https://doi.org/10.48047/AFJBS.4.4.2022.396-407

AFJBS African Journal of Biological Sciences

Research Paper Open Access

Biomedical importance & Properties of Length Biased Modified Aradhana Distribution

Ravikumar K[1], Thomas Mathew[2] and Prasanth C B[3*]

[1] *Associate Professor, Department of Statistics, Government Victoria College, Palakkad, Kerala,India.*
[2]*Principal, Associate Professor and Research Guide, M.D. College, Pazhanji, Thrissur, Kerala,India.*
[3]*Assistant Professor, Department of Statistics, Sree Kerala Varma College, Thrissur, Kerala. India*
prasanthwarriercb@keralavarma.ac.in (*Corresponding author)

Volume 4, Issue 4, Dec 2022

Received: 15 Oct 2022

Accepted: 25 Nov 2022

Published: 05 Dec 2022

doi: 10.48047/AFJBS.4.4.2022.396-407

Abstract
Here we have proposed a new distribution by applying the technique of transformation, length biased to the parent distribution. The new distribution is termed Length Biased Modified Aradhana distribution (LBMAD) has been presented. Also detailed the important statistical properties like moments, survival function, hazard function, and moment generating function. Also, the approach of maximum likelihood estimation was used to figure out the parameters of the new distribution. Lastly, a set of real-time data has been shown and analyzed to show how useful a new distribution is. A randomly selected 180 patients from a hospital at Chennai, which shows the mean reduction (mg/dL) of low-density lipoproteins (LDL) from the Near Optimal range of LDL(100 – 129) after taking the prescribed diet and physical exercise as prescribed by their consultant dietician for four weeks continuously

Keywords: Order statistics, New quasi-Aradhana distribution, Maximum likelihood estimation, Length biased distribution, Survival analysis

1. INTRODUCTION

There are many real life situations where known probability distributions failed to show a good fit for the data. For example, a common but an important biomedical data is based on LDL cholesterol. Direct LDL cholesterol testing measures the amount of cholesterol found inside low-density lipoproteins (LDL) in a sample of blood. Mostly this type of data set shows lack of symmetry, that is most of them are skewed. Dealing with such non-symmetrical data for analysis we can make use of the modified distributions. The concept of weighted distribution is one of the reputed and powerful tools in probability and statistics because it provides a comprehensive and informative approach for modeling, analyzing, and interpreting complicated statistical information. Fisher (1934) was the first person to use weighted distributions to demonstrate how ascertainment influences frequency estimation. Later, Rao (1965) indicated that recorded observations should not be regarded as a random sample from the standard distribution for various reasons. Reliability, ecology, biomedicine, family data analysis, meta-analysis, and many other fields of study have improved and expanded the use of the weighted distribution. The result of this has been the development of common statistical models. If the weight function only looks at the length of the units of interest, it is clear that the weighted distribution will be skewed toward longer units. The term length bias is a type of weighted distribution, so it can happen if the sample observations are not chosen correctly. The number of length-biased data points, on the other hand, is proportional to their length. This type of distribution is referred to as a length-biased distribution. It was Cox who first thought of a length-biased distribution in 1962. A length-biased distribution refers to sample data where the chance of getting an observation depends on how big that observation is.But Cox (1969) and Zelen (1974) also came up with the idea of length-biased sampling. It happens when there isn't a proper sampling frame, which leads to length-biased sampling. The idea of length bias can be used in many biomedical fields, including a history of families, analysis of reliability, geological sciences, evaluation of survival, disease, intermediate events, and population studies. A lot of researchers have looked into biased distributions and how they can be used to handle complicated data sets from a wide range of applied fields. In 2021, Ganaie and Rajagopalan created the length-biased power quasi-Lindley distribution and showed how it could be used. In 2016, Bodhisuwan et al. suggested a way to figure out the parameters of a Beta-Pareto distribution that is biased by length. Akanbi and Oyebanjo (2021) found the length-biased Gumbel distribution by employing it to look at wind speed data. Ahmad and Tripathi (2021) discussed power size-biased Maxwell distribution with engineering applications. Shanker and Shukla (2017) came up with the idea of the size-biased poisson Garima distribution and showed how it could be used. It was estimated by Salama et al. (2023) that the length-biased weighted exponentiated inverted exponential distribution would look like. It was Kousar and Memon (2017) who talked about their study of the length-biased weighted Nakagami distribution. Wani et al. created the size-skewed Lindley-quasi Xgamma distribution in 2022. It was used to study survival times.

A new quasi-Aradhana distribution that is a particular case of Aradhana distribution was proposed by Shanker et al. (2023). The suggested distribution has two parameters and is a lifetime distribution. Various statistical properties of it have been discussed, including the hazard function, survival function, mean residual life function, reverse hazard function, stochastic ordering, Lorenz and Bonferroni curves, and deviation from the mean and median. Further, its moments and moments-based measures have also been obtained. The maximum likelihood method was used to find the parameters of the suggested distribution.

2. MATERIAL AND METHODS

2.1 Length Biased Modified Aradhana (LBMA) Distribution

The probability density function (PDF) of the modified Aradhana distribution, as projected by Shanker et al., in their 2023 publication, is “expressed as:

$$f(x;\theta,\alpha) = \frac{\theta^3}{\theta^4 + 2\theta^2\alpha + 2\alpha^2}\left(\theta^2 + 2\theta\alpha x + \alpha^2 x^2\right)e^{-\theta x};\ x > 0,\ \theta > 0,\ \alpha > 0 \quad (1)$$

the distribution's cumulative distribution function (CDF) is provided as:

$$F(x;\theta,\alpha) = 1 - \left(1 + \frac{\theta\alpha x(\theta\alpha x + 2\theta^2 + 2\alpha)}{\theta^4 + 2\theta^2\alpha + 2\alpha^2}\right)e^{-\theta x};\quad x > 0,\ \theta > 0, \alpha > 0. \quad (2)$$

Let X be the unknown variable followed by a non-negative condition with PDF= $f(x)$. Let the non-negative weight function be $w(x)$.The PDF of a weighted random variable X_w can be described as:

$$f_w(x) = \frac{w(x)\, f(x)}{E(w(x))},\quad x > 0.$$

Where the non - negative weight function be *w(x)* and $E(w(x)) = \int w(x) f(x)dx < \infty.$

For various choices of weight function *w(x)* obviously if *w(x)* = x^c, the proposed distribution is termed a weighted distribution. In the currentresearch, the length-biased version of the” modified Aradhana distribution, LBMAD, has to be determined. So, consequently, the weight function considered at *w(x)* = *x*, the subsequent distribution is termed a length-biased distribution and its PDF is “given by

$$f_l(x) = \frac{x\, f(x)}{E(x)} \quad (3)$$

Where $E(x) = \int_0^\infty x\, f(x)dx = \frac{\theta^4 + 4\alpha\theta^2 + 6\alpha^2}{\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)} \quad (4)$

We will now get the necessary PDF of LBMAD by applying equations (1) and (4) to equation (3).

$$f_l(x) = \left(\frac{x\theta^4}{\theta^4 + 4\alpha\theta^2 + 6\alpha^2}\right)\left(\theta^2 + 2\theta\alpha x + \alpha^2 x^2\right)e^{-\theta x} \quad (5)$$

and the CDF of LBMADis derived as:

$$F_l(x) = \int_0^x f_l(x)dx$$

$$F_l(x) = \left(\frac{1}{\theta^4 + 4\alpha\theta^2 + 6\alpha^2}\right)\left(\theta^6\int_0^x x e^{-\theta x}dx + 2\alpha\theta^5\int_0^x x^2 e^{-\theta x}dx + \alpha^2\theta^4\int_0^x x^3 e^{-\theta x}dx\right) \quad (6)$$

Put $\theta x = t \Rightarrow \theta dx = dt \Rightarrow dx = \frac{dt}{\theta}$, Also $x = \frac{t}{\theta}$ As $x \to x,\ t \to \theta x$ and as $x \to 0,\ t \to 0$

Following eq". (6)'s simplification, the LBMAD's cumulative distribution function will be as:

$$F_l(x) = \frac{1}{\left(\theta^4 + 4\alpha\theta^2 + 6\alpha^2\right)}\left(\theta^4\gamma(2, \theta x) + 2\alpha\theta^2\gamma(3, \theta x) + \alpha^2\gamma(4, \theta x)\right) \tag{7}$$

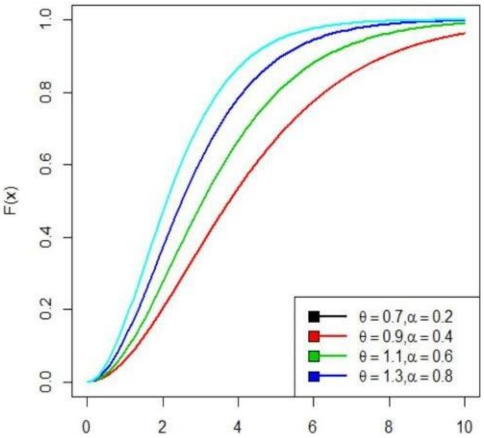


Figure 1. pdf of LBMAD

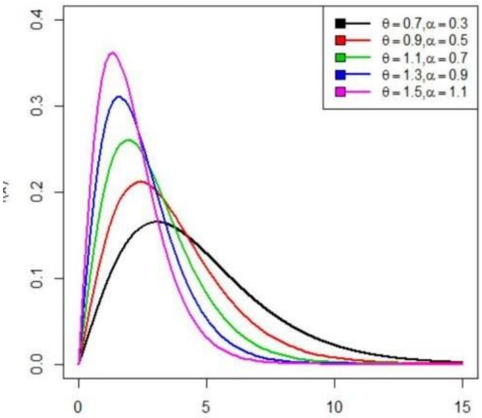


Figure 2. cdf of LBMAD

The nature of pdf and cdf of LBMAD is clear From the figure 1 and Figure 2.

3. RESULTS AND DISCUSSION

The hazard rate function, Mills ratio, survival function, along with the reverse hazard rate function of the LBMAD will all be obtained in this section.

3.1 Survival function

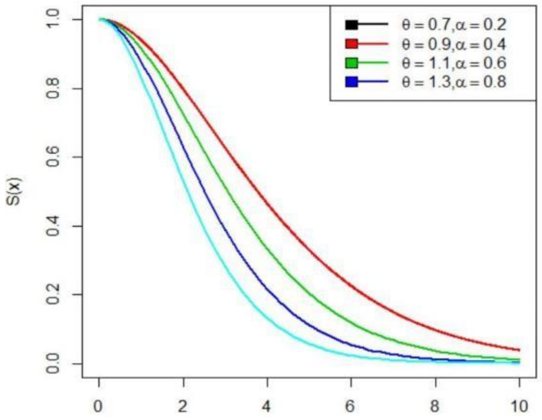
The LBMAD's survival/reliability function will be ascertained to be

$$S(x) = 1 - F_l(x) = 1 - \frac{1}{\left(\theta^4 + 4\alpha\theta^2 + 6\alpha^2\right)}\left(\theta^4\gamma(2, \theta x) + 2\alpha\theta^2\gamma(3, \theta x) + \alpha^2\gamma(4, \theta x)\right)$$

3.2 Hazard function

The "hazard function/the failure rate/hazard rate, is provided by

$$h(x) = \frac{f_l(x)}{1 - F_l(x)} = \frac{x\theta^4(\theta^2 + 2\theta\alpha x + \alpha^2 x^2)e^{-\theta x}}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2) - (\theta^4\gamma(2, \theta x) + 2\alpha\theta^2\gamma(3, \theta x) + \alpha^2\gamma(4, \theta x))}$$



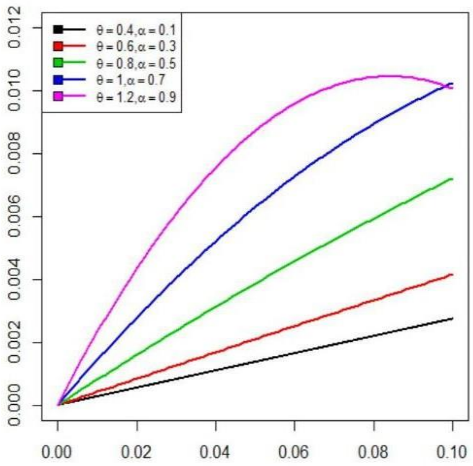


Figure 3. Reliability function of LBMAD **Figure 4. Hazard function of LBMAD**

The nature of Reliability function and Hazard function of LBMAD is clear From the figure 1 and Figure 2.

3.3 Test for Length Biasedness of LBMAD

Consider "the random sample $X_1, X_2, \ldots, X_n$ of size n drawn from the LBMAD. To study its flexibility the hypothesis is to be analyzed and investigated.

$$H_0 : f(x) = f(x;\theta,\alpha) \quad \text{Vs} \quad H_1 : f(x) = f_l(x;\theta,\alpha)$$

In order to explore and find out, if the random sample of size n comes from the LBMAD, the given rule of test statistic is to be applied.

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \frac{f_l(x;\theta,\alpha)}{f(x;\theta,\alpha)} = \prod_{i=1}^{n} \left(\frac{x\,\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \right) = \left(\frac{\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \right)^n \prod_{i=1}^{n} x_i$$

It should" be rectified that the null hypothesis should not be retained, if

$$\Delta = \left(\frac{\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \right)^n \prod_{i=1}^{n} x_i > k$$

Equivalently, it should also be pointed out that the null hypothesis is to be rejected where

$$\Delta^* = \prod_{i=1}^{n} x_i > k \left(\frac{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)}{\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)} \right)^n \quad \Delta^* = \prod_{i=1}^{n} x_i > k^*, \text{Where } k^* = k \left(\frac{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)}{\theta(\theta^4 + 2\theta^2\alpha + 2\alpha^2)} \right)^n$$

If the variable $2log\Delta$ follows a "chi-square distribution with one degree of freedom, then the sample size, n, is large and the p-value is obtained by applying the chi-square distribution. Therefore, it is evident that the null hypothesis should not be maintained if the probability value is" provided.

$p\left(\Delta^* > \gamma^*\right)$, Where $\gamma^* = \prod_{i=1}^{n} x_i$ is smaller than a particular level of significance and $\prod_{i=1}^{n} x_i$ is the observed value of the statistic Δ^*.

3.4 Structural Properties: Moments and its related measures

This section will cover the different statistical characteristics of LBMAD, including moments, moment “generating function, harmonic mean, and characteristic function.

3.4.1 Moments

Consider the random variable *X* following LBMAD with parameters θ and α, then the r^{th} order moment $E(X^r)$ of introduced distribution will be determined as

$$E(X^r) = \mu_r' = \int_0^\infty x^r f_l(x)dx = \int_0^\infty x^r \left(\frac{x\theta^4}{\theta^4+4\alpha\theta^2+6\alpha^2}\right)\left(\theta^2+2\theta\alpha x+\alpha^2x^2\right)e^{-\theta x}dx$$

$$= \int_0^\infty \left(\frac{x^{r+1}\theta^4}{\theta^4+4\alpha\theta^2+6\alpha^2}\right)\left(\theta^2+2\theta\alpha x+\alpha^2x^2\right)e^{-\theta x}dx = \int_0^\infty \left(\frac{x^{r+1}\theta^4}{\theta^4+4\alpha\theta^2+6\alpha^2}\right)\left(\theta^2+2\theta\alpha x+\alpha^2x^2\right)e^{-\theta x}dx$$

$$E(X^r) = \mu_r' = \left(\frac{\theta^4}{\theta^4+4\alpha\theta^2+6\alpha^2}\right)\left(\theta^2\int_0^\infty x^{r+1}e^{-\theta x}dx + 2\theta\alpha\int_0^\infty x^{r+2}e^{-\theta x}dx + \alpha^2\int_0^\infty x^{r+3}e^{-\theta x}dx\right)$$

$$E(X^r) = \mu_r' = \left(\frac{\theta^4}{\theta^4+4\alpha\theta^2+6\alpha^2}\right)\left(\theta^2\int_0^\infty x^{(r+2)-1}e^{-\theta x}dx + 2\theta\alpha\int_0^\infty x^{(r+3)-1}e^{-\theta x}dx + \alpha^2\int_0^\infty x^{(r+4)-1}e^{-\theta x}dx\right)$$

After simplification,

$$E(X^r) = \mu_r' = \frac{\left(\theta^4\Gamma(r+2)+2\alpha\theta^2\Gamma(r+3)+\alpha^2\Gamma(r+4)\right)}{\theta^r(\theta^4+4\alpha\theta^2+6\alpha^2)} \tag{8}$$

Now by replacing r = 1, 2, 3 and 4 in eq (8), we will determine the required first 4 moments of LBMAD as

$$E(X) = \mu_1' = \frac{2\theta^4+12\alpha\theta^2+24\alpha^2}{\theta(\theta^4+4\alpha\theta^2+6\alpha^2)} \quad E(X^2) = \mu_2' = \frac{6\theta^4+48\alpha\theta^2+120\alpha^2}{\theta^2(\theta^4+4\alpha\theta^2+6\alpha^2)}$$

$$E(X^3) = \mu_3' = \frac{24\theta^4+240\alpha\theta^2+720\alpha^2}{\theta^3(\theta^4+4\alpha\theta^2+6\alpha^2)} \quad E(X^4) = \mu_4' = \frac{120\theta^4+1440\alpha\theta^2+5040\alpha^2}{\theta^4(\theta^4+4\alpha\theta^2+6\alpha^2)}$$

$$\text{Variance} = \frac{6\theta^4+48\alpha\theta^2+120\alpha^2}{\theta^2(\theta^4+4\alpha\theta^2+6\alpha^2)} - \left(\frac{2\theta^4+12\alpha\theta^2+24\alpha^2}{\theta(\theta^4+4\alpha\theta^2+6\alpha^2)}\right)^2$$

$$S.D(\sigma) = \sqrt{\left(\frac{6\theta^4+48\alpha\theta^2+120\alpha^2}{\theta^2(\theta^4+4\alpha\theta^2+6\alpha^2)} - \left(\frac{2\theta^4+12\alpha\theta^2+24\alpha^2}{\theta(\theta^4+4\alpha\theta^2+6\alpha^2)}\right)^2\right)}$$

3.4.2. Moment generating function and characteristic function

Consider “the random variable *X* following LBMAD with parameters θ and α, then the moment generating function of proposed distribution willbe determined as

$$M_X(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_l(x)dx$$

Using Taylor series, we obtain

$$M_X(t) = \int_0^\infty \left(1 + tx + \frac{(tx)^2}{2!} + ...\right) f_l(x)dx = \int_0^\infty \sum_{j=0}^\infty \frac{t^j}{j!} x^j f_l(x)dx = \sum_{j=0}^\infty \frac{t^j}{j!} \mu_j'$$

$$M_X(t) = \sum_{j=0}^\infty \frac{t^j}{j!} \left(\frac{\theta^4 \Gamma(j+2) + 2\alpha\theta^2 \Gamma(j+3) + \alpha^2 \Gamma(j+4)}{\theta^j (\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \right) \tag{9}$$

$$M_X(t) = \frac{1}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \sum_{j=0}^\infty \frac{t^j}{j!\theta^j} \left(\theta^4 \Gamma(j+2) + 2\alpha\theta^2 \Gamma(j+3) + \alpha^2 \Gamma(j+4) \right)$$

Likewise, the characteristic function" of LBMAD,

$$\varphi_X(t) = M_X(it) = \frac{1}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \sum_{j=0}^\infty \frac{it^j}{j!\theta^j} \left(\theta^4 \Gamma(j+2) + 2\alpha\theta^2 \Gamma(j+3) + \alpha^2 \Gamma(j+4) \right)$$

3.5 Parameter Estimation and Fisher's Information Matrix

This section will explain the process of estimating the parameters of LBMAD and constructing its Fisher's information matrix using the estimation of the maximum likelihood technique. Consider the random sample $X_1, X_2, ..., X_n$ of size n from the LBMAD, then the likelihood function is" written as

$$L(x) = \prod_{i=1}^n f_l(x)$$

$$L(x) = \prod_{i=1}^n \left(\left(\frac{x_i \theta^4}{\theta^4 + 4\alpha\theta^2 + 6\alpha^2} \right) \left(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2 \right) e^{-\theta x_i} \right)$$

$$L(x) = \frac{\theta^{4n}}{\left(\theta^4 + 4\alpha\theta^2 + 6\alpha^2 \right)^n} \prod_{i=1}^n \left(x_i \left(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2 \right) e^{-\theta x_i} \right)$$

The "log likelihood function is given by

$$\log L = 4n \log\theta - n \log\left(\theta^4 + 4\alpha\theta^2 + 6\alpha^2\right) + \sum_{i=1}^n \log x_i +$$

$$+ \sum_{i=1}^n \log\left(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2 \right) - \theta \sum_{i=1}^n x_i \tag{10}$$

Now differentiating the log likelihood equation (10) with respect to parameters θ and α. We establish the following normal equations as

$$\frac{\partial \log L}{\partial\theta} = \frac{4n}{\theta} - n\left(\frac{\left(4\theta^3 + 8\alpha\theta\right)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)} \right) + \sum_{i=1}^n \left(\frac{(2\theta + 2\alpha x_i)}{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)} \right) - \sum_{i=1}^n x_i = 0$$

$$\frac{\partial \log L}{\partial \alpha} = -n\left(\frac{(4\theta^2 + 12\alpha)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)}\right) + \sum_{i=1}^{n}\left(\frac{(2\theta x_i + 2\alpha x_i^2)}{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)}\right) = 0$$

It is important to remember that algebra cannot easily solve the likelihood equations indicated above due to their complexity. For this reason, we use numerical methods such as the Newton-Raphson method to estimate the required parameters of the proposed" distribution. To obtain the confidence interval, apply the asymptotic normality results. We have $\hat{\lambda} = (\hat{\theta}, \hat{\alpha})$ denotes the MLE *of* $\lambda = (\theta, \alpha)$. We can determine the results as

$$\sqrt{n}(\hat{\lambda} - \lambda) \to N_2(0, I^{-1}(\lambda))$$

Where $I^{-1}(\lambda)$ is Fisher's information matrix.i.e., $I(\lambda) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta\, \partial \alpha}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha\, \partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) \end{pmatrix}$

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{4n}{\theta^2} - n\left(\frac{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)(12\theta^2 + 8\alpha) - (4\theta^3 + 8\alpha\theta)(4\theta^3 + 8\alpha\theta)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)^2}\right)$$

$$+ \sum_{i=1}^{n}\left(\frac{\left((\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)(2) - (2\theta + 2\alpha x_i)(2\theta + 2\alpha x_i)\right)}{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -n\left(\frac{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)(12) - (4\theta^2 + 12\alpha)(4\theta^2 + 12\alpha)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)^2}\right)$$

$$+ \sum_{i=1}^{n}\left(\frac{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)(2x_i^2) - (2\theta x_i + 2\alpha x_i^2)(2\theta x_i + 2\alpha x_i^2)}{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)^2}\right)$$

$$E\left(\frac{\partial^2 \log L}{\partial \theta\, \partial \alpha}\right) = -n\left(\frac{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)(8\theta) - (4\theta^3 + 8\alpha\theta)(4\theta^2 + 12\alpha)}{(\theta^4 + 4\alpha\theta^2 + 6\alpha^2)^2}\right)$$

$$+ \sum_{i=1}^{n}\left(\frac{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)(2x_i) - (2\theta + 2\alpha x_i)(2\theta x_i + 2\alpha x_i^2)}{(\theta^2 + 2\theta\alpha x_i + \alpha^2 x_i^2)^2}\right)$$

Since λ being unknown,we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and this can be used to obtain asymptotic confidence intervalsfor θ and α.

3.6 Simulation Analysis

The simulated data from the PDF of LBMAD is analyzed. The descriptive Statistics of the same is shown at Table 1.

Table 1. Descriptive Statistics of the simulated data with MLE of θ = 0.0.001 and α=0.2099

Mean	8.56111	Range	19
Standard Error	0.393762	Minimum	0.9
Median	6.9	Maximum	19.9
Mode	2.1	Sum	1513.1
Standard Deviation	5.282872	Count	5000
Sample Variance	27.90873	Largest(1)	19.9
Kurtosis	-0.96724	Smallest(1)	0.9
Skewness	0.440727	Confidence Level(95.0%)	0.777013

Table 1. Shows a clear positive skewness of LBMAD. Hence some new distributions are applied to check the goodness of fit.

4. APPLICATION

This section examines the goodness of fit of a real-world data set in LBMAD and compares it to New Quasi-Aradhana(NQA), quasi-Garima, and quasi-Akash distributions.

The real lifetime data, from 180 randomly selected patients from a hospital at Chennai, which shows the mean reduction (mg/dL) of low-density lipoproteins (LDL) after taking the particular diet and physical exercise as prescribed by their consultant dietician for four weeks continuously.(Table 2).

Table 2: The mean reduction (mg/dL) of low-density lipoproteins (LDL)

1.5	1.3	3.3	4.7	6.2	18.2	8.6	11.2	19	2.6	4.2	5.3
1.8	1.3	3.3	4.7	6.2	11	8.6	11.2	19.9	2.7	4.3	5.5
1.8	1.5	3.5	4.7	6.2	7.1	8.8	11.2	19.9	2.7	4.3	5.5
1.9	0.9	3.1	4.4	6.1	11.5	8.6	11	18.4	1.9	4.1	4.9
13	0.9	3.1	4.4	6.1	11.9	8.6	11	18.9	2.1	4.2	5
13	0.9	3.2	4.6	6.2	11.9	8.6	11.1	18.9	2.1	4.2	5
1.5	0.9	3.2	4.6	6.2	12.4	8.6	11.1	19	2.6	4.2	5.3
12.5	1.5	3.5	4.7	6.2	12.4	8.8	11.2	13.1	2.9	4.3	5.7
12.5	1.8	3.6	4.8	6.3	13.7	8.8	11.5	18.1	2.9	4.3	5.7
12.9	1.8	3.6	4.8	6.3	7.4	8.8	11.5	13.1	13.9	4.4	5.7
12.9	1.9	4	4.9	6.7	7.4	8.9	11.9	13.3	14.1	4.4	5.7
1.9	1.9	4	4.9	6.7	7.4	8.9	11.9	13.3	14.1	15.4	17.3
13	1.9	4.1	4.9	6.7	7.4	8.9	12.4	13.6	15.4	17.3	18.1
13	15.4	17.3	18.2	11	7.6	13.7	12.4	13.6	15.4	17.3	18.2
1.5	13	15.4	17.3	17.3	13.9	13.9	13.7	12.4	13.6	15.4	11

Here the sample mean = 8.4061, mode = 1.9 and median = 6.9, hence the data is non-symmetric, positively skewed.

To calculate the unknown parameters and determine the values of the model comparison criterion, R software is utilized. We used criteria values such as AIC (Akaike Information Criterion), BIC (Bayesian Information Criterion), AICC (Akaike Information Criterion Corrected), and -2logL to compare the LBMAD over new quasi-Aradhana, Garima, and Akash distributions. The distribution is better if its corresponding criterion values such as *AIC*, *BIC*, *AICC* and *-2logL* are lesser over compared distributions. The given formulas are applied for determining the criterion values.

$$AIC = 2k - 2\log L, \qquad BIC = k\log n - 2\log L \quad \text{and} \quad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

Where–2log$L$ is maximized value of log-likelihood function under considered model, n is the sample size and k is the number of parameters in the statistical model.

Table 3: MLE, S.E, Criterions (AIC, BIC, AICC, -2logL)

Distribution	MLE	S.E	-2logL	AIC	BIC	AICC
LBMAD	$\hat{\alpha} = 0.0010$ $\hat{\theta} = 0.2114$	$\hat{\alpha} = 0.0064$ $\hat{\theta} = 0.0533$	634.61	638.61	643.82	638.73
NQA	$\hat{\alpha} = 0.1443$ $\hat{\theta} = 0.2596$	$\hat{\alpha} = 0.0950$ $\hat{\theta} = 0.0247$	637.50	641.50	646.71	641.63
Garima	$\hat{\theta} = 0.1515$	$\hat{\theta} = 0.0130$	649.63	651.63	654.24	651.67
Akash	$\hat{\theta} = 0.2952$	$\hat{\theta} = 0.0168$	641.92	643.92	646.53	643.97

From table 3 given above, it is quite clear from the results that the LBMAD has the lesser *AIC*, *BIC*, *AICC* and *-2logL* values as compared to the NQA, Garima and Akash distributions, which implies that the LBMAD leads to a better fit over NQA, Garima and Akash distributions. Hence, it can be revealed and explored thoroughly that the LBMAD provides a quite satisfactory results over new quasi-Aradhana, Garima and Akash distributions.

5. CONCLUSION

In the present article, we have developed a novel class of a modified Aradhana distribution termed as LBMAD. By employing the length biased technique to its baseline distribution, we have introduced a new distribution which has different characteristics and properties. After a thorough analysis, the proposed new distribution was presented with various structural properties. These included the shape of the pdf and cdf, moments, mean and variance, hazard rate function, survival function, moment generating function, additionally, the maximum likelihood estimation technique has been used to estimate the parameters of the new distribution. Finally, the practical applicability and superiority of the LBMAD has been examined with real life data set by comparing its fit over other well-known distributions like NQA, Garima and Akash distributions. The distribution of the mean reduction (mg/dL) of low-density lipoproteins (LDL) is studied and its characterisation is derived with respect to some probability distributions. Hence, it has been realized from the results that the LBMAD performs quite well over NQA, Garima and Akash distributions.

Author'scontribution: All authors equally contributed

Funding: This study was not supported by grants, funds or any other kind of support from any organization in the public, commercial, or not-for-profit sectors.

Ethical statement: No ethical approval is required for the study.

Conflict of interest: The authors have no competing interests relevant to this manuscript.

REFERENCES

1. Ahmad, A. and Tripathi, R. (2021). Power size-biased Maxwell distribution with applications related to engineering data. *International Journal of Statistics and Applied Mathematics*, 6(6), 137-147.
2. Akanbi, O. B. and Oyebanjo, A. K. (2021). Length biased Gumbel distribution and its application to wind speed data. *Mathematical Theory and Modeling*, 11(5), 14-39.
3. Bodhisuwan, W., Nanuwong, N. and Pudprommarat, C. (2016). Parameter estimation for the length biased Beta-Pareto distribution and application. *Walailak J Sci & Tech*, 13(5), 301-315.
4. Cox, D. R. (1969), Some sampling problems in technology, In New Development in Survey Sampling, Johnson, N. L. and Smith, H., Jr .(eds.) New York Wiley-*Interscience*, 506-527.
5. Cox, D. R. (1962). Renewal theory. Barnes and Noble, New York, USA.
6. Fisher, R. A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. *Ann. Eugenics*, 6, 13-25.
7. Ganaie, R. A. and Rajagopalan, V. (2021). A new extension of power quasi Lindley distribution with properties and applications of the life time data. *International Journal of Statistics and Reliability Engineering*, 8(1), 171-183.
8. Ghitany, M. E., Atieh, B. and Nadarajah, S. (2008). Lindley distribution and its applications. *Mathematics Computing and Simulation*, 78(4), 493-506.

9. Kousar, F. and Memon, A. Z. (2017). A study of length biased weighted Nakagami distribution. *International Journal of Mathematics and Statistics*, 18(1), 42-59.
10. Rao, C. R. (1965), On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete, G.P. Patiled; Pergamum Press and Statistical publishing Society,Calcutta.320-332.
11. Salama, M. W., Elsherpieny, E. A. and A. Abd-elaziz, A. E. (2023). The length-biased weighted exponentiated inverted exponential distribution: Properties and estimation. *Computational Journal of Mathematical and Statistical Sciences*, 2(2), 181-196.
12. Shanker, R. and Shukla, K. K. (2017). Size-biased poisson-garima distribution with applications. *Biometrics & Biostatistics International Journal*, 6(3), 335-340.
13. Shanker, R., Soni, N. K., Shanker, R. and Prodhani, H. R. (2023). A new quasi Aradhana distribution with properties and applications. *Journal of Xidian University*, 17(11), 472-493.
14. Wani, S. A., Hassan, A., Sheikh, B. A. and Dar, S. A. (2022). Size biased Lindley-quasi Xgamma distribution applicable to survival times. *Thailand Statistician*, 20(2), 452-474.
15. Zelen, M. (1974). Problems in cell kinetic and the early detection of disease, in Reliability and Biometry, F. Proschan & R. J. Sering, eds, *SIAM*, *Philadelphia*, 701-706