_2(x)$ is level of reliability of this information.

4 Mathematical Formulation

Let S be a set of symptoms /observations $S = \{s_1, s_2, s_3, \ldots, s_n\}$.
Let D be reference set of possible diagnoses $D = \{d_1, d_2, d_3, \ldots, d_m\}$.

Example 4.1:

$$S = \{\text{high fever, cough, ...}\}$$
$$D = \{\text{Malaria, Typhoid, ...}\}$$

When a patient describes his symptoms it may be vague. For example, he may say he had pain for three or four days. That is the data could be fuzzy. Another patient may describe 'high fever'. Now if the patient had a thermometer at home and taken proper readings that data would be highly reliable. On the other, hand if they had just decided the patient had 'high fever' merely by touching the patient's forehead it may not be so reliable.

Hence since fuzziness and reliability of data have to be taken into account it is appropriate to consider the 'patient's symptoms' as a z-fuzzy subset of S.

5 Z-Fuzzy Relationship between Symptoms and Diagnosis

With the help of medical experts the Z-FR matrix $\left(R_1(i,j), R_2(i,j)\right)$ is constructed. Here $R_1(i,j)$ indicates how likely the symptoms 's_i' will indicate the diagnosis 'd_j'. $R_2(i,j)$ of course indicates the reliability of this information.

Let $P: S \to [0,1] \times [0,1]$ be the Z-FSS of S denoting the symptoms/observations for a particular patient.

Let

$$P(s_i) = \left(P_1(i), P_2(i)\right)$$

Then to find the possible diagnosis we use the following procedure.

Step 1

Calculate$\Delta = P \circ R$. Then Δ is a Z-FSS of D. It gives the possible diagnoses which are related to the given patients symptoms

$$\Delta(j) = (\left(\Delta_1(j), \Delta_2(j)\right))$$

Here $\Delta_1(j)$ shows the possibility of diagnosis d_j being the correct diagnosis, $\Delta_2(j)$ indicates the reliability of this information.

Here '$\circ$' can be any of the following composition methods

(i) Sup-min , Overall-min
(ii) Sup-avg , Overall-min
(iii) Avg-product , Avg-product

(i) **Sup-min, Overall-min**

$$\Delta_1(j) = \max_i\left\{\min\left(P_1(i), R_1(i,j)\right)\right\}$$

$$= \max\left\{\min\left(P_1(1), R_1(1,j)\right), \min\left(P_1(2), R_1(2,j)\right), \dots, \min\left(P_1(n), R_1(n,j)\right)\right\}$$

$$\Delta_2(j) = \min_i\left\{\min\left(P_2(i), R_2(i,j)\right)\right\}$$

$$= \min\{P_2(1), P_2(2), \dots, P_2(n), R_2(1,j), R_2(2,j), \dots, R_2(n,j)\}$$

(ii) **Sup-avg, Overall-min**

$$\Delta_1(j) = \max_i\left\{\frac{P_1(i) + R_1(i,j)}{2}\right\}$$

$$\Delta_2(j) = \min_i\left\{\min\left(P_2(i), R_2(i,j)\right)\right\}$$

(iii) **Avg-product, Avg-product**

$$\Delta_1(j) = \frac{1}{n}\{P_1(i) \cdot R_1(i,j)\}$$

$$\Delta_2(j) = \frac{1}{n}\{P_2(i) \cdot R_2(i,j)\}$$

Step 2

Calculate the rank matrix $M(\Delta)$ by using the formula

$$M(j) = ((\Delta_1(j).\Delta_2(j)))[27]$$

With help of rank matrix, the possible diagnoses can be ordered.

Example 5.1:

$$S = \{s_1, s_2, s_3, s_4, s_5\};\ D = \{d_1, d_2, d_3, d_4\}$$

$$P = \left((.8,.9), (.3,.8), (0,1), (0,.9), (.9,.8)\right)$$

$$R = \begin{bmatrix} (.9,1) & (0,.9) & (.1,.9) & (.8,.9) \\ (.3,.9) & (.8,1) & (.1,.9) & (.1,.9) \\ (.3,.9) & (.2,.9) & (.9,.9) & (.1,.9) \\ (0,1) & (.1,1) & (.9,.9) & (.8,.9) \\ (.9,1) & (.1,1) & (.1,1) & (.8,1) \end{bmatrix}$$

(i) Sup-min, Overall-min

$$\Delta_1(1) = \max\{\min(.8,.9), \min(.3,.3), \min(0,.3), \min(0,0), \min(.9,.9)\}$$
$$= \max\{.8,.3,0,0,.9\}$$
$$= .9$$

$$\Delta_1(2) = \max\{\min(.8,0), \min(.3,.8), \min(0,.2), \min(0,.1), \min(.9,.1)\}$$
$$= \max\{0,.3,0,0,.1\}$$
$$= .3$$

$$\Delta_1(3) = \max\{\min(.8,.1), \min(.3,.1), \min(0,.9), \min(0,.9), \min(.9,.1)\}$$

$$= \max\{.1,.1,0,0,.1\}$$
$$= .1$$
$$\Delta_1(4) = \max\{\min(.8,.8), \min(.3,.1), \min(0,.1), \min(0,.8), \min(.9,.8)\}$$
$$= \max\{.8,.1,0,0,.8\}$$
$$= .8$$
$$\Delta_2(1) = \min\{\min(.9,1), \min(.8,.9), \min(1,.9), \min(.9,1), \min(.8,1)\}$$
$$= \min\{.9,.8,.9,.9,.8\}$$
$$= .8$$
$$\Delta_2(2) = \min\{\min(.9,.9), \min(.8,1), \min(1,.9), \min(.9,1), \min(.8,1)\}$$
$$= \min\{.9,.8,.9,.9,.8\}$$
$$= .8$$
$$\Delta_2(3) = \min\{\min(.9,.9), \min(.8,.9), \min(1,.9), \min(.9,.9), \min(.8,1)\}$$
$$= \min\{.9,.8,.9,.9,.8\}$$
$$= .8$$
$$\Delta_2(4) = \min\{\min(.9,.9), \min(.8,.9), \min(1,.9), \min(.9,.9), \min(.8,1)\}$$
$$= \min\{.9,.8,.9,.9,.8\}$$
$$= .8$$

So $\Delta = ((.9,.8),(.3,.8),(.1,.8),(.8,.8))$

Therefore

$M(\Delta) = (.72,.24,.08,.64)$.

So the order of possible diagnosis is d_1, d_4, d_2, d_3.

(ii) Sup-avg, Overall-min

$$\Delta_1(1) = \max\left\{\frac{.8+.9}{2}, \frac{.3+.3}{2}, \frac{0+.3}{2}, \frac{0+0}{2}, \frac{.9+.9}{2}\right\}$$
$$= \max\left\{\frac{1.7}{2}, \frac{.6}{2}, \frac{.3}{2}, \frac{0}{2}, \frac{1.8}{2}\right\}$$
$$= \max\{.85,.3,.15,0,.9\}$$
$$= .9$$
$$\Delta_1(2) = \max\left\{\frac{.8+0}{2}, \frac{.3+.8}{2}, \frac{0+.2}{2}, \frac{0+.1}{2}, \frac{.9+.1}{2}\right\}$$
$$= \max\left\{\frac{.8}{2}, \frac{.11}{2}, \frac{.2}{2}, \frac{.1}{2}, \frac{1}{2}\right\}$$
$$= \max\{.4,.55,.1,.05,.5\}$$
$$= .55$$
$$\Delta_1(3) = \max\left\{\frac{.8+.1}{2}, \frac{.3+.1}{2}, \frac{0+.9}{2}, \frac{0+.9}{2}, \frac{.9+.1}{2}\right\}$$
$$= \max\left\{\frac{.9}{2}, \frac{.4}{2}, \frac{.9}{2}, \frac{.9}{2}, \frac{1}{2}\right\}$$
$$= \max\{.45,.2,.45,.45,.5\}$$
$$= .5$$

$$\Delta_1(4) = \max\left\{\frac{.8+.8}{2}, \frac{.3+.1}{2}, \frac{0+.1}{2}, \frac{0+.8}{2}, \frac{.9+.8}{2}\right\}$$

$$= \max\left\{\frac{1.6}{2}, \frac{.4}{2}, \frac{.1}{2}, \frac{.8}{2}, \frac{1.7}{2}\right\}$$

$$= \max\{.8, .2, .05, .4, .85\}$$

$$= .85$$

$$\Delta_2(1) = \min\{\min(.9,1), \min(.8,.9), \min(1,.9), \min(.9,1), \min(.8,1)\}$$

$$= \min\{.9, .8, .9, .9, .8\}$$

$$= .8$$

$$\Delta_2(2) = \min\{\min(.9,.9), \min(.8,1), \min(1,.9), \min(.9,1), \min(.8,1)\}$$

$$= \min\{.9, .8, .9, .9, .8\}$$

$$= .8$$

$$\Delta_2(3) = \min\{\min(.9,.9), \min(.8,.9), \min(1,.9), \min(.9,.9), \min(.8,1)\}$$

$$= \min\{.9, .8, .9, .9, .8\}$$

$$= .8$$

$$\Delta_2(4) = \min\{\min(.9,.9), \min(.8,.9), \min(1,.9), \min(.9,.9), \min(.8,1)\}$$

$$= \min\{.9, .8, .9, .9, .8\}$$

$$= .8$$

So $\Delta = ((.9,.8), (.55,.8), (.5,.8), (.85,.8))$

Therefore

$M(\Delta) = (.72, .44, .40, .68)$.

So the order of possible diagnosis is d_1, d_4, d_2, d_3.

(iii) Avg-product, Avg-product

$$\Delta_1(1) = \frac{1}{5}\{.72 + .09 + 0 + 0 + .81\}$$

$$= \frac{1}{5} \times 1.62$$

$$= .33$$

$$\Delta_1(2) = \frac{1}{5}\{0 + .24 + 0 + 0 + .09\}$$

$$= \frac{1}{5} \times 0.33$$

$$= .07$$

$$\Delta_1(3) = \frac{1}{5}\{.08 + .03 + 0 + 0 + .09\}$$

$$= \frac{1}{5} \times 0.2$$

$$= .04$$

$$\Delta_1(4) = \frac{1}{5}\{.64 + .03 + 0 + 0 + .72\}$$

$$= \frac{1}{5} \times 1.39$$
$$= .28$$
$$\Delta_2(1) = \frac{1}{5}\{.9 + .72 + .9 + .9 + .8\}$$
$$= \frac{1}{5} \times 4.22$$
$$= .84$$
$$\Delta_2(2) = \frac{1}{5}\{.81 + .8 + .9 + .9 + .8\}$$
$$= \frac{1}{5} \times 4.21$$
$$= .84$$
$$\Delta_2(3) = \frac{1}{5}\{.81 + .72 + .9 + .81 + .8\}$$
$$= \frac{1}{5} \times 4.04$$
$$= .81$$
$$\Delta_2(4) = \frac{1}{5}\{.81 + .72 + .9 + 0.81 + .8\}$$
$$= \frac{1}{5} \times 4.04$$
$$= .81$$

So $\Delta = ((.33, .84), (.07, .84), (.04, .81), (.28, .81))$
Therefore
$M(\Delta) = (.28, .06, .03, .23)$.
So the order of possible diagnosis is d_1, d_4, d_2, d_3.
However in the above methods, we have the same order of diagnosis.

Conclusion

We investigate, develop and enrich the concept of ZFR by the illustration of the practical example. Then we deal with application of z-fuzzy relationship to medical diagnosis. It has more potential for application in different fields like Agriculture, Industry, commerce, etc. Hence essentially each composition here leads to different mathematical modeling.

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References

[1] Adlassnig K.P ‘A Fuzzy Logical Model of Computer-Assisted Medical Diagnosis’, A Fuzzy Logical Model of Medical Diagnosis, 19, pp.141-148, 1980.

[2] Adlassnig K.P, Fuzzy set theory in medical diagnosis, IEEE Trans. On systems, Man and cybernetics, 16, pp.260-265, 1986.

[3] George J. Klir, Bo Yuan, Fuzzy sets and Fuzzy logic, theory and applications, PHI Learning Private Ltd., New Delhi, pp.119-149, 2012.

[4] Kerre E.E A walk through fuzzy relations and their application to information retrieval, medical diagnosis and expert systems, seminar for Mathematical Analysis, State University of Gent, Belgium.1990.

[5] Latha Devi R.A,Velammal G ‘The concept of z-fuzzy relations’ Utilitas Mathematica, 120, pp. 1146 – 1154, 2023.

[6] Latha Devi R.A, Velammal G ‘The properties of compositions of z-fuzzy relations’ Utilitas Mathematica, 120, pp. 1176 – 1183, 2023.

[7] Sanchez, E ‘Medical diagnosis and composite fuzzy relations’. In: Gupta, M.M., Ragade, R.K. and Yeager, R.R., Eds., Advances in Fuzzy Sets and Applications, North-Holland, New York, pp.437-447, 1979.

[8] Shahilabhanu.M.,Velammal.G, Operations on Zadeh’s Z-number, IOSR Journal of Mathematics , 11 (3), pp. 88-94, 2015.

[9] Zadeh.L.A Fuzzy sets, Information and Control, pp.338-353, 1965.

[10] Zadeh.L.A A note on Z-number, information science 181, pp.2923-2932, 2011

[11] Zimmermann H J, Fuzzy set theory &its applications, fourth edition, Springer international edn. pp.79, 2001.