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Supply Chain Inventory System Modeling Using Fuzzy Systems

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ABSTRACT

In a supply chain system, deteriorating production systems implement inventory models to account for transportation costs based on the quantity ordered and the distance between the supplier's manufacturing facility and the retailer's warehouse. A crisp model is first created, and then an appropriate fuzzy model is created, taking into account various fuzzy parameters that affect inventory cost. Transportation costs are rarely considered in inventory modelling, as distinct models have been built for suppliers and retailers. The current methodology minimizes the overall average inventory cost of the supply chain, which includes suppliers and retailers jointly, in a clear and fuzzy environment. The retailer's demand parameter is said to be determined by the level of inventory. At the warehouses of suppliers and retailers, degradation happens continuously during production and storage.The objective of this article is to establish an inventory model that, when combined with the signed distance method to defuzzify the fuzzy numbers, optimizes the supply chain's overall average inventory cost. Sensitivity analysis has been done on the model's fuzzy parameters, and a numerical example is provided to confirm the applicability of the model.

Key words: Fuzzy system, Inventory model , fuzzy numbers demand, Transportation costs, warehouses

1.0 INTRODUCTION

Inventory management is crucial in commerce, involving various operational facets within an organization to efficiently distribute goods throughout the business process. The primary goal of most inventory models is to reduce inventory costs for business operators focused on specific components of their operations. In the sector, commodities typically flow through four main distribution layers: producers/manufacturers, distributors/suppliers, retailers, and end users, particularly consumers. Each participant plays a role in the business operations, aiming to streamline inventory costs. Research in inventory systems often aims to minimize costs for manufacturers, suppliers, and retailers dealing with single or multiple commodities. In the realm of supply chain networks (SCNs), which involve multiple business entities across different levels, scholars have developed models to optimize the movement of goods between end users and businesses. Studies have explored various factors affecting inventory systems or SCNs, with a particular focus on cost parameters. Efficient modeling of these aspects is critical to minimizing overall inventory costs or supply chain network costs, ultimately enhancing profitability for businesses.

Various inventory models included four types of demand: stock-dependent, time-dependent, exponential, and constant. They don't, however, always function effectively for optimizing inventory systems. Throughout the marketing process of commodities in a corporation, items are created or manufactured at different levels and finished goods are supplied to the market to be used by end customers. Certain objects deteriorate while others hold up well enough to be used until they are delivered to the end user, including during the production process, stock holding process, and inventory in transit system, among other stages of this process. Deterioration is the process through which products lose value in the supply chain, either chemically or physically, and costs the owner of the business money. The main factor impacting inventory costs during storage and transit is degradation, a topic that has long drawn the attention of scholars. Uncertainty in the market during the business process contributes to both price inflation and the production system. Fixed-value parameters are not enough to deal with the unpredictability of the market. Forecasts are subject to change and cannot be seen as providing an exact amount of items necessary due to several unknown factors specific to the country. Comparably, cost parameter inflation is similarly erratic and dependent on the state of affairs and emerging threats. It is essential to assess the uncertain future state and model the system appropriately in order to closely monitor the level of market uncertainty with respect to the specific quantity of commodities. In this topic, a great lot of study publications dealing with the unknown condition and amount have been generated. When this type of uncertainty was first observed and studied in the seventeenth century, the Zadeh [1] looked into interval-based membership functions that specified a graded value. A large body of literature applies fuzzy set theory in a fuzzy setting [2,3,4,5,6,7,8,9,10,11, 12, 27]. Some modern scholars have included uncertain market scenarios and have constructed inventory system models based on their research [13, 14, 15, 16, 17, 18, 19, 20, 21, 28, 29].Almost all of these studies focus only on inventory modeling of an inventory system in a fuzzy environment, excluding the expenses and activities related to transportation.

Many researchers are currently working on supply chain system modeling that takes into account the transportation network of commodities, but very few are taking into account the fuzzy environment to deal with uncertain future situations that may not be exact but may be very similar to the situation that is in demand at the time. By including the transportation system into inventory modeling, Konur [22] has created a carbon-constrained integrated inventory control and truckload transportation with heterogeneous freight trucks.

Mohammed et al. have presented a tri-level location model for forward and reverse supply chains [23 , 25]. Sinha and Anand developed the improved bacteria foraging algorithm (IBFA) to optimize supply chain networks for perishable products [24 ,26 29,30,31]. They compared the outcomes of IBFA with those of the current BFA. Nevertheless, in a fuzzy environment, these models have not been taken into account for optimizing the supply chain's overall inventory cost.

Keeping an inventory system that works efficiently is crucial for manufacturing firms in order to meet client demands and sustain operational excellence. However, inventory management becomes much more complex and challenging when defective items are incorporated into the production process. Products that, as a result of mistakes, defects, or other flaws in the production process, are not up to the required standards of quality or specifications are considered imperfect. If these defective products are not managed well, costs may increase, customer satisfaction may suffer, and overall inefficiencies may occur. Inspired by a number of research studies on inventory modeling and a model of the transportation supply chain system involving commodities in the current competitive business era, an inventory model is proposed for the supply chain system involving the supplier and the retailer in order to optimize the total average inventory cost of both parties involved in the business process.Initially, a clear model was created for the inventory level demand rate, and a fuzzy model was created to address the unpredictability of the production, demand, deterioration, and purchase rates.

The underlying assumption is that demand is always met and that production and supply happen instantly. By employing the Signed Distance Method to defuzzify triangular fuzzy integers, fuzzy parameters are taken into account.It is the only object in the research. If the model is fuzzy, sensitivity analysis has been done on the fuzzy parameters.

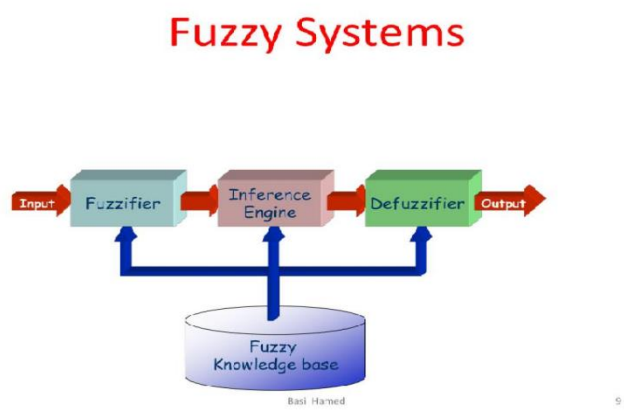

Fig. 1 Geometrical Representation of Fuzzy System

2.0 Preliminaries of Fuzzy Set

In order to manipulate and express the imprecise data that drives decision-making in a quantitative manner, fuzzy set theory has become a very useful tool. To deal with inaccurate data, fuzzy triplets of trapezoidal or other fuzzy numbers are typically utilized in inventory modeling, where input parameter values are set as functions of these fuzzy triplets. Present here are some fuzzy definition sets.

Fuzzy Set: A fuzzy set V˜ on a given universal set X is denoted and defined by

$$V˜ = \{(x, \lambda_{V˜}(x)) : x \,\varepsilon\, X\}$$

Where $\lambda_{V˜}$: X→ [0,1], is the membership function and $\lambda_{V˜}$(x) describes degree of x in V˜.

Fuzzy Triangular Number: A fuzzy number is specified by the triplet $(a_1 a_2 a_3)$ is known as triangular fuzzy if $a_1 < a_2 < a_3$ and is defined by its continuous membership function $\lambda_{V˜}$: X→ [0,1]as follows:

$$\lambda_{V˜}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & if\ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} & if\ a_2 \le x \le a_3 \\ 0 & otherwise \end{cases}$$

Signed Distance: Suppose V˜ be the fuzzy set defined on the $\Re$ (set of real numbers), then the signed distance of V˜ is given by

$$d(V˜,0) = \frac{1}{2}\int_0^1 [V_L(\alpha) + V_R(\alpha)]\, d_\alpha$$

where $V_\alpha = [V_L(\alpha) + V_R(\alpha)]$= [$a$ + b- a) α, d -(d-c) , a] , a ε [0,1] is a a-cut of a fuzzy set V˜.

If V˜= $(a_1 a_2 a_3)$ is a triangular fuzzy number then the signed distance of V˜ is defined as

d (V˜, 0) =$\frac{1}{4}(a_1 + 2\ a_2 + a_3)$

3.0 Assumptions and Notations

The following presumptions and notations serve as the foundation for the model's formulation:

3.1 Premises

1. The rate of replenishment happens instantly
2. The wait time for supplies is minimal.
3. You cannot allow shortages.
4. The rate of degradation in warehouses is steady.
5. The holding cost rate is consistent throughout both warehouses.
6. During the evaluation time, damaged units were neither restored nor replicated.
7. Items begin to deteriorate as soon as they are added to the inventory management system.
8. In a fuzzy model, parameters are seen as triangular fuzzy numbers.
9. The supplier's demand (D) is such that D=Q/T, where Q is the retailer's ordered quantity for the given time period (T).
10. The retailer's demand rate is deterministic, determined by the retailer's in-hand inventory at any given moment t. It is represented as such that

3.2 Notation

d_r	Deterioration cost of retailer per unit item
d_s	Deterioration cost of supplier per unit item
h_r	Inventory holding cost of retailer per unit item
h_s	Inventory holding cost of supplier per unit item
p_r	Purchasing cost of retailer per unit item
a	Fix transportation cost per trip of transportation
b	Variable transportation cost per ordered quantity per unit distance
d	Distance of retailer's warehouse from production unit of supplier
γ	Rate of deterioration of inventory at both warehouse of supply chain
T	Cycle length of retailers
T_1	Production time period of supplier
T_2	Time at which supplier's inventory vanishes
$I_R(t)$	Inventory level of retailer at any time t in the interval $0 \le t \le T$
$I_{Si}(t)$	Inventory level of supplier at any time t in the interval $0 \le t \le T_2$ where $i = 1,2$
N	Number of deliveries supplied by supplier to the retailers
Q	Order size per order of retailer
$\tilde{\alpha}$	Fuzzy fixed demand parameter of retailer
$\tilde{\beta}$	Fuzzy variable demand parameter base on inventory level
$\tilde{\gamma}$	Fuzzy deterioration rate in both warehouses

$\tilde{p_r}$ Fuzzy purchasing cost parameter of retailer
$\tilde{S_c}$ Fuzzy set up cost parameter of supplier
$\tilde{P}$ Fuzzy production rate of supplier

4.0 Mathematical Model

Section 4.1 introduces separate inventory models for both retailers and suppliers. The subsequent section demonstrates how various parameters affect the combined supply chain through numerical examples.

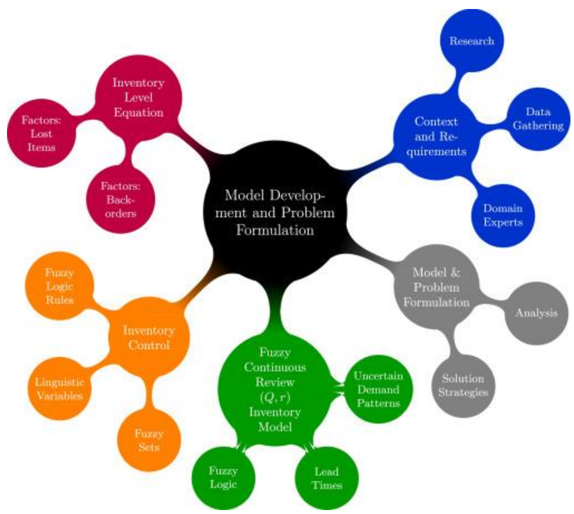

Figure 2 Pictorial diagrams of Inventory and fuzzy continuous review model.

The retailer's inventory model for the jth cycle is illustrated in Figure 3. Initially, Q units of inventory are added to the warehouse. Over time, inventory levels decrease due to consumer demand and deterioration. This is governed by a differential equation describing the inventory level dynamics.

$$\frac{dI_R(t)}{dt} = -\gamma I_R(t) - d(t); \qquad (j-1)T \le t \le jT \qquad (1)$$

At the beginning of inventory system $I_R((j-1)T) = Q$, using as boundary condition solution of eq. (1) is

$$I_{Rj}(t) = \left(Q + \frac{\alpha}{\beta+\gamma}\right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}; \quad (j-1)T \le t \le jT \qquad (2)$$

Where,

$$Q = e^{(\beta+\gamma)T} - 1$$

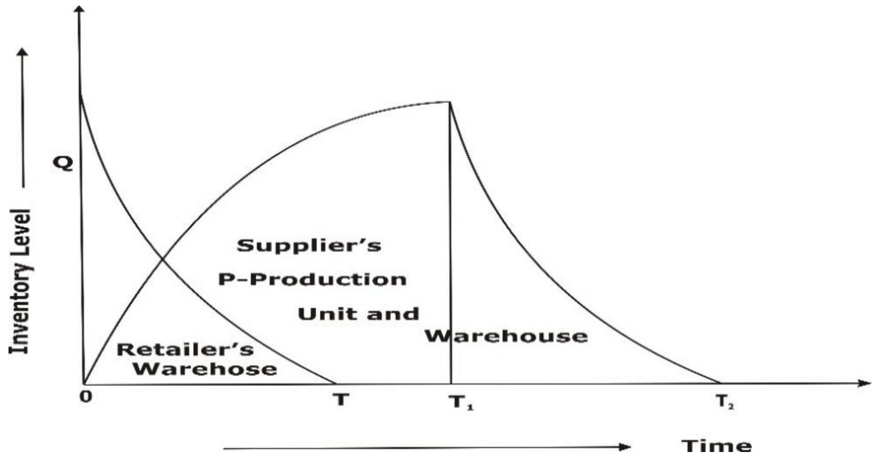

Figure-3 Graph represention depletion of inventory level in the warehouses

The retailer's inventory system consists following costs to be included in minimizing average total inventory cost

I. Ordering cost
II. Inventory holding cost
III. Deterioration cost
IV. Purchasing cost

Now above costs are:
Ordering cost for j[th]cycle$(j-1)T \le t \le jT$

$$CO_{Ij} = j\ o_r \tag{3}$$

Ordering cost for complete N–cycle as given blow:

$$CO_{In} = \sum_{j=1}^{n} j\ o_r \tag{4}$$

Inventory holding cost for j[th]cycle$(j-1)T \le t \le jT$ as given below :

$$IH_{crj} = h_r \left\{ \int_{(j-1)T}^{jT} I_{Rj}(\mathrm{t})\, dt \right\}$$

$$= h_r \left\{ \int_{(j-1)T}^{jT} \left(Q + \frac{\alpha}{\beta+\gamma} \right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}\ dt \right\} \tag{5}$$

Inventory holding cost for complete N-cycle are given below

$$IH_{crn} = \sum_{j=1}^{n} IH_{crj} \tag{6}$$

Inventory deterioration cost for j[th]cycle$(j-1)T \le t \le jT$ is given by

$$ID_{crj} = d_r \left\{ \int_{(j-1)T}^{jT}\ \gamma\, I_{Rj}(\mathrm{t})\, dt \right\}$$

$$= d_r \left\{ \int_{(j-1)T}^{jT} \gamma \left(Q + \frac{\alpha}{\beta+\gamma} \right) e^{-(\beta+\gamma)(t-(j-1)T)} - \frac{\alpha}{\beta+\gamma}\ \ dt \right\} \tag{7}$$

Inventory deterioration cost for complete N-cycle is given by

$$ID_{crn} = \sum_{j=1}^{n} ID_{crj} \tag{8}$$

Inventory purchase cost for j[th] cycle is given by

$$IP_{crj} = p_c\ (Q_j)$$

$$= P_c\ (e^{(\beta+\gamma)T} - 1) \quad (9)$$

Inventory purchase cost for complete N-cycle is given by

$$IP_{crn} = \sum_{j=1}^{n} IP_{crj} \quad (10)$$

Hence the total average inventory cost per unit of time of retailer for complete n-cycle is given by

$$TI_{cr}(N,T) = \frac{1}{T}[CO_{In} + IH_{crn} + ID_{crn} + IP_{crn}]$$

$$= \frac{1}{T}\left[\sum_{j=1}^{n} j\ o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} - IP\right] \quad (11)$$

4.2: Supplier's Inventory Model

Figure 2 shows that a graphical representation of the inventory level produced and supplied to the retailer. Initially, P units of inventory are produced, which then decrease due to supplier demand and deterioration over time. The remaining products decrease further due to supplier demand and continued deterioration. Throughout this period, the inventory level is governed by the following differential equation.

$$\frac{dI_{s1}(t)}{dt} = P - a\ d(t) - \gamma I_s(t); \qquad (0 \le t \le T_1) \qquad (12)$$

$$\frac{dI_{s2}(t)}{dt} = -\gamma I_R(t) - d(t); \qquad (T_1 \le t \le T_2) \qquad (13)$$

At the beginning of inventory system$I_{s1}(0) = 0$, apply this condition as boundary condition solution of equation (12) is given below :

$$I_{s1}(t) = \left(\frac{P-D}{\gamma}\right) a(1 - e^{-\gamma t}); \qquad (0 \le t \le T_1) \qquad (14)$$

According to continuity at $t = T_1$,$I_{s2}(T_2) = I_{s1}(T_1)$, using this condition solution of equation (13) is given below

$$I_{s2}(t) = \left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma}; \qquad (T_1 \le t \le T_2) \qquad (15)$$

The supplier's inventory system consists following costs to be included in minimizing average total inventory cost

- Setup cost
- Inventory holding cost
- Deterioration cost
- Production cost
- Transportation cost

Now above costs are illustrated as under given below:

Setup cost

$$S_{sc} = s_c \quad (16)$$

Inventory holding cost as given below:

$$IH_{cs} = h_s \left\{ \int_0^{T_1} I_{s1}(\mathrm{t})\, dt + \int_{T_1}^{T_2} I_{s2}(\mathrm{t})\, dt \right\}$$

$$= h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) \; dt \right\} \quad (17)$$

Inventory deterioration cost as given below:

$$ID_{cs} = d_s \left\{ \int_0^{T_1} \gamma\, I_{s1}(\mathrm{t})\, dt + \int_{T_1}^{T_2} \gamma\, I_{s2}(\mathrm{t})\, dt \right\}$$

$$= d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} \quad (18)$$

Inventory production cost as given below:

$$IP_{cs} = C_p \int_0^{T_2} P\, dt \quad (19)$$

Inventory transportation cost for complete N-cycle as given below:

$$IT_{csn} = \; N\,(a + b\, Q\, d\,) \quad (20)$$

Thus the total average inventory cost per unit of time for complete n-cycle is given by

$$TI_{cs}(N, T_1, T_2) = \; \frac{1}{T}[S_{sc} + IH_{cs} + ID_{cs} + IP_{cs} + IT_{csn}]$$

$$= \frac{1}{T}\Bigg[s_c + h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) \; dt \right\} +$$
$$d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} +$$
$$C_p \int_0^{T_2} P\, dt + N\,(a + b\, Q\, d\,) \Bigg] \quad (21)$$

Present worth of total inventory cost of supply chain taken together is given by

$$TIC_{csr}(N, T, T_1, T_2)$$
$$= \frac{1}{T}\Bigg[\sum_{j=1}^{n} j\; o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} \; + \sum_{j=1}^{n} IP_{crj} + S_{sc}$$
$$+ h_s \left\{ \int_0^{T_1} \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t})\, dt + \int_{T_1}^{T_2} \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) \; dt \right\}$$
$$+ d_s \left\{ \int_0^{T_1} \gamma \left(\frac{P-D}{\gamma}\right)(1 - e^{-\gamma t}) dt \right.$$
$$\left. + \int_{T_1}^{T_2} \gamma \left(\left(I_{s1}(T_1) + \frac{D}{\gamma}\right) e^{-\gamma(t-T_1)} - \frac{D}{\gamma} \right) dt \right\} \; + C_p \int_0^{T_2} P\, dt + N\,(a + b\, Q\, d\,) \Bigg]$$

(22)

The combined inventory average cost is function of T, T_1, T_2 only and the optimal value of T, T_1, T_2 can be obtained by equating $\frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T} = 0$; $\frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T_1} = 0$; $\frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T_2} = 0$ and necessary condition of Hessian matrix H which are as under satisfied.

$$H = \begin{pmatrix} \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial T^2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T \partial T_1} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T \partial T_2} \\ \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1 \partial T} & \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial {T_1}^2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1 \partial T_2} \\ \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_1 \partial T_2} & \frac{\partial^2 TIC_{csr}(T,T_1,T_2)}{\partial T_2 \partial T} & \frac{\partial TIC_{csr}(T,T_1,T_2)}{\partial {T_2}^2} \end{pmatrix}$$

The necessary condition for T, T_1, T_2 to be optimal is that the first principal minor determinant of H , $H_{11} > 0$, second principal minor $H_{22} > 0$ and third principal minor $H_{33} > 0$ and optimal value of N (discrete value) is found in such a way that optimal value of $TIC_{csr}(T^*, T_1^*, T_2^*)$is achieved.Here $T^*, T_1^*, T_2^*$are optimal values.

5.0 Formulation of Inventory Model (Fuzzy Model) Development

In an uncertain environment, dealing with parameters that have a clear-cut value is challenging. In today's global market, factors such as costs, demand rates, production rates, and deterioration rates can fluctuate due to various reasons like natural disasters. These fluctuations in parameters cannot be predicted in advance until they actually occur. Therefore, the approach is to consider potential ranges of fluctuation.To address such uncertainty, a corresponding fuzzy model has been developed. This model takes into account the vagueness of parameters that impact total inventory costs, treating these parameters as triangular fuzzy numbers. The Signed Distance Method is employed to solve this model, aiming to minimize the total inventory cost. Sensitivity analysis is conducted on the fuzzy parameters to understand their impact, and deviations are observed accordingly.

Using equation (22) and fuzzy parameters we have,
$\tilde{\alpha}_r = (\tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{\alpha}_3)$,$\tilde{\beta}_r = (\tilde{\beta}_1, \tilde{\beta}_2, \tilde{\beta}_3)$,$\tilde{\gamma}_{rs} = (\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3)$,$\tilde{S}_{sc} = (\tilde{S}_1, \tilde{S}_2, \tilde{S}_3)$,
$\tilde{p}_r = (\tilde{p}_1, \tilde{p}_2, \tilde{p}_3)$and $\tilde{P} = (\tilde{P}_1, \tilde{P}_2, \tilde{P}_3)$
Therefore, fuzzy model is given by
$$\tilde{TIC}_{csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2) = (\tilde{TIC}_{1csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2), \tilde{TIC}_{2csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2), \tilde{TIC}_{3csr}(\tilde{N}, \tilde{T}, \tilde{T}_1, \tilde{T}_2)) \quad (23)$$

Where,

$$\tilde{TIC}_{1csr}(\tilde{T}, \tilde{T}_1, \tilde{T}_2) = \frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S}_1 + h_s \left\{\int_0^{T_1} \left(\frac{\tilde{P}_1 - D}{\tilde{\gamma}_1}\right)(1 - e^{-\tilde{\gamma}_1 t})\, dt + \int_{T_1}^{T_2} \left(\left(\left(Q + \frac{\tilde{\alpha}_1}{\tilde{\beta}_1 + \tilde{\gamma}_1}\right) e^{-(\tilde{\beta}_1 + \tilde{\gamma}_1)(T_1 - (j-1)T)} - \frac{\tilde{\alpha}_1}{\tilde{\beta}_1 + \tilde{\gamma}_1} + \frac{D}{\tilde{\gamma}_1}\right) e^{-\tilde{\gamma}_1 (t - T_1)} - \frac{D}{\tilde{\gamma}_1}\right) dt\right\} + d_s \left\{\int_0^{T_1} \tilde{\gamma}_1 \left(\frac{\tilde{P}_1 - D}{\tilde{\gamma}}\right)(1 - e^{-\tilde{\gamma}_1 t}) dt + \int_{T_1}^{T_2} \tilde{\gamma}_1 \left(\left(\left(Q + \frac{\tilde{\alpha}_1}{\tilde{\beta}_1 + \tilde{\gamma}_1}\right) e^{-(\tilde{\beta}_1 + \tilde{\gamma}_1)(T_1 - (j-1)T)} - \frac{\tilde{\alpha}_1}{\tilde{\beta}_1 + \tilde{\gamma}_1} + \frac{D}{\gamma}\right) e^{-\tilde{\gamma}_1 (t - T_1)} - \frac{D}{\tilde{\gamma}_1}\right) dt\right\} + C_p \int_0^{T_2} \tilde{P}_1 dt + N\,(a + b\, Q\, d\,)\Bigg]$$

$$\tilde{TIC}_{2csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2) = \frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S}_2 + h_s \Bigg\{\int_0^{T_1}\left(\frac{\tilde{P}_2 - D}{\tilde{\gamma}_2}\right)(1-e^{-\tilde{\gamma}_2 t})\,dt + \int_{T_1}^{T_2}\left(\left(\left(Q+\frac{\tilde{\alpha}_2}{\tilde{\beta}_2+\tilde{\gamma}_2}\right)e^{-(\tilde{\beta}_2+\tilde{\gamma}_2)(T_1-(j-1)T)} - \frac{\tilde{\alpha}_2}{\tilde{\beta}_2+\tilde{\gamma}_2} + \frac{D}{\tilde{\gamma}_2}\right)e^{-\tilde{\gamma}_2(t-T_1)} - \frac{D}{\tilde{\gamma}_2}\right)dt\Bigg\} + d_s\Bigg\{\int_0^{T_1}\tilde{\gamma}_2\left(\frac{\tilde{P}_2-D}{\tilde{\gamma}_2}\right)(1-e^{-\tilde{\gamma}_2 t})dt + \int_{T_1}^{T_2}\tilde{\gamma}_2\left(\left(\left(Q+\frac{\tilde{\alpha}_2}{\tilde{\beta}_2+\tilde{\gamma}_2}\right)e^{-(\tilde{\beta}_2+\tilde{\gamma}_2)(T_1-(j-1)T)} - \frac{\tilde{\alpha}_2}{\tilde{\beta}_2+\tilde{\gamma}_2} + \frac{D}{\gamma}\right)e^{-\tilde{\gamma}_2(t-T_1)} - \frac{D}{\tilde{\gamma}_2}\right)dt\Bigg\} + C_p\int_0^{T_2}\tilde{P}_2 dt + N\,(a+b\,Q\,d)\Bigg]$$

$$\tilde{TIC}_{3csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2) = \frac{1}{T}\Bigg[\sum_{j=1}^{n} j\, o_r + \sum_{j=1}^{n} IH_{crj} + \sum_{j=1}^{n} ID_{crj} + \sum_{j=1}^{n} IP_{crj} + \tilde{S}_1 + h_s \Bigg\{\int_0^{T_1}\left(\frac{\tilde{P}_3 - D}{\tilde{\gamma}_3}\right)(1-e^{-\tilde{\gamma}_3 t})\,dt + \int_{T_1}^{T_2}\left(\left(\left(Q+\frac{\tilde{\alpha}_3}{\tilde{\beta}_3+\tilde{\gamma}_3}\right)e^{-(\tilde{\beta}_3+\tilde{\gamma}_3)(T_1-(j-1)T)} - \frac{\tilde{\alpha}_3}{\tilde{\beta}_3+\tilde{\gamma}_3} + \frac{D}{\tilde{\gamma}_3}\right)e^{-\tilde{\gamma}_3(t-T_1)} - \frac{D}{\tilde{\gamma}_3}\right)dt\Bigg\} + d_s\Bigg\{\int_0^{T_1}\tilde{\gamma}_3\left(\frac{\tilde{P}_3-D}{\tilde{\gamma}_3}\right)(1-e^{-\tilde{\gamma}_3 t})dt + \int_{T_1}^{T_2}\tilde{\gamma}_3\left(\left(\left(Q+\frac{\tilde{\alpha}_3}{\tilde{\beta}_3+\tilde{\gamma}_3}\right)e^{-(\tilde{\beta}_3+\tilde{\gamma}_3)(T_1-(j-1)T)} - \frac{\tilde{\alpha}_3}{\tilde{\beta}_3+\tilde{\gamma}_3} + \frac{D}{\gamma}\right)e^{-\tilde{\gamma}_3(t-T_1)} - \frac{D}{\tilde{\gamma}_3}\right)dt\Bigg\} + C_p\int_0^{T_2}\tilde{P}_3 dt + N\,(a+b\,Q\,d)\Bigg]$$

Using Signed Distance Method total average inventory can be calculated by

$$\tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2) = \frac{1}{4T}\left[\tilde{TIC}_{1csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2) + 2\tilde{TIC}_{2csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2) + \tilde{TIC}_{3csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)\right] \quad (24)$$

The objective is to

minimize: $\tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)$
Subject to: $(\tilde{T} > 0, \tilde{T}_1 > 0, \tilde{T}_2 > 0)$

The combined inventory average cost is function of $\tilde{T},\tilde{T}_1,\tilde{T}_2$ only and the optimal value of $\tilde{T},\tilde{T}_1,\tilde{T}_2$ can be obtained by equating $\frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}} = 0$; $\frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_1} = 0$; $\frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_2} = 0$ under the necessary condition of Hessian matrix $\tilde{H}$ which are as under is satisfied.

$$\tilde{H} = \begin{pmatrix} \frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}^2} & \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}\partial \tilde{T}_1} & \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}\partial \tilde{T}_2} \\ \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_1\partial \tilde{T}} & \frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_1^2} & \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_1\partial \tilde{T}_2} \\ \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_1\partial \tilde{T}_2} & \frac{\partial^2 \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_2\partial \tilde{T}} & \frac{\partial \tilde{TIC}_{csr}(\tilde{T},\tilde{T}_1,\tilde{T}_2)}{\partial \tilde{T}_2^2} \end{pmatrix}$$

The necessary condition for $\tilde{T},\tilde{T}_1,\tilde{T}_2$ to be optimal is that the first principal minor determinant of $\tilde{H}$, $\tilde{H}_{11} > 0$, second principal minor $\tilde{H}_{22} > 0$ and third principal minor $\tilde{H}_{33} > 0$ and optimal value of N (discrete value) is found in such a way that optimal value of $TIC_{csr}(N,\tilde{T}^*,\tilde{T}_1^*,\tilde{T}_2^*)$ is achieved. Here $\tilde{T}^*,\tilde{T}_1^*,\tilde{T}_2^*$ are optimal values.

6.0 computational for Illustration of models:

To analyze the model, we consider the following example where the exponential function is solved up to the second approximation. The parameter values, though not derived from specific real-life case studies, are chosen realistically and randomly to illustrate and validate the model. These parameter values are appropriately scaled and displayed in Table-1 for the crisp model and Table-3 for the fuzzy model. Using suitable mathematical software, we compute the optimal average inventory cost based on these parameter values, as shown in Table-2 and Table-4 for the two models respectively. Sensitivity analysis is conducted exclusively on the fuzzy model's fuzzy parameters, exploring changes in the upper bounds of triangular fuzzy numbers.

Table-1

Parameter	h_r	h_s	d_r	d_s	α	B	γ	o_r	p_r	s_c	a	b	d	P	C_s	N
Example	0.81	0.72	0.91	0.96	50	0.21	0.021	91	21	200	40	0.03	95	150	5.0	2

Table-2: Crisp Model

T^*	T_1^*	T_2^*	$TIC_{csr}(T^*, T_1^*, T_2^*)$
1.0689	3.0965	39.7485	191.795

Table-3:

Parameter	h_r	h_s	d_r	d_s	α	B	γ	o_r	p_r	s_c	a	b	d	P	C_s	N
Example	0.8	0.71	0.91	0.95	50	0.20	0.02	90	20	180	40	0.03	95	150	5.0	2

Parameter	α	B	γ	p_r	s_c	P
Triangular fuzzy number	(49, 50,51)	(0.19,0.2, 0.21)	(0.01, 0.02,0.03)	(15,20,25)	(180,200,220)	(100, 150,200)

Table-4: Fuzzy Model

Method	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9412	2.5961	41.4928	190.183

7. Sensitivity

Fuzzy Model

Table-5.1: Variation in total inventory cost with respect to production unit

Variation in total inventory cost with respect to $\tilde{P}$ (100,150,250)				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.08193	2.2814	41.426	207.937

Table-5.2: Variation in total inventory cost with respect to purchase cost

Variation in total inventory cost with respect to

$\widetilde{p_r}(15,20,30)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9814	2.6708	40.835	212.11

Table-5.3: Variation in total inventory cost with respect to fixed demand

Variation in total inventory cost with respect to $\tilde{\alpha}(49,50,60)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9847	2.7614	40.9868	213.509

Table-5.4: Variation in total inventory cost with respect to scale factor of demand

Variation in total inventory cost with respect to $\tilde{\beta}(0.19, 0.20, 0.23)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9814	2.6743	40.8189	187.208

Table-5.5: Variation in total inventory cost with respect to deterioration rate

Variation in total inventory cost with respect to $\tilde{\gamma}(0.01, 0.02, 0.035)$				
Parameter→ Method ↓	T^*	T_1^*	T_2^*	$\widetilde{TIC}_{csr}(T^*, T_1^*, T_2^*)$
Signed Distance	0.9116	2.5303	40.1561	191.458

Computational Analysis of Crisp and Fuzzy model

Upon satisfying all provided conditions and constraints, the optimal solution is achieved. Tables 2 and 4 demonstrate that the average total inventory cost, measured appropriately, is minimized in the fuzzy model compared to the crisp model. In the fuzzy model, the cycle length for retailers and the production period for suppliers are shorter, but the total supplier cycle length is moderately higher. Therefore, the fuzzy model proves more beneficial in handling uncertain situations due to its flexibility in parameter selection for future planning. The trend in the total average inventory cost of the supply chain under the fuzzy inventory model is illustrated through a 2-D graphical representation in Figure 3, depicting changes for selected parameters.

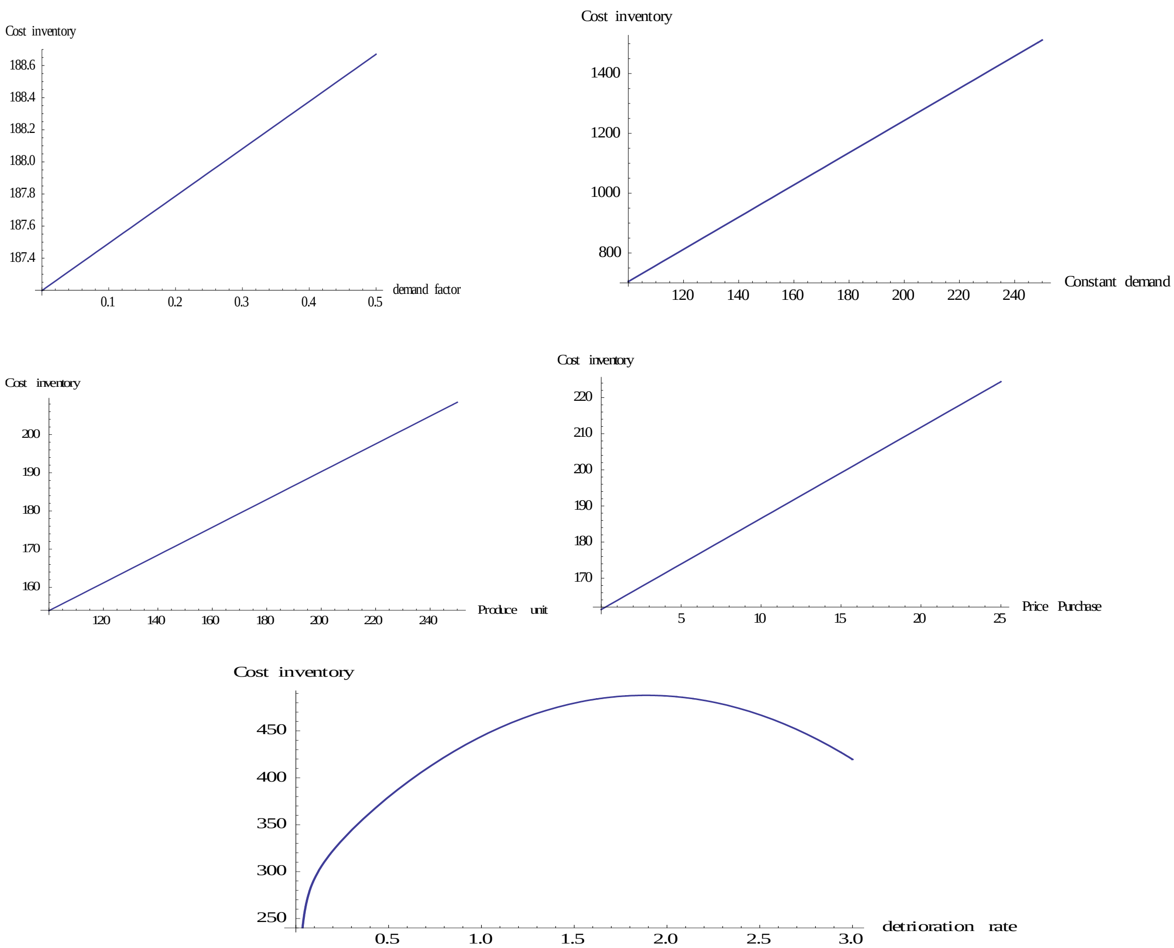

Figure-3: Geometrical trend of change in Average Inventory cost

8. Sensitivity Analysis:

From sensitivity performance, the following are observations have been made: -

1. If there is increase in the upper value of triangular fuzzy number of scale parameter of demand, keeping other same, the cycle length of retailer and production period of supplier and total cycle length of supplier slightly increases but the average inventory cost moderately decreases.
2. If there is increase in the upper value of triangular fuzzy number of deterioration parameter, keeping other same, the cycle length of retailer and production period of supplier and total cycle length of supplier slightly decreases but the average inventory cost overall have no effect.
3. If there is increase in the upper value of triangular fuzzy number of purchased cost parameter, keeping other same, the cycle length of retailer and production period of supplier have no more effect and total cycle length of supplier slightly increases but the average inventory cost moderately increases.

4. If there is increase in the upper value of triangular fuzzy number of production parameter, keeping other same, the cycle length of retailer moderately decreases and there is slight decrease in the production period of supplier but total cycle length of supplier have no effect while average inventory cost moderately increases.

9. Conclusion and Future Prospects:

This paper presents an interdisciplinary research project that integrates inventory management models with optimization techniques. Its main goal is to advance the field of inventory management by creating solutions that work, especially when products are declining across their life cycle. A corporation's perspective and decision-making processes are greatly influenced by inventory management. In order to maximize the overall inventory cost encompassing suppliers and retailers, this study presents an integrated supply chain model. It is assumed that the pace of degradation is the same for both parties and will never change. While per-order demand is set, retailer demand fluctuates according to inventory levels. Numerical examples are used to validate the study's development of both crisp and fuzzy models. An optimization problem is addressed and the outcomes are compared using the proper mathematical software. Sensitivity analysis investigates variations in the upper bounds of triangular fuzzy numbers on a subset of fuzzy parameters. The results demonstrate that the fuzzy model is more useful than the crisp model for handling uncertainties in the future.Additionally, the model can be expanded by considering other demand patterns and incorporating trade credit payments, along with varying combinations of deterioration rates and inflation.In conclusion, this study opens up new avenues for investigation and development of the production inventory model, which now includes imperfect products. This advances the field of manufacturing systems going forward.

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