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Recursive Pole Clustering Technique for Model Order Reduction using Improved Pade Approximations

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AbstractThis paper introduced a novel order reduction method for large- scale linear dynamic single-input and single-output (SISO) systems to the lower order models. The coefficients of the reduced numerator are determined by using the Improved Pade approximation approach, while the reduced order denominator polynomial has been computed by recursive pole clustering technique. The proposed method assures the stability of the reduced order model, if the original model of high order is stable. The efficacy of the proposed method tested on few systems taken from the literature. The graphical and analytical comparison has been done with the help of MATLAB/Simulink. ***Keywords:*** Recursive Pole Array, Stability, Order Reduction, Improved Pade approximations,

Dynamic systems.

Introduction

In the field of science and engineering, modelling of linear and nonlinear complex dynamic systems has a significant role in analysis and design of the control systems. The mathematical modelling of the complex dynamic systems consist of high order differential equations and simultaneous equations, which results a complex model in the terms of either a transfer function or a state model. These equations are not easy to use to analyze and synthesize the control systems. In order to have a better understanding of the system, and to reduce the complexity in terms of hardware and computations, it is very often necessary to convert these high-order equations into low-order equations, which reflect the same characteristics as that of the system under consideration.

Previously, many order reduction methods [1-5] have been suggested in the literature for order reduction of the frequency and time domain linear dynamic systems. Further, many researchers have suggested mixed reduction methods which retains the features of two different reduction methods [6-12]. Sinha and Pal [3] suggested clustering techniques based on the Inverse Distance Measure (IDM) criterion, in which separate clusters of the poles and zeros are formed to get a cluster centre. Further, Vishwakarma [13] suggested a modified Pole cluster technique based on IDM criterion, which generates more dominant cluster centres as compared to other methods [3]. Abha [14], in her work suggested a method to get a better cluster centre, if the poles of the original systems are properly grouped to form the clusters.

The proposed method based on recursive pole clustering technique is a mixed method to reduce the high-order linear system into the lower order models. In this method, the denominator polynomial of the reduced model is determined by the proposed recursive clustering technique, in which the required number of pole clusters are formed and their effective centres are computed. The numerator coefficients are obtained by using improved Pade Approximations. The graphical and analytical comparison of the proposed method with the other well-known reduction methods is presented with the help of the examples taken from the literature. The step responses are plotted for comparison in time domain while the Bode /Nyquist diagrams are plotted for comparison in the frequency domain.

1. Formulation of the Problem

2.1 Single-Input Single-Output (SISO) System

Consider the n^{th} – order original linear dynamic system represented by its transfer function as

$$G(s) = \frac{N(s)}{D(s)} = \frac{e_o + e_1 s + e_2 s^2 + \ldots + e_{n-1} s^{n-1}}{f_o + f_1 s + f_2 s^2 + \ldots + f_n s^n} \tag{1}$$

Let the k^{th} – order reduced model of the original dynamic system $G(s)$ be

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{x_o + x_1 s + x_2 s^2 + \ldots + x_{k-1} s^{k-1}}{y_o + y_1 s + y_2 s^2 + \ldots + y_k s^k} \tag{2}$$

The purpose of the proposed method is to realize the k^{th} – order reduced model $R_k(s)$ from the original system $G(s)$, such that it retains the important characteristics of the original system.

2.2 Multiple-Inputs Multiple-Outputs (MIMO) Systems

Consider a large scale MIMO system with $'p'$ number of inputs and $'q'$ number of outputs represented by its transfer function matrix as

$$[G(s)] = \frac{1}{D_n(s)} \begin{bmatrix} a_{11}(s) & a_{12}(s) & \dots & a_{1p}(s) \\ a_{21}(s) & a_{22}(s) & \dots & a_{2p}(s) \\ \dots & \dots & \dots & \dots \\ a_{q1}(s) & a_{q2}(s) & \dots & a_{qp}(s) \end{bmatrix} \qquad (3)$$

Where $D_n(s) = f_o + f_1 s + f_2 s^2 + \dots + f_n s^n$ is the common denominator of the original system.

The general form of the original transfer function matrix (3) can be written as

$[G(s)] = [g_{ij}(s)]$ for $i = 1, 2, 3, \dots, q$ and $j = 1, 2, 3, \dots, p$

$$\text{Where} \quad g_{ij}(s) = \frac{a_{ij}(s)}{D_n(s)} = \frac{e_o + e_1 s + e_2 s^2 + \dots + e_{n-1} s^{n-1}}{D_n(s)} \qquad (4)$$

Let the k^{th} – order reduced MIMO model of the original system (3) with the same number of inputs and outputs, be represented as

$$[R_k(s)] = \frac{1}{D_k(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) & \dots & b_{1p}(s) \\ b_{21}(s) & b_{22}(s) & \dots & b_{2p}(s) \\ \dots & \dots & \dots & \dots \\ b_{q1}(s) & b_{q2}(s) & \dots & b_{qp}(s) \end{bmatrix} \qquad (5)$$

The general form of the reduced model of the original system (3) can be written as

$[R_k(s)] = [h_{ij}(s)]$ for $i = 1, 2, 3, \dots, q$ and $j = 1, 2, 3, \dots, p$

$$\text{Where} \quad h_{ij}(s) = \frac{b_{ij}(s)}{D_k(s)} = \frac{x_o + x_1 s + x_2 s^2 + \dots + x_{k-1} s^{k-1}}{D_k(s)} \qquad (6)$$

The objective is to determine the k^{th} – order reduced MIMO model of the original system (3), such that it contains the important features of the original system.

3. Detailed description of the proposed method

The mathematical procedure of the proposed method consists of the following two steps:

Step-1: **Determination of the k^{th} – order reduced denominator**

Pole clusters are formed from the original high order system, separately for real and imaginary poles. The poles on the imaginary axis shall be retained in the reduced models for marginally stable system.

Case-1: Let 'z' number be the poles in a cluster containing with real poles, like $(p_1, p_2, ..., p_z)$, then computer oriented algorithm can be written as

Step-1a: Set $i = 1$

Step-2b: Compute the pole cluster centre from the poles { $|p_1| < |p_2| < ...|p_z|$ }

$$p_{ci} = -\left[\left(\sum_{i=1}^{z}\sqrt{|p_i|}\right) \div z\right]^2$$

Step-3c: Counter increment as $i = i + 1$

Step-4c: Compute pole cluster centre as $p_{ci} = -\left[\left(\sqrt{|p_{c(i-1)}|} + \sqrt{|P_1|}\right) \div 2\right]^2$

Step-5d: If $i = z + 1$, Then, final effective pole cluster centre is $p_{ei} = p_{ci}$, otherwise go to *Step-3c*

Case-2: Consider a pole cluster having only complex conjugate poles such as $[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_2), ..., (\alpha_m \pm j\beta_m)]$ then the effective complex cluster centre ($A_c \pm jB_c$) can be determined by the same algorithm as discussed in Step-1, separately for real and imaginary parts of the poles.

Where, $\overset{*}{p}_e = A_c + jB_c$ and $\overset{\bullet}{p}_e = A_c - jB_c$

After getting the effective cluster centres, the reduced denominator of the order 'k' can be obtained by one of the following cases [13] as:

Case-(a): The reduced k^{th}-order denominator can be obtained, if all pole cluster centre are real as:

$$D_k(s) = (s - p_{e1})(s - p_{e2})...(s - p_{ek}) \tag{7}$$

Where $p_{e1}, p_{e2}, ...p_{ek}$ are effective pole cluster centres of the 1^{st}, 2^{nd}, ... k^{th} Cluster.

Case-(b): If one cluster centre pairs are complex conjugate and $(k-2)$ cluster centres are real, then $D_k(s)$ can be obtained as:

$$D_k(s) = (s - p_{e1})(s - p_{e2})...(s - p_{e(k-2)})(s - \overset{*}{p}_{e1})(s - \overset{\bullet}{p}_{e1}) \tag{8}$$

Case-(c): If all the pole cluster centres are complex conjugate, then $D_k(s)$ can be obtained as:

$$D_k(s) = (s - \overset{*}{p}_{e1})(s - \overset{\bullet}{p}_{e1})(s - \overset{*}{p}_{e2})(s - \overset{\bullet}{p}_{e2})...(s - \overset{*}{p}_{ek/2})(s - \overset{\bullet}{p}_{ek/2}) \tag{9}$$

Step-2: Determination of the numerator of the reduced model using improved Pade approximation [10]

The original high order system (1) can be expanded in power series as

$$\left.\begin{aligned} G(s) &= \sum_{i=0}^{\infty} M_i s^{-i-1} \qquad (\text{About } s = \infty) \\ &= -\sum_{i=0}^{\infty} T_i s^i \qquad (\text{About } s = 0) \end{aligned}\right\} \tag{10}$$

Where, $'M_i'$ and $'T_i'$ are i^{th} – markov parameter and time moment respectively of the original system.

The numerator coefficients of the reduced order model can be obtained by using the following equations:

$$\left.\begin{aligned} x_0 &= y_0 T_0 \\ x_1 &= y_0 T_1 + y_1 T_0 \\ x_2 &= y_0 T_2 + y_1 T_1 + y_2 T_0 \\ &\cdots \\ &\cdots \\ x_{\alpha-1} &= y_0 T_{\alpha-1} + y_1 T_{\alpha-2} + ... + y_{\alpha-2} T_1 + y_{\alpha-1} T_0 \\ x_{\alpha-\beta} &= y_k M_{\beta-1} + y_{k-1} M_{\beta-2} + ... + y_{k-\beta+2} M_1 + y_{k-\beta+1} M_0 \\ &\cdots \\ &\cdots \\ x_{k-2} &= y_k M_1 + y_{k-1} M_0 \\ x_{k-1} &= y_k M_0 \end{aligned}\right\} \tag{11}$$

Here, $k = \alpha + \beta$, $\alpha =$ Number of Time moments, β=Number of markov parameters

The above $(k-1)$ equations can be solved to get the coefficients of the reduced numerator $N_k(s)$.

2. Methods for Performance Comparison

To check the goodness of the reduced order models obtained by the proposed method the following methods have been used in this paper.

- *Graphical Methods*
- *Analytical Methods*

In graphical method of comparison, time response (step) and frequency response (Bode diagram, Nyquist plot) comparison between the original system and the reduced model is carried out with the help of MATLAB/simulink.

In analytical method, the following performance indices have been computed with the help of Matlab/Simulink.

$$\text{Integral Square Error (ISE)} = \int_0^\infty [g(t) - r(t)]^2 \, dt \quad (12)$$

$$\text{Integral Amplitude of Error (IAE)} = \int_0^\infty |g(t) - r(t)| \, dt \quad (13)$$

$$\text{Relative Integral Square Error (RISE)} = \frac{\int_0^\infty [g(t) - r(t)]^2 \, dt}{\int_0^\infty [g(t) - r(\infty)]^2 \, dt} \quad (14)$$

Where, $g(t)$ and $r(t)$ are the unit step response of the original high order and reduced order models respectively and $r(\infty)$ is the steady-state value of the reduced model.

4. Validation, Results and Analysis

Example-1: Consider the 6th –order system [15] described by the following transfer function as

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.9s^4 + 209.5s^3 + 102.4s^2 + 18.3s + 1}$$

The poles are: -0.1, -0.2, -0.5, -1, -5, -10

Using the Step-1 of the proposed method, the following two clusters are formed and their effective cluster centre are found as

Pole Cluster	Effective cluster centre
(-0.1, -0.2, -0.5)	$p_{e1} = 0.1142$
(-1, -5, -10)	$p_{e2} = 1.303$

Therefore, second order denominator of the reduced model is obtained as

$$D_2(s) = (s+0.1142)(s+1.303)$$
$$= s^2 + 1.4172s + 0.1488$$

Few initial time moments and markov parameters are computed from the original system and given as

$$T_0 = 1.0$$
$$T_1 = -10.298$$
$$M_0 = 1.0$$
$$M_1 = -15.3$$

Here, two reduced models are possible, which also known as biased models according to combination of time moments and markov parameters as

$R_k(s)$	$k = \alpha + \beta$	Remarks
$k = 2$	$k = 2 + 0$	Two initial time moments are retained
$k = 2$	$k = 1 + 1$	One initial markov parameter and one initial time moment are retained

$$R_{k=2+0}(s) = \frac{-0.1151s + 0.1488}{s^2 + 1.4172s + 01488}$$

$$R_{k=1+1}(s) = \frac{s + 0.1488}{s^2 + 1.4172s + 01488}$$

The step response comparison of the reduced models with the original system is shown in Fig.1. The step response of the second order reduced model $R_{k=2+0}(s)$ is shown by red colour while the response of $R_{k=1+1}(s)$ is shown by blue colour. It may be seen that the reduced model with red colour is better in quality.

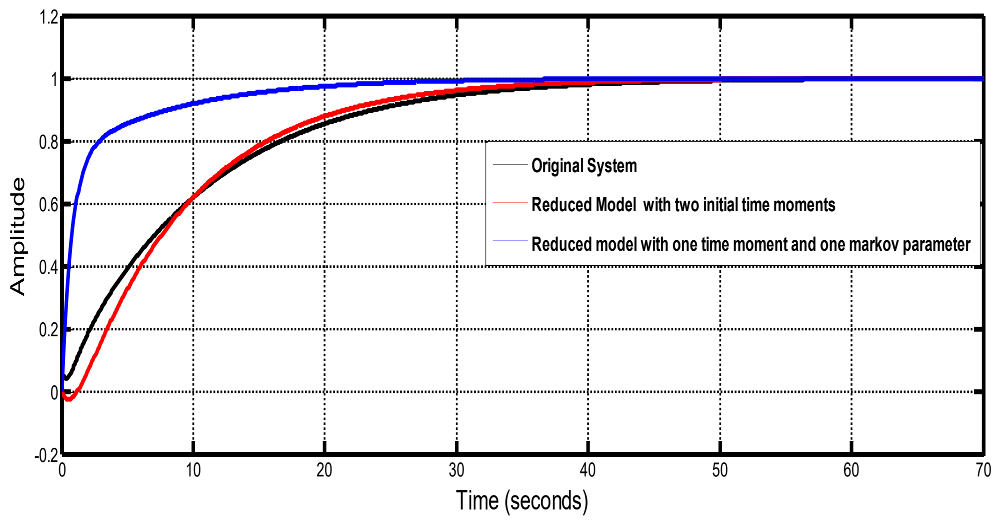

Fig.1: Step response comparison with the original system

The ISE, IAE and RISE indices are computed from the simulink model. The reduced models are compared tabulated in Table-I.

Table-I: Qualitative comparison for example-1

Reduction Methods	Reduced Model	ISE	IAE	RISE
Proposed Method	$R_{k=2+0}(s)=\frac{-0.1151s+0.1488}{s^2+1.4172s+01488}$	0.05102	0.5982	0.0117
	$R_{k=1+1}(s)=\frac{s+0.1488}{s^2+1.4172s+01488}$	1.933	4.274	0.4434

Example-2: Consider a 8th -order large-scale system taken from literature [14].

$$G(s)=\frac{N(s)}{D(s)}$$

$$N(s)=18s^7+514s^6+5982s^5+36380s^4+122664s^3$$
$$+222088s^2+185760s+40320$$

$$D(s)=s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3$$
$$+118124s^2+109584s+40320$$

The poles are: -1,-2,-3,-4,-5,-6,-7,-8

First two Time moments $T_o=1,\quad T_1=1.889$

First Markov parameter $M_o=18$

For finding 2nd order reduced model, two possible pole clusters are made as

Cluster-1: (-1,-2,-3) gives effective cluster centre $p_{e1}=1.097$

Cluster-2: (-4,-5,-6,-7,-8) gives effective cluster centre $p_{e2}=4.109$

Hence, 2nd -order reduced denominator is obtained as

$D_2(s)=s^2+5.206s+4.508$

$$R_{k=2+0}(s)=\frac{13.722s+4.508}{s^2+5.206s+4.508}$$

$$R_{k=1+1}(s)=\frac{18s+4.508}{s^2+5.206s+4.508}$$

Hence, reduced model $R_{k=2+0}(s)$ may be chosen for further analysis and design of the system because it is the best model.

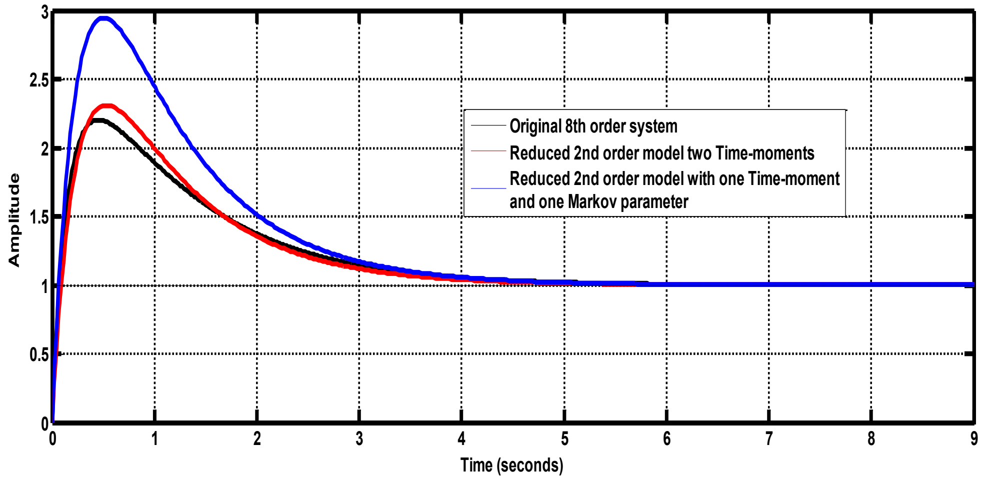

Fig.2: Step responses comparison with the original 8th order system

Table-II: Qualitative comparison for example-2

Reduction Methods	Reduced Model	ISE	IAE	RISE
Proposed Method	$R_{k=2+0}(s)=\frac{13.722s+4.508}{s^2+5.206s+4.508}$	**0.01831**	**0.2136**	**0.01249**
	$R_{k=1+1}(s)=\frac{18s+4.508}{s^2+5.206s+4.508}$	0.4792	0.9548	0.3269
Pal J.[16]	$R_2(s)=\frac{40320+151776.576s}{40320+75600s+65520s^2}$	1.6509	2.385	1.127
Mittal *et al.*[2]	$R_2(s)=\frac{1.9906+7.0908s}{2+3s+s^2}$	0.2689	0.8099	0.1834
Lucas [17]	$R_2(s)=\frac{2+6.7786s}{2+3s+s^2}$	0.2792	0.7628	0.1905
Prasad and Pal [18]	$R_2(s)=\frac{500+17.98561s}{500+13.24571s+s^2}$	1.4584	1.926	0.9948
Abha Thesis [15]	$R_2(s)=\frac{6.5688+18.6703s}{6.5688+6.2600s+s^2}$	0.08887	0.5001	0.06061
Hutton, Friedland [19]	$R_2(s)=\frac{0.43184+1.98955s}{0.43184+1.1736s+s^2}$	1.932	2.722	1.318
Krishnamurthy, *et al.*[20]	$R_2(s)=\frac{40320+155658.6152s}{40320+75600s+65520s^2}$	1.655	2.424	1.129

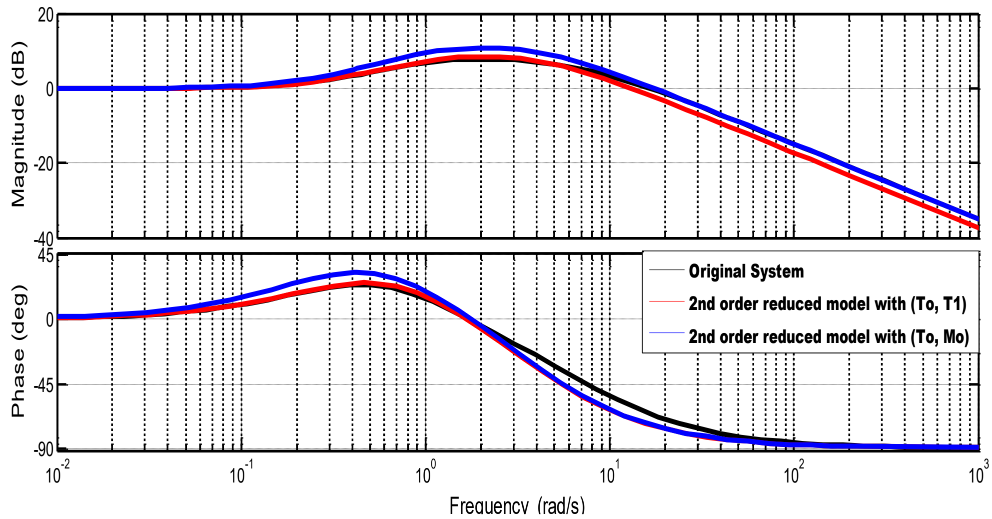

Fig.3: Frequency domain comparison using Bode plot

From the Fig.2, It is very much clear that red response is closely matching with the original response. Hence, it may be chosen for analysis of the original system. As we know that the red response represents the second order reduced model with two initial time moments of the original system. In frequency response comparison shown in Fig. 3, it is seen that reduced model $R_{k=2+0}(s)$ is closely matching the original system in the both magnitude as well as phase plots.

Example-3: Consider a MIMO 4th-order system [15] as shown below:

$$G(s)=\begin{bmatrix}\dfrac{s+20}{(s+1)(s+10)}\\ \dfrac{s+10}{(s+2)(s+5)}\end{bmatrix}=\begin{bmatrix}g_{11}(s)\\ g_{21}(s)\end{bmatrix}$$

$$=\frac{1}{D_4(s)\ =\ 100\ +\ 180s\ +\ 97s^2+18s^3+\ s^4}\begin{bmatrix}a_{11}(s)\\ a_{21}(s)\end{bmatrix}$$

Where, $D_4(s) = 100 + 180s + 97s^2 + 18s^3 + s^4$

$a_{11}(s) = 200 + 150s + 27s^2 + s^3$

$a_{21}(s) = 100 + 120s + 21s^2 + s^3$

Here, $D_4(s)$ is the common part of the denominator of polynomial.

The poles are at: -1, -2, -5, -10

Consider two pole clusters as

Cluster-1: (-1,-2) and Cluster-2: (-5, -10)

Effective cluster centres are obtained using the proposed algorithm as

$p_{e1} = 1.218$ and $p_{e2} = 6.089$

Hence, reduced denominator is obtained as

$$D_2(s) = s^2 + 7.307s + 7.416$$

Finally, using the proposed method, following reduced models are obtained as

With $T_o = 2, T_1 = -2.1$	With $T_o = 1, T_1 = -0.6$	With $T_o = 2, M_o = 1$	With $T_o = 1, M_o = 1$
$R_2(s) = \frac{1}{s^2 + 7.307s + 7.416}\begin{bmatrix} a_{11}(s) \\ a_{21}(s) \end{bmatrix},$			
$a_{11}(s) = 14.832 - 0.9596s$	$a_{21}(s) = 7.416 + 2.857s$	$a_{11}(s) = 14.832 + s$	$a_{21}(s) = 7.416 + 2.857s$

Therefore, two possible reduced models are obtained as

$$R_{k=2+0}(s) = \frac{1}{s^2 + 7.307s + 7.416}\begin{bmatrix} 14.832 - 0.9596s \\ 7.416 + 2.857s \end{bmatrix} = \begin{bmatrix} r_{11}(s) \\ r_{21}(s) \end{bmatrix}$$

$$R_{k=1+1}(s) = \frac{1}{s^2 + 7.307s + 7.416}\begin{bmatrix} 14.832 + s \\ 7.416 + s \end{bmatrix} = \begin{bmatrix} r_{11}(s) \\ r_{21}(s) \end{bmatrix}$$

The step responses of the original and reduced models are plotted and shown in Fig.4, from this figure, it may be concluded that the reduced model's performance is quite better. Here, two reduced models are synthesized from the proposed method as $R_{k=2+0}(s)$ and $R_{k=1+1}(s)$. The performance indices are computed from the simulink model/ Matlab is tabulated and compared with the other well-known methods.

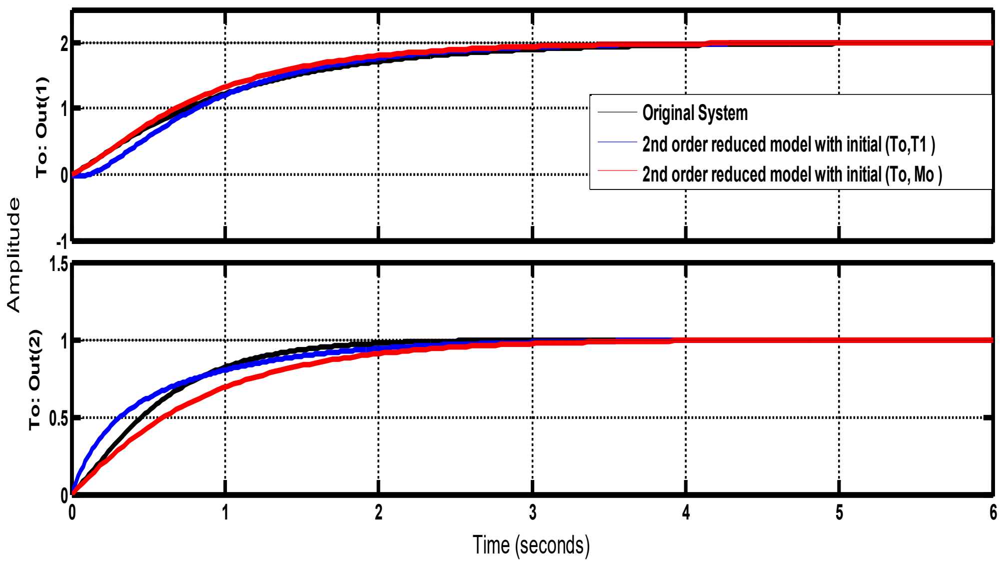

Fig.4: Step response comparion with the original system

Table-III: Qualitative comparison for example-3

Reduction Method	$k=\alpha+\beta$	**Reduced model elements**	**ISE**	**IAE**	**RISE**
Proposed method	$k=2+0$	$r_{11}(s)$	0.02258	0.2457	0.01033
		$r_{21}(s)$	0.0103	0.1459	0.03136
	$k=1+1$	$r_{11}(s)$	0.01865	0.2646	0.008532
		$r_{21}(s)$	0.02234	0.2505	0.06799
Abha [15] Thesis		$r_{11}(s)$	0.05398	0.368	0.02468
		$r_{21}(s)$	0.007383	0.1209	0.02246

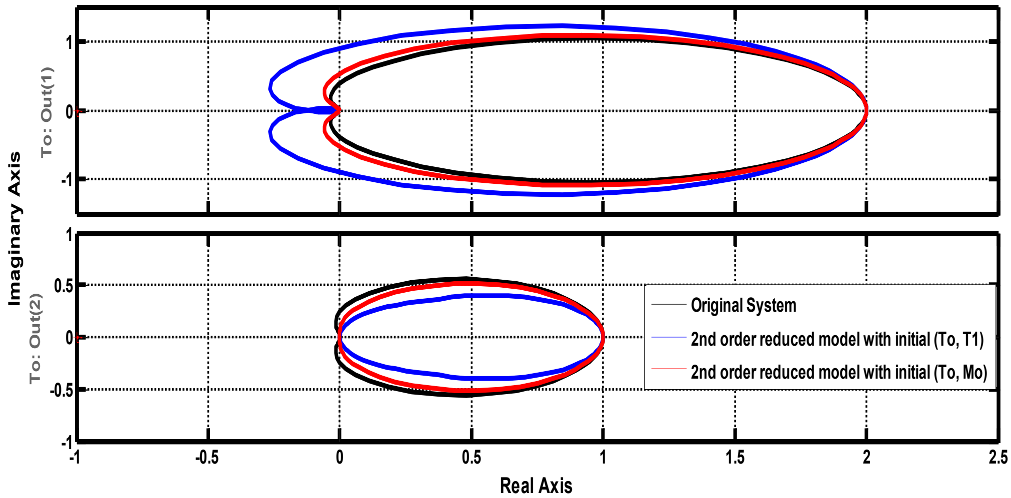

Fig.5: Nyquist plot comparison of the reduced models with original system

The Nyquist plot (freqeumcy response) compariosn of the reduced models with the original system has been shown in Fig.5, to check the performance in the frequency domain. It may be concluded the perfomance of the reduced model $R_{k=1+1}(s)$ in frequeny domain compariosn in quite better. Hence, the major advantage of the proposed method is to provide an option to choose a better model.

5. Conclusion

In this paper, the authors proposed a new order reduction method based on the recursive pole clustering technique and improved Pade approximations. The suggested method is computationally simple and conceptually easy to understand. The Inverse Distance measure (IDM) criterion has been modified to get efficient pole cluster centres. Then, an improved Pade approximation has been applied to get the reduced numerator coefficients. The algorithmic view/flow chart and step by step procedure has been incorporated in the paper to understand the mathematical procedure of the proposed method. Three numerical problems have been taken from the literature to check the effectiveness of the proposed method. The major advantage of the improved Pade approximation method is to create $'k'-$number of models for the $k^{th}-$order reduction. Therefore, the best one can be chosen for further analysis and design of the system. To show efficacy of the proposed method, two SISO and one MIMO systems are considered and reduced to second order models. The reduced order models are compared analytically and graphically such as step response and Bode plot/Nyquist plot which are shown in various figures and ISE, IAE and RISE indices are computed with the help of Simulink/Matlab. The following are the merits of the proposed methods:

- Retention of stability, if the original system is stable.
- Satisfactory approximation in transient and steady-state region.
- Simple computer oriented mathematical procedure.
- Applicable for linear unstable systems as well.

The proposed method has the following scope for future work:

- Scope to get better approximation
- Clustering technique may be implemented for nonlinear system

Conflicts of Interest

The authors declare that they have no conflicts of interest.

6. References

[1] S. Mukherjee. Order reduction of linear systems using eigen spectrum analysis. Journal of Electrical Engineering. 1996;77; 76-79.

[2] A. K. Mittal, R. Prasad and S.P. Sharma. Reduction of linear dynamic systems using an error minimization technique. J. Inst. Engg. India (EL*)*. March 2004; 84; 201– 206.

[3] J. Pal, A.K. Sinha and N.K. Sinha. Reduced order modelling using pole clustering and time moments matching. J. Inst. Eng. India (EL*)*. 1995; 76;1-6.

[4] C.B. Vishwakarma, R. Prasad. Time domain model order reduction using Hankel matrix approach. Journal of Franklin Institute. June 2004; 351(6); 3445-3456.

[5] Therapos C. P. Balanced Minimal Realization of SISO Systems. Electronics Letters. 1983; 19(11); 424-426.

[6] R. Prasad, A.K. Pant et al. Multivariable systems approximation using polynomial derivatives. Journal of Institution of Engineers. India, IE(I), Journal-EL. 1995; 76; 186-188.

[7] Y. Shamash. Linear system reduction using Pade approximations to allow retention of dominant poles. Int. J. control. 1975; 21(2); 257-272.

[8] A. K. Prajapati et al. Reduced order modelling of LTI systems by using Routh approximation and Fraction division methods. Journal of Circuits Systems and Signal Processing, Jan. 2019; 38(7):3340–3355.

[9] Sharad K. et al. Improved reduced order modelling using clustering method with dominant pole retention. IETE Journal of research, 2020; 66 (1), 42-52.

[10] J. Singh, C.B. Vishwakarma and K. Chatterjee. Biased reduction method by combing improved modified pole clustering and improved Pade approximations. Applied Mathematical Modelling. 2016; 40(2); 1418-1426.

[11] T. N. Lucas. Continued-traction expansion about two or more points: A flexible approach to linear system reduction. Journal of the Franklin Institute, 1986; 321(1); 49-60.

[12] M. Hutton et al. Routh approximations for reducing order of linear time invariant systems. IEEE Transaction on Automation Control. 1975; 20(3); 329-337.

[13] C.B. Vishwakarma, R. Prasad. MIMO System Reduction using modified pole clustering and genetic algorithm. Modelling and simulation in Engineering. 2009; 2009;1-9.

[14] Abha Kumari and C.B. Vishwakarma. Order abatement of Linear Dynamic Systems Using Renovated Pole Clustering and Cauer Second Form Techniques. Circuit, Systems and Signal Processing. 2021; 40(9); 4212-4229.

[15] Abha Kumari. Order Reduction of Linear Dynamic Systems for Controller Design. PhD Thesis, Gautam Buddha University Greater Noida. March 2022.

[16] J. Pal. Stable reduced order pade approximants using the Routh Hurwitz array. Electron. Lett. 1979;15(8); 225–226.

[17] Lucas T.N. Factor division: a useful algorithm in model reduction. Proc. IEE. 1983; 130(6);. 362–364.

[18] R. Prasad and J. Pal. Stable reduction of linear systems by continued fractions. J. Inst. Eng. IE. 1991; 72; 113–116.

[19] M. F. Hutton and B. Friedland. Routh Approximations for Reducing Order of Linear Time-Invariant Systems. IEEE Trans. Automat. Control. 1975; 20(3); 329–337.

[20] V. Krishnamurthy and V. Seshadri. Model Reduction Using the Routh Stability Criterion. IEEE Trans. Automat. Contr. 1978; 23(4); 729–731.

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