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## RISK MODELLING AND SURVIVAL ANALYSIS

**Author: U. Kumar,**

**Assistant Professor, Mathematical Sciences**

**Email id: [uk9545505rrr@gmail.com](mailto:uk9545505rrr@gmail.com)**

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### **Abstract:-**

Survival or time to event analysis comprises a collection of methods to model the time until an event of interest occurs. A typical element of survival data is censoring where events times are unknown. Survival analysis assumes that censoring occurs independently of risk of the outcome of interest. The paper provides introduction, theoretical basics and statistical approaches to survival analysis with competing risks.

**Keywords: Statistical methods, survival analysis, censoring, Estimators**

### **Introduction:-**

Survival analysis also known as time to event analysis is a branch of statistics that studies the amount of time it takes before a particular event of interest occurs.

Survival analysis is the analysis about the data measured from a specific time of origin until an event of interest or specified end points collected et al [1994]. The benefits of survival analysis is that it can better tackle the issue of censoring as its main variable other than time that addresses whether the expected event happened or not.

Cumulative incidence function is a proper summary of statistics for analysing competing risk data.

The Method of Fine and Gray [1999] Extends the Cox regression to Model the Cumulative Incidence Function that marks a milestone in development of Modelling Technique for Competing Risks data.

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Despite Achieving great popularity the Fine and Gray Model has been Criticized for weakness of interpretation . On the other hands Proportional Hazard Model for Cause- Specific Hazards are easy to Fit and offer a more Natural interpretation in terms of rate ratio.

Review on Survival Models Methods are provided by Cox and Oakes [1984] Klein and Moeschberger[2006] and more recently Wang et al[2019].

Cumulative Incidence Function is Estimated by Modeling the Cause Specific Hazard Function of all Causes.

The topics of Competing Risk Events and Estimation of Cumulative Incidence of an Event of Interest have been Discussed by several authors. [Gail,1975] Reviews the Theoretical Concepts Underlying the Estimation of Cumulative Incidence of an Event using a variety of models.

The cumulative incidence Accounting for Competing Risks Events is Estimated in a Two Step - Processes Kalbfleisch and Prentice et al [1980].

The Cause- Specific Approach with a Long -history of Applications and have Proportional Hazards Models are Typically Assumed [Prentice et al, 1978].

Tai et al [2001] Developed a Method to Estimate the Cumulative incidence of a Specific Events Based on an Extension of Cox Proportional Hazard Regression Model.

Clark et al [2003 ], Bradburn et al [2003]Provides a Detailed Review on various Survival Concepts.

In Contrast to the Model of Fine and Gray [1999] that targets to the Subdistribution Function directly and has become a popular choice in Practice. The Subdistribution might be consistent in restricted time ranges but such an assumption need to be Formally Justified Latouche et al [2013]

Hua and Xiang [2013] Cater for interval Censoring using transformed baseline hazard function using splines.

### **Survival Analysis:-**

Survival Analysis is the Analysis of data measured from a specific time of origin until an Event of Interest or specified End- Point [Collett,1994]

Competing risk arise in studies in which individual are subject number of potential failure events and the occurrence of one event might impede the occurrence of other events.

When there are no Competing risk events that is when there is only one type of failure the estimation of the Cumulative Incidence of the event derived using the Kaplan-Meier Approach and the Competing risk approach are identical.

Competing Events (or risks ) Preclude the Observation of an Event of Interest or after the probability of the Events occurrences. The Presence of Competing events Violates the Assumptions of Independent Censoring which is the basis of Standard survival Analysis.

**Censoring:-**

The difficulties relating to Survival Analysis largely from the fact that only some individual have experienced the event and thus survival time will be unknown for a subset of study groups. This Phenomena is called Censoring.

Censoring can also occur if we observe the presence of a state or condition but do not know where it began.

In general feature of censoring means special method for analysis are needed.

Let  $T$  and  $C$  denotes failure times and censoring time for data that have  $k$  competing risks the pair  $(X, f)$  is observed where  $X = \min(T, C)$  and  $f = 1, 2, 3, \dots, k$  is an indicator that has a value of 0 for censoring other values that designate specific failure causes for analysing competing risks data two are full quantities are the Cause specific hazard function and the Cumulative incidence functions.

Survival time:-

Survival time is also called survival time, failure time etc which refers to the time experienced from a certain point to the end point event of the observed object.

i.e the survival at a given time is the conditional probability of surviving to a specific time given that the individual is at risk for the event at that

time.

$$S(t) = \lim_{\Delta t} \frac{\Pr(t \leq T \leq t + \Delta t)}{\Delta t} = \frac{F(t)}{S(t)} = -\frac{d}{dt} \ln S(t)$$

Survival data are often summarised using the Cumulative incidence function for an event.

**Cumulative distribution function :-**

The cumulative distribution function is expressed by  $F(t)$  referring to the probability of survival time being less than or equal to

$$F(t) = P(T \leq t) = \int_0^t f(s) ds \text{ from } 0 \text{ to } t \text{ Survival function.}$$

Survival function is generally expressed as  $S(t)$  which refers to the probability of Survival time  $T$  being greater than  $t$  corresponding to the cumulative distribution function and is expressed by the formula  $S(t) = P[T \geq t] = 1 - F(t)$

Two types of hazards have been described in the presence of competing risks the cause specific hazard and the subdistribution hazard. The hazard is the probability that an individual who is under observation at a time has an event at that time.

The cause specific hazard function generates the classical concepts of hazard functions to the competing risk setting and it describes the rate failure from one event type in the presence of the others

The cause specific hazard function  $hc(t)$  at time  $t$  is the instantaneous rate of failure due to cause  $C$  conditional on survival until time  $t$  or later it is defined as

$$hc(t) = \lim_{\Delta t} \frac{[P(t < T < t + \Delta t, \delta = c \mid T > t)]}{\Delta t}, \quad c = 1, 2, 3, \dots, C.$$

**Cumulative hazard rate function:-**

The cumulative hazard rate function is an integral of the hazard rate function

That means that the cut off to time  $t$  is the probability of a state change. It is expressed as follows

$$H(t) = \int_0^t h(s) ds = -\log S(t)$$

It is of interest because it provides insight into the conditional failure rates so hazard relates to the incident event rate while survival reflects the cumulative non occurrence.

The cumulative incidence function denoted by  $F_c(t)$  is the probability of failure due to cause  $C$

Prior to time  $t$ .

It is defined as

$$F_c(t) = P(T \leq t, \delta = c), c = 1, 2, 3, \dots, C$$

The cumulative incidence function quantifies the risk of failure from a particular event type when there are competing risks.

Cox proportional hazard model Cox et al [1972] is essentially a regression model commonly used in statistics for investigating the association between the survival time and one or more predictor variables

Cox model provide estimates of cause specific hazards in contrast to Fine and Gray Model, provides the subdistribution hazards.

Mathematically the  $k$ th cumulative incidence function can be expressed in terms of all the cause specific hazards functions via the integral as follows

$$F_k(t) = \int_0^t S(u) h_k(u) du =$$

$$\int_0^t S(u) dH_k(u) \quad k = 1, 2, 3, \dots, K =$$

$$\text{where} \quad H_k(t) = \int_0^t h_k(u) du$$

is the cause specific cumulative hazard function and

$$S(t) = \exp\left(-\sum_k H_k(t)\right) \quad k = 1, 2, \dots, K.$$

is the overall survival function which is the probability of surviving beyond time  $t$ . It is clear that  $F_k(t)$  involves not only  $h_k(t)$  but also all the competing cause specific hazard function when  $K > 1$ . both models Cox Regression and Fine and Gray regression require proportionality of hazards.

For semiparametric regression approaches there are two prevailing methods to construct the model that conditionally explain survival in the presence of competing risks.

Cai et al[2012] extends long hazard using linear splines for interval censored data extended to the estimate the baseline hazard using linear splines to model the log hazard with smoothing parameters estimated by restricted maximum likelihood.

**Conclusion:**

In this paper Survival time, the common hazard rate function ,cumulative risk rate function ,censoring are studied in theoretical form. The method based on the constructions of survival models for the risk rate are studied.

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