 ISSN: 2663-2187

https://doi.org/10.48047/AFJBS.6.10.2024.6172-6181

African Journal of Biological Sciences

Journal homepage: http://www.afjbs.com

Research Paper Open Access

Common Fixed Point Theorems in Fuzzy Metric Space

Hargovind Dubey[1*], Kamal Wadhwa[1] Sanjay Choudhary[1], Ramakant Bhardwaj[2]

[1*]hari131281@gmail.com ,[1]wadhwakamal68@gmail.com, [1]drsanjay0702@gmail.com, [2]rkbhardwaj100@gmail.com,

[1]Department of Mathematics, Govt. Narmada, PG College, Narmada Puram (M.P)

[2]Department of Mathematics, amity University Kolkata, WB.

Article Info
Volume 6, Issue 10, 2024
Received 16 May 2024
Accepted 10 Jun 2024
Doi: : 10.48047/AFJBS.6.10.2024.6172-6181

Abstract

The purpose of this paper is to establish common fixed point theorems for six and seven self-mappings in fuzzy metric space using E.A. like property [15]

AMS (2000) Subject Classification: 54H25, 47H10.

Keywords: coincidence point, fuzzy metric space, weakly compatible mappings, E.A.like property.

1.Introduction

The concept of fuzzy set was introduced in 1965 by Zadeh [1]as a new way to represent vagueness in everyday life. A large number of renowned mathematicians worked with fuzzy sets in different branches of Mathematics, Fuzzy Metric Space is one of them. This paper uses the concept of fuzzy metric space introduced by Kramosil and Michalek [2] and modified by George and Veeramani [3] with the help of t-norm. Grabiec [4] obtained the fuzzy version of Banach contraction principle, which is a milestone in developing fixed point theory in fuzzy metric space. Jungck [5] proposed the concept of compatibility.The concept of compatibility in fuzzy metric space was proposed by Mishra et al.[6]. Later on, Jungck [7] generalized the concept of compatibility by introducing the concept of weak compatibility. Singh and Chauhan [8] and Cho [9] provided fixed point theorems in fuzzy metric space for four self-maps using the concept of compatibility where two mappings needed to be continuous. In 2017 Govery A. and Singh M.[10] proved a common fixed point theorem for six self-mappings in fuzzy metric space using the concept of compatibility and weak compatibility where one map is needed to be continuous.

In this paper, two theorems has been proved on common fixed point, one for six and another for seven self-mappings in fuzzy metrics space, using E.A.like property[15] which relaxes the condition of continuity , containment of ranges and completeness of space, generalizing the result of Govery A. and Singh M.[10]. Some related recent work is also done in this field can be seen in [16-26]

2. Definition

2.1 [11] - A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous t-norms if $*$ satisfying conditions:

i. $*$ is commutative and associative;
ii. $*$ is continuous;
iii. $a*1=a$ for all $a\in[0,1]$;
iv. $a*b\le c*d$ whenever $a\le c$ and $b\le d$, and $a,b,c,d\in[0,1]$.

Examples: $a*b=$ min{a, b}, $a*b=$ a.b

Definition 2.2[3] - A 3-tuple $(X,M,*)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions , $\forall$ $x,y,z\in X, s,t>0$,

(f1) $M(x,y,t)>0$;

(f2) $M(x,y,t)=1$ if and only if $x=y$.

(f3) $M(x,y,t)=M(y,x,t)$;

(f4) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$

(f5) $M(x,y,.):(0,\infty)\rightarrow(0,1]$ is continuous.

Then M is called a fuzzy metric on X. Then $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Example 2.3 (Induced fuzzy metric [3]) – Let (X,d) be a metric space. Denote a $*$ b =ab for all $a,b\in[0,1]$ and let M_d be fuzzy sets on $X^2\times(0,\infty)$ defined as follows:

$$M_d(x,y,t)=\frac{t}{t+d(x,y)}$$

Then $(X,M_d,*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Definition 2.4[8]: Two self mappings f and g of a fuzzy metric space $(X,M,*)$ are called compatible if $\lim_{n\to\infty}M(fgx_n,gfx_n,t)=1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}fx_n=\lim_{n\to\infty}gx_n=x$ for some $x\in X$.

Lemma 2.5[6] : Let $(X, M, *)$ be fuzzy metric space. If there exists $q \in (0,1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$,then $x = y$

Definition 2.6: Let X be a set, f and g selfmaps of X . A point $x \in X$ is called a coincidence point of f and g iff $fx = gx$. We shall call $w = fx = gx$ a point of coincidence of f and g .

Definition 2.7 [14] **:** A pair of maps f and g is called weakly compatible pair if they commute at coincidence points.

$$fx = gx \rightarrow fgx = gfx$$

Definition 2.8[15]:Let f & g be two self-maps of a fuzzy metric space $(X, M, *)$ .we say that f & g satisfy the E.A. Like property if there exists a sequence $\{x_n\}$ such that,

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in f(X)$ or $z \in g(X)$ i.e. $z \in f(X) \cup g(X)$

Definition 2.9[15] (Common E.A. like Property) : Let A, B, S and T be self maps of a fuzzy metric space $(X, M, *)$, then the pairs (A,S) and (B,T) said to satisfy common E.A. Like property if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} By_n = z$

where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.

Example:let $X = [0, 2)$ and $M(x, y, t) = \dfrac{t}{t + d(x, y)}$ for all $x, y \in X$ then $(X, M, *)$ is a fuzzy metric space. Where $a * b = \min\{a, b\}$.

$$A(x) = \begin{cases} .25, 0 \leq x \leq .52 \\ \dfrac{x}{2}, x > .52 \end{cases} \qquad S(x) = \begin{cases} .25, 0 \leq x \leq .6 \\ x - .25, x > .6 \end{cases}$$

$$T(x) = \begin{cases} .25, 0 \leq x \leq .6 \\ \dfrac{x}{4}, x > .6 \end{cases} \qquad B(x) = \begin{cases} .25, 0 \leq x \leq .95 \\ x - .75, x > .95 \end{cases}$$

We define $x_n = .5 + \dfrac{1}{n}$ and $y_n = 1 + \dfrac{1}{n}$

We have $A(X) = \{.25\} \cup (.26, 1]$

$S(X) = \{.25\} \cup (.35, 1.75]$

$T(X) = (.15, .5]$ and

$B(X) = \{.25\} \cup (.20, 1.25]$

Also $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \dfrac{1}{2}[.5 + \dfrac{1}{n}] = .25 \in S(X)$

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} .5 + \frac{1}{n} - .25 = .25 \in A(X)$$

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} \frac{1}{4}[1 + \frac{1}{n}] = .25 \in B(X)$$ and

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} 1 + \frac{1}{n} - .75 = .25 \in T(X)$$

Govery. A. and Singh M.[10] proved the following results:

Theorem: Let $(X, M,*)$ be a complete fuzzy metric space and let $A, B, S, T\ , P\ and\ Q$ be mappings from X into itself such that the following conditions are satisfied:

(i) $P(X) \subset ST(X), Q(X) \subset AB(X)$

(ii) $AB = BA, ST = T\,S, PB = BP, QT = T\,Q$;

(iii) $Either\ AB$ or P is continuous

(iv) $(P, AB) is\ compatible\ and\ (Q, ST\ )\ is\ weakly\ \ compatible$;

(v) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq M(ABx, STy, t) * M(Px, ABx, t) * M(Qy, STy, t) * M(Px, STy, t)$$

Then $A, B, S, T\ , P\ and\ Q$ have a unique common fixed point in X.

3. **Main Results**

Theorem3.1 Let $(X, M,*)$ be a fuzzy metric space and let $A, B, S, T\ , P\ and\ Q$ be mappings from X into itself such that the following conditions are satisfied:

(i) $AB = BA, ST = T\,S, PB = BP, QT = T\,Q$;

(ii) $(P, AB)\ and\ (Q, ST\ )\ is\ weakly\ \ compatible$;

(iii) Pairs (P, AB) and $(Q, ST\ )$ follows E. A. like property

(iv) There exists $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq min\{M(ABx, STy, t), M(Px, ABx, t), M(Qy, STy, t), M(Px, STy, t)\}$$

Then $A, B, S, T\ , P\ and\ Q$ have a unique common fixed point in X.

Proof: since pair $(P, AB)$and (Q, ST) satisfy common E.A. Like property therefore there exists two sequences $\{x_n\}$ and $\{y_n\}$ in Xsuch that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STy_n = z$$

Where $z \in AB(X) \cap ST(X)$ or $z \in P(X) \cap Q(X)$

Suppose that $z \in AB(X) \cap ST(X)$, now we have

$\lim_{n\to\infty} Px_n = z \in AB(X)$ than there exist some $u \in X\ such\ that\ \ z = ABu$

We claim that $Pu = ABu$

$put\ x = u, y = y_n$ in (iv)

$$M(Pu, Qy_n, qt) \geq min\{M(ABu, STy_n, t), M(Pu, ABu, t), M(Qy_n, STy_n, t), M(Pu, STy_n, t)\}$$

Taking $n \to \infty$

$$M(Pu, z, qt) \geq min\{M(z, z, t), M(Pu, z, t), M(z, z, t), M(Pu, z, t)\}$$

$$M(Pu, z, qt) \geq M(Pu, z, t)$$

$$Pu = z$$

Therefore $Pu = z = ABu$

Since pair $(\boldsymbol{P}, AB)$ is weakly compatible therefore

$$Pz = PABu = ABPu = ABz$$

Now, we claim that $Pz = z$

Put $x = z\ and\ y = y_n$ in (iv)

$$M(Pz, Qy_n, qt) \geq min\{M(ABz, STy_n, t), M(Pz, ABz, t), M(Qy_n, STy_n, t), M(Pz, STy_n, t)\}$$

Taking $n \to \infty$

$$M(Pz, z, qt) \geq min\{M(Pz, z, t), M(Pz, Pz, t), M(z, z, t), M(Pz, z, t)\}$$

$$M(Pz, z, qt) \geq M(Pz, z, t)$$

$$Pz = z$$

Therefore $Pz = z = ABz$

Putting $x = Bz$ and $y = y_n$ in (iv)

$$\mathrm{M(PBz, Qy_n, qt) \geq min\{M(ABBz, STy_n, t), M(PBz, ABBz, t), M(Qy_n, STy_n, t),}$$
$$\mathrm{M(PBz, STy_n, t)\}}$$

Taking $n \to \infty$

Also, $BP = PB, AB = BA$, so we have $P(Bz) = B(Pz) = Bz$

and $(AB)(Bz) = (BA)(Bz) = B(ABz) = Bz$

$$\mathrm{M(Bz, z, qt) \geq min\{M(Bz, z, t), M(Bz, Bz, t), M(z, z, t),}$$
$$\mathrm{M(Bz, z, t)\}}$$

$$\mathrm{M(Bz, z, qt) \geq M(Bz, z, t)}$$

$$Bz = z$$

Since $Bz = z$,

we also have $ABz = z \to Az = z.$

Therefore, $Az = Bz = Pz = z.$

Again,

$\lim_{n\to\infty} Qy_n = z \in ST(X)$ than for some $v \in X, z = STv$

We claim that $Qv = STv$

$put\ x = x_n, y = v$ in (iv)

$$M(Px_n, Qv, qt) \geq min\{M(ABx_n, STv, t), M(Px_n, ABx_n, t), M(Qv, STv, t), M(Px_n, STv, t)\}$$

Taking $n \to \infty$

$$M(z, Qv, qt) \geq min\{M(z, z, t), M(z, z, t), M(Qv, z, t), M(z, z, t)\}$$

$$M(Qv, z, qt) \geq min\{M(z, z, t), M(z, z, t), M(z, z, t), M(Qv, z, t)\}$$

$$M(Qv, z, qt) \geq M(Qv, z, t)$$

$$Qv = z = STv$$

Since pair $(\boldsymbol{Q}, \boldsymbol{ST})$ **is weakly compatible therefore**

$$Qz = QSTv = STQv = STz$$

Now, we claim that $Qz = z$

Put $y = z\ and\ x = x_n$ in (iv)

$$\boldsymbol{M(Px_n, Qz, qt) \geq min\{M(ABx_n, STz, t), M(Px_n, ABx_n, t), M(Qz, STz, t), M(Px_n, STz, t)\}}$$

Taking $n \to \infty$

$$\boldsymbol{M(Qz, z, qt) \geq min\{M(z, Qz, t), M(z, z, t), M(Qz, Qz, t), M(z, Qz, t)\}}$$

$$\boldsymbol{M(Qz, z, qt) \geq M(Qz, z, t)}$$

$$Qz = z = STz$$

Putting $x = x_n$ and $y = Tz$ in (iv)

$$\boldsymbol{M(P\,x_n, QTz, qt) \geq min\{M(ABx_n, STTz, t), M(Px_n, ABx_n, t), M(QTz, STTz, t),}$$
$$\boldsymbol{M(Px_n, STTz, t)\}}$$

Since $$Q(Tz) = T(Qz) = Tz$$

$$(ST)(Tz) = (TS)(Tz) = T(STz) = Tz$$

$$\boldsymbol{M(z, Tz, qt) \geq min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t),}$$
$$\boldsymbol{M(z, Tz, t)\}}$$

$$\mathrm{M(Tz, z, qt) \geq M(Tz, z, t)}$$

$$Tz = z$$

we get $Tz = z$ and also we have $STz = z \to Sz = z$.

Therefore, $$Tz = Sz = Qz = z.$$

$$Az = Bz = Tz = Sz = Qz = Pz = z.$$

Hence A, B, T, S, Q, P have common fixed point.

To show uniqueness of fixed point, Put x = z and y = u

$$\mathrm{M(Pz, Qu, qt) \geq min\{M(ABz, STu, t), M(Pz, ABz, t), M(Qu, STu, t), M(Pz, STz, t)\}}$$

$$\mathrm{M(z, u, qt) \geq min\{M(z, u, t), M(z, z, t), M(u, u, t), M(z, z, t)\}}$$

$$\mathrm{M(z, u, qt) \geq M(z, u, t)}$$

$$\mathrm{z = u}$$

Hence A, B, T, S, Q, P have unique common fixed point.

Theorem3.2: Let $(X, M, *)$ be a fuzzy metric space and let A, B, R, S, T, P and Q be mappings from X into itself such that the following conditions are satisfied:

(i) $AB = BA, ST = TS, PB = BP, QT = TQ, PR = RP, TR = RT, BR = RB$

(ii) (P, ABR) and (Q, STR) $are\ weakly\ compatible$;

(iii) Pairs (P, ABR) and (Q, STR) follows E. A. like property

(iv) There exists $q \in (0, 1)$ such that for every $x, y \in X$ and $t > 0$

$$M(Px, Qy, qt) \geq min\{M(ABRx, STRy, t), M(Px, ABRx, t), M(Qy, STRy, t), M(Px, STRy, t)\}$$

Then A, B, R, S, T, P and Q have a unique common fixed point in X.

Proof: since pair $(P, ABR)$and (Q, STR) satisfy common E.A. Like property therefore there exists two sequences$\{x_n\}$ and $\{y_n\}$in Xsuch that

$$\lim_{n\to\infty} Px_n = \lim_{n\to\infty} ABRx_n = \lim_{n\to\infty} Qy_n = \lim_{n\to\infty} STRy_n = z$$

Where $z \in ABR(X) \cap STR(X)$ or $z \in P(X) \cap Q(X)$

Suppose that $z \in ABR(X) \cap STR(X)$,now we have

$\lim_{n\to\infty} Px_n = z \in STR(X)$ than for some $u \in X, z = STRu$

We claim that $Qu = STRu$

$put\ x = x_n, y = u$ in (iv)

$$M(Px_n, Qu, qt)$$
$$\geq min\{M(ABRx_n, STRu, t), M(Px_n, ABRx_n, t), M(Qu, STRu, t), M(Px_n, STRu, t)\}$$

Taking $n \to \infty$

$$M(z, Qu, qt) \geq min\{M(z, z, t), M(z, z, t), M(Qu, z, t), M(z, z, t)\}$$
$$M(Qu, z, qt) \geq M(Qu, z, t)$$
$$Qu = z = STRu$$

Since pair (Q, STR) is weakly compatible therefore

$$Qz = QSTRu = STRQu = STRz$$

Now, we claim that $Qz = z$

Putting $x = x_n$ and $y = z$ in (iv)

$$M(Px_n, Qz, qt)$$
$$\geq min\{M(ABRx_n, STRz, t), M(Px_n, ABRx_n, t), M(Qz, STRz, t), M(Px_n, STRz, t)\}$$
$$M(z, Qz, qt) \geq min\{M(z, Qz, t), M(z, z, t), M(Qz, Qz, t), M(z, Qz, t)\}$$

$$M(z, Qz, qt) \geq M(z, Qz, t)$$
$$Qz = z$$
$$Qz = z = STRz$$

Again , $\lim_{n\to\infty} Qy_n = z \in ABR(X)$, therefore there exist $v \in X$ such that $ABRv = z$

Putting $x = v$ & $y = y_n$ in (iv)

$$M(Pv, Qy_n, qt)$$
$$\geq min\{M(ABRv, STRy_n, t), M(Pv, ABRv, t), M(Qy_n, STRy_n, t), M(Pv, STRy_n, t)\}$$
$$M(Pv, z, qt) \geq min\{M(z, z, t), M(Pv, z, t), M(z, z, t), M(Pv, z, t)\}$$
$$M(Pv, z, qt) \geq M(Pv, z, t)$$
$$Pv = z$$
$$Pz = PABRv = ABRPv = ABRz$$

Putting $x = z$, and $y = y_n$ in (iv)

$$M(Pz, Qy_n, qt)$$
$$\geq min\{M(ABRz, STRy_n, t), M(Pz, ABRz, t), M(Qy_n, STRy_n, t), M(Pz, STRy_n, t)\}$$

$$M(Pz, z, qt) \geq min\{M(Pz, z, t), M(Pz, Pz, t), M(z, z, t), M(Pz, z, t)\}$$
$$M(Pz, z, qt) \geq M(Pz, z, t)$$
$$Pz = z$$
$$Pz = z = ABRz$$

Again putting $x = Rz$, and $y = z$ in (iv)

$M(PRz, Qz, qt)$
$\geq \ min\{M(ABRRz, STRz, t), M(PRz, ABRRz, t), M(Qz, STRz, t), M(PRz, STRz, t)\}$

$M(RPz, Qz, qt)$
$\geq \ min\{M(RABRz, STRz, t), M(RPz, RABRz, t), M(Qz, STRz, t), M(RPz, STRz, t)\}$

$$M(Rz, z, qt) \ \geq \ min\{M(Rz, z, t), M(Rz, Rz, t), M(z, z, t), M(Rz, z, t)\}$$
$$M(Rz, z, qt) \ \geq M(Rz, z, t)$$
$$Rz = \ z$$

$STRz = z => STz = z$ and $ABRz = z => ABz = z$

$$STz = ABz = z$$

Putting $x = Bz$ and $y = z$ in (iv)

$M(PBz, Qz, qt)$
$\geq \ min\{M(ABRBz, STRz, t), M(PBz, ABRBz, t), M(Qz, STRz, t), M(PBz, STRBz, t)\}$
$M(BPz, Qz, qt)$
$\geq \ min\{M(BABRz, STRz, t), M(BPz, BABRz, t), M(Qz, STRz, t), M(BPz, BSTRz, t)\}$

$$M(Bz, z, qt) \ \geq \ min\{M(Bz, z, t), M(Bz, Bz, t), M(z, z, t), M(Bz, Bz, t)\}$$
$$M(Bz, z, qt) \ \geq M(Bz, z, t)$$
$$Bz = z$$

Therefore $ABz = z => Az = z$

$$Az = Bz = z$$

$x = z$ and $y = Tz$ in (iv)

$M(Pz, QTz, qt)$
$\geq \ min\{M(ABRz, STRTz, t), M(Pz, ABRz, t), M(QTz, STRTz, t), M(Pz, STRTz, t)\}$
$M(Pz, TQz, qt)$
$\geq \ min\{M(ABRz, TSTRz, t), M(Pz, ABRz, t), M(TQz, TSTRz, t), M(Pz, TSTRz, t)\}$

$$M(z, Tz, qt) \ \geq \ min\{M(z, Tz, t), M(z, z, t), M(Tz, Tz, t), M(z, Tz, t)\}$$
$$M(z, Tz, qt) \ \geq M(z, Tz, t)$$
$$Tz = \ z$$
$$STz = \ z => Sz = z$$
$$Sz = Tz = z$$
$$Pz = Qz = Rz = Sz = Tz = Az = Bz = z$$

Therefore z is common fixed point of P, Q, R, S, T, A, B

Uniqueness of common fixed point z can easily be shown as in Theorem3.1.

4. Conclusion

This paper is generalization of the result of Govery A. and Singh M.[10] in the sense of using the weak compatible mapping for both pairs which is lighter condition than compatible and continuity of mappings, containment of ranges, completeness of space is not required for the existence of fixed point for six and seven self-mappings in fuzzy metric space

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