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# Dynamics of Prey- Predator Model with Holling type-II Response

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**ABSTRACT:** In this paper we investigate the dynamics of prey-predator model of holling type II response function. The system is described by a system of ordinary differential equations. The boundedness properties, long term behaviour of the system, equilibrium points are identified. Local stability analysis is discussed at each of its equilibrium points. Global stability is studied by constructing suitable Lyapunov's function. We proved that the system is both locally and globally asymptotically stable. Further Numerical simulation is performed and in support of analytical study.

**Keywords:** prey-predator, local stability, global stability, Simulation

**Mathematics Subject Classification:** 34DXX

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# 1. INTRODUCTION

Prey-predator model are building blocks of Eco system. It Is the significant relationship in ecology Many researchers' attention has been capture in this interaction. The Dynamics of prey-predator model are explored by Carlos [2], Freedman [4], Kot [6], Lakshmi Narayan [7], Lokta [8], May.R.M [11], Murry [12,13], lima [10], Ranjith Kumar [15] and Sita Rambabu [18]. The models with different holling time functional responses are also included in the study. Later harvesting of prey-predator models are also explored by [1,3,16]. The harvesting prey-predator models with holling type response of type1+kNi is included in the general prey-predator model with harvesting of Prey with different harvesting efforts considered for investigation. The dynamics of this relationship can help us to protect the diversity of species in large scale. An important component in this relation is functional response of the predator. The classical type of functional responses has the following forms (Holling types I, II, III &IV).

The basic holing types functional response are available in literature [14,17]. Authors [1,5,9] studied the dynamics of prey-predator model with the holling type functional responses I,II,III and IV with the Following basic model as

$$\frac{dx}{dt} = rx\left(1-\frac{x}{K}\right) - \varphi(x)y$$
$$\frac{dy}{dt} = ry\left(1-\frac{y}{K}\right) + +\epsilon\varphi(x)y \qquad (1.1)$$

Here $\varphi(x)$ can be defined as functional responses mentioned as I, II,III &IV type.

Liu,W[9] studied the Michaelis-Menten type harvesting in prey-predator model and bifurcation analysis. Xiao [19] analysed the prey-predator dynamics with constant harvesting rate. In spite of the above we proposed the with holling type response$1+kN_1$ in prey-predator model with logistics growth .We studied the dynamics of the model includes the boundedness properties, long term behaviour of the system, stability analysis at co-existing state. Finally, the analytical results are supported by numerical simulation..

## 2. Materials and Methods

### 2.1 FORMATION OF MODEL

The system of equations for the proposed model with holling type-II response function is taken for investigation. The system of equations for the proposed model is

$$\frac{dN_1}{dt} = a_1 N_1 [1 - \frac{N_1}{L_1}] - \frac{\alpha_{12} N_1 N_2}{1+kN_1}$$
$$\frac{dN_2}{dt} = a_2 N_2 [1 - \frac{N_2}{L_2}] + \frac{\alpha_{21} N_1 N_2}{1+kN_1} \qquad (2.1)$$

With initial conditions $N_1(0) = N_{10} > 0$ &$N_2(0) = N_{20} > 0$ (2.2)

**Nomenclature:**

| S.No. | Parameter | Description |
|---|---|---|
| 1 | $N_1, N_2$ | Populations of the prey and predator respectively |
| 2 | $a_1, a_2$ | Natural growth rates of prey and predator |
| 3 | $L_1, L_1$ | Carrying capacities of two species |
| 4 | K | Proportion constant |
| 5 | $\alpha_{12}$ | Rate of decrease of the prey due to inhibition by the predator |
| 6 | $\alpha_{21}$ | Rate of increase of the predator due to successful attacks on the prey |

**2.2. Positivity and Bounded ness of the Solutions.**

In this section we prove the positivity and bounded ness of the solutions of system of equations (2.1) along with the initial conditions (2.2). To prove the results, we use the following two lemmas.

**Lemma1:** if $a, b > 0$ and $\frac{dx}{dt} \leq (\geq) x(t)\big(a - bx(t)\big)$ with $x(0) > 0$ then

$$\log_{t\to\infty} sup\, x(t) \leq \frac{a}{b} \quad (\log_{t\to\infty} inf\, x(t) \geq \frac{a}{b})$$

**Lemma2:** if $a, b > 0$ and $\frac{dx}{dt} \leq x(t)\big(a - bx(t)\big)$ with $x(0) > 0$, then for all $t \geq 0$

$$x(t) \leq \frac{a}{b - ce^{-at}} \text{ with } c = b - \frac{a}{x(0)} \text{ in particular } x(t) \leq \max\{x(0), \frac{a}{b}\} \text{ for all } t \geq 0$$

**Theorem 2.2.1:** All the solutions $N_1(t)$&$N_2(t)$ of the system (2.1) with initial conditions (2.2) are positive i.e., $N_1(t) > 0$ &$N_2(t) > 0$

**Proof:** From the system of equations (2.1) the prey equation is given by

$\frac{dN_1}{dt} = a_1 N_1 [1 - \frac{N_1}{L_1}] - \frac{\alpha_{12} N_1 N_2}{1+kN_1}$ it follows that $N_1(t) = 0$ is an invariant set. This implies and $N_1(t) > 0$ for all $t \geq 0$. We apply similar argument for the predator equations $\frac{dN_2}{dt} =$

$a_2N_2[1-\frac{N_2}{L_2}]+\frac{\alpha_{21}N_1N_2}{1+kN_1}\ \ N_2(t)=0$ is an invariant set and hence $N_2(t)>0$ for all $t\geq 0$. Thus, all the trajectories $R_+^2$ cannot cross the co-ordinate axis. Hence all the solutions $N_1(t)\&N_2(t)$ are positive.

**Theorem 2.2.2:** All the solutions $N_1(t)\&N_2(t)$ of the system (2.1) with initial conditions (2.2) are bounded for all $t\geq 0$.

**Proof:** From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt}=a_1N_1[1-\frac{N_1}{L_1}]-\frac{\alpha_{12}N_1N_2}{1+kN_1}\leq a_1N_1[1-\frac{N_1}{L_1}]$$

using lemma 2 where both $a_1\&L_1>0$

And also $N_1(t)\leq\frac{a_1L_1}{a_1-cL_1e^{-a_1t}}$ and $c=\frac{a_1}{L_1}-\frac{a_1}{N_{10}}$

In particular $N_1(t)\leq\max\{N_{10},L_1\}=M_1$ for all $t\geq 0$. (2.2.2.1)

From the predator equation from (2.1) we have equations

$$\frac{dN_2}{dt}=a_2N_2[1-\frac{N_2}{L_2}]+\frac{\alpha_{21}N_1N_2}{1+kN_1}\leq N_2\left(a_2+\frac{\alpha_{21}N_1}{1+kN_1}-\frac{a_2N_2}{L_2}\right)$$ again, using lemma2

Which implies $N_2\left(a_2+\frac{\alpha_{21}N_1}{1+kN_1}-\frac{a_2N_2}{L_2}\right)\leq\ N_2\left(a_2+\frac{\alpha_{21}M_1}{1+kM_1}-\frac{a_2N_2}{L_2}\right)$

from equation (2.2.2.1)

In particular

$N_2(t)\leq\max\left\{N_{20},\frac{L_2a_2(1+kM_1)+L_2\alpha_{21}M_1}{a_2(1+kM_1)}\right\}=M_2$ for all $t\geq 0$. (2.2.2.3)

Hence the system (2.1) possesses bounded solutions.

**2.3. Permeance:**

The long-term behaviour of the dynamical system in particular the positive solutions of the system approach the boundary of the positive orthant. If the system includes two spices the positive solutions approach the boundary of positive quadrant (two-dimensional space) and for three species the positive solutions approach the boundary of positive octant (three-dimensional space).

The permeance of the system exist if there exist two positive constants $\eta_1\&\eta_2$ such that each positive solutions of $N_1(t)\&N_2(t)$ with initial conditions $N_{10}\&N_{20}$ in $R_+^2$ satisfies,

$\min\{\ \lim_{t\to\infty}inf\ N_1(t,N_{10},N_{20}),\lim_{t\to\infty}inf\ N_2(t,N_{10},N_{20})\}\geq\ \eta_1$ and

$\max\{\lim_{t\to\infty} sup\, N_1(t, N_{10}, N_{20}), \lim_{t\to\infty} sup\, N_2(t, N_{10}, N_{20})\} \le \eta_2$

**Theorem 2.3.1.** The system (2.1) with initial conditions (2.2) is permanent if

$$a_2\alpha_{22}(1+kM_1) > \alpha_{12}[a_2(1+kM_1)+\alpha_{12}M_1]$$

**Proof:** From the system of equations (2.1) the prey equation is given by

$$\frac{dN_1}{dt} = a_1N_1[1-\frac{N_1}{L_1}] - \frac{\alpha_{12}N_1N_2}{1+kN_1} \ge N_1(a_1 - \frac{a_1N_1}{L_1} - \alpha_{12}N_2) \ge N_1\left(a_1 - \frac{a_1N_1}{L_1} - \frac{\alpha_{12}L_2\{a_2(1+kM_1)+\alpha_{21}M_1\}}{a_2(1+kM_1)}\right)$$

using the bounded ness property from equation (2.2.2.3)

Hence $\frac{dN_1}{dt} \ge N_1\left(A_1 - \frac{a_1N_1}{L_1}\right)$ where

$$A_1 = a_1 - \frac{\alpha_{12}L_2\{a_2(1+kM_1)+\alpha_{21}M_1\}}{a_2(1+kM_1)}$$

Using lemma1 $\lim_{t\to\infty} inf\, N_1(t) \ge \frac{A_1L_1}{a_1}$ and $\lim_{t\to\infty} sup\, N_1(t) \le L_1$ from equation (2.2.2.1)

From the system of equations (2.1) the prey equation is given by

$$\frac{dN_2}{dt} = a_2N_2[1-\frac{N_2}{L_2}] + \frac{\alpha_{21}N_1N_2}{1+kN_1} \ge N_2\left(a_2 + \alpha_{21}N_1 - \frac{a_2N_2}{L_2}\right) \ge N_2\left(a_2 + \alpha_{21}M_1 - \frac{a_2N_2}{L_2}\right)$$

Using lemma1 $\lim_{t\to\infty} inf\, N_2(t) \ge \frac{A_2L_2}{a_2}$ where

$A_2 = a_2 + \alpha_{21}M_1$ and $\lim_{t\to\infty} sup\, N_2(t) \le \frac{\alpha_{12}L_2\{a_2(1+kM_1)+\alpha_{21}M_1\}}{a_2(1+kM_1)}$ from equation (2.2.2.3)

Now choose $\eta_1 = min\left(\frac{A_1L_1}{a_1}, \frac{A_2L_2}{a_2}\right)$ and $\eta_2 = max\left(L_1, \frac{\alpha_{12}L_2\{a_2(1+kM_1)+\alpha_{21}M_1\}}{a_2(1+kM_1)}\right)$ we get the permanence of the system (2.1).

**2.4. Equilibrium states:**

By equating $\frac{dN_i}{dt} = 0$, $i$ =1, 2 we get the following equilibrium states

**I. The Extinct state** $E_1$: $\overline{N}_1 = 0, \overline{N}_2 = 0$ (2.4.1)

**II. Semi Extinct: The state in which one of two species Extinct and one survive**

Case A. $E_2$: $\overline{N_1} = L_1$, $\overline{N_2} = 0$ (2.4.2)

Case B: $E_3$: $\overline{N}_1 = 0, \overline{N}_2 = L_2$ (2.4.3)

**III: Two species are survived**

Solve the system of equations (2.1) we get the cubic equation in $\overline{N_1}$ is given by

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$$aN_1^{\ 3} + bN_1^{\ 2} + cN_1 + d = 0 \qquad (2.4.4)$$

Where

$a = a_1 a_2 k^2$

$b = 2ka_1a_2 - k^2 a_1 a_2 L_1$

$c = a_1a_2 + ka_2L_1L_2\alpha_{12} + \alpha_{12}\alpha_{21}L_1L_2 \ - \ 2ka_1a_2L_1$

$d = a_2L_1L_2\alpha_{12} - a_1a_2L_1$

On solving equation (2.4.4), three possible roots exist for $\overline{N_1}$. The second equilibrium point $\overline{N_2}$ is obtained from the following equation

$$\overline{N}_2 = \frac{(1+kN_1)(a_1L_1 - a_1N_1)}{\alpha_{12}L_1} \qquad (2.4.5)$$

The three possible equilibria for this case are obtained by solving equation (2.4.4) and for each value of $\overline{N_1}$ and corresponding $\overline{N_2}$ value is obtained from equation (2.4.5).

**2.5. Local stability analysis:**

The Jacobean matrix for the system of equations (2.1) is given by

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

where

$$J_{11} = \frac{\partial f_1}{\partial N_1}, J_{12} = \frac{\partial f_1}{\partial N_2}, \quad J_{21} = \frac{\partial f_2}{\partial N_1}, J_{22} = \frac{\partial f_2}{\partial N_2} \qquad (2.5.1)$$

Here $\quad f_1(N_1, N_2) = \frac{dN_1}{dt} = a_1N_1\left[1 - \frac{N_1}{L_1}\right] - \frac{\alpha_{12}N_1N_2}{1+kN_1}$

$$f_2(N_1, N_2) = \frac{dN_2}{dt} = a_2N_2\left[1 - \frac{N_2}{L_2}\right] + \frac{\alpha_{21}N_1N_2}{1+kN_1} \qquad (2.5.2)$$

Calculate the Jacobean matrix i.e.

$$J = \begin{bmatrix} -\frac{a_1N_1}{L_1} - \frac{k\alpha_{12}N_1N_2}{(1+kN_1)^2} & -\frac{\alpha_{12}N_1}{1+kN_1} \\ \frac{\alpha_{21}N_2}{(1+kN_1)^2} & -\frac{a_2N_2}{L_2} \end{bmatrix} \qquad (2.\ 5.3)$$

The characteristic equation is given by $det(J - \lambda I) = 0$

The system is stable if the Eigen roots of equation (2.5.3) are negative,

in case of real roots or negative real parts in case complex roots, otherwise unstable.

**Case (i)** $E_1(0,0)$ is unstable

**Case (ii) :** The characteristic equation for case A:$E_2$ is $\lambda^2 + a_1\lambda = 0$ ,where roots are

$\lambda = 0$ & $\lambda = -a_1$ hence the system neutrally stable.

The characteristic equation for case B: $E_3$ is

$\lambda^2 + a_2\lambda = 0$ ,where roots are

$\lambda = 0$ & $\lambda = -a_2$ hence the system neutrally stable.

**Case (iii): Co-existing case $E_4$:**

The characteristic equation is given by $a\lambda^2 + b\lambda + c = 0$ (2.5.4)

where a=1,

$$b = \frac{a_1 N_1}{L_1} + \frac{a_2 N_2}{L_2} - \frac{k\alpha_{12} N_1 N_2}{(1 + kN_1)^2}$$

$$c = \frac{a_1 a_2 N_1 N_2}{L_1 L_2} - \frac{k a_2 \alpha_{12} N_1 {N_2}^2}{L_2 (1+kN_1)^2} + \frac{\alpha_{12}\alpha_{21} N_1 N_2}{(1+kN_1)^3} \tag{2.5.5}$$

The system is stable if the sums of roots are negative and products of roots are positive i.e

If $\frac{a_1 N_1}{L_1} + \frac{a_2 N_2}{L_2} > \frac{k\alpha_{12} N_1 N_2}{(1+kN_1)^2}$ and

$$\frac{a_1 a_2 N_1 N_2}{L_1 L_2} + \frac{\alpha_{12}\alpha_{21} N_1 N_2}{(1+kN_1)^3} > \frac{k a_2 \alpha_{12} N_1 {N_2}^2}{L_2 (1+kN_1)^2} \tag{2.5.6}$$

$E_4$ is locally asymptotically stable condition (2.5.6) is satisfied otherwise unstable

## 2. 6. Global stability:

**Theorem 2.6.1:** The axial equilibrium point $E_4(\overline{N_1}, \overline{N_2})$ is globally asymptotically stable

**Proof:** Let the Lyapunov function be

$$V(\overline{N_1}, \overline{N_2}) = l_1 \left[ (N_1 - \overline{N_1}) - \overline{N_1} \log\left(\frac{N_1}{\overline{N_1}}\right) \right] + l_2 \left[ (N_2 - \overline{N_2}) - \overline{N_2} \log\left(\frac{N_2}{\overline{N_2}}\right) \right] \tag{2.6.1}$$

The time derivate of V along the solutions of equations (2.1) is

$$\frac{dV}{dt} = l_1 \frac{dN_1}{dt}\left[1 - \frac{\overline{N_1}}{N_1}\right] + l_2 \frac{dN_2}{dt}\left[1 - \frac{\overline{N_2}}{N_2}\right] \tag{2. 6.2}$$

$$= l_1[N_1 - \overline{N_1}]\left[a_1\left[1 - \frac{N_1}{L_1}\right] - \frac{\alpha_{12} N_2}{1+kN_1}\right] +$$

$$l_2[N_2 - \overline{N_2}]\left[a_2\left[1 - \frac{N_2}{L_2}\right] + \frac{\alpha_{21} N_1}{1+kN_1}\right] \tag{2.6.3}$$

By proper choice of

$$a_1 = \frac{a_1 \overline{N_1}}{L_1} + \frac{\alpha_{12} \overline{N_2}}{1 + k\overline{N_1}}, a_2 = \frac{a_2 \overline{N_2}}{L_2} + \frac{\alpha_{21} \overline{N_1}}{1 + k\overline{N_1}} = \frac{\alpha_{12}}{\alpha_{21}}$$

$$l_1 = \frac{1 + k\overline{N_1}}{\alpha_{12}}, l_2 = \frac{1 + k\overline{N_1}}{\alpha_{21}}$$

We get

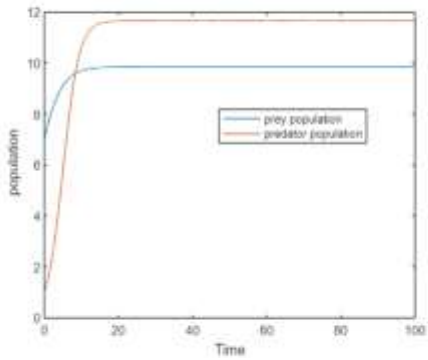
$$\frac{dV}{dt} = -\frac{\alpha_1(1+k\overline{N_1})}{L_1\alpha_{12}}\left(N_1 - \overline{N_1}\right)^2 - \frac{\alpha_2(1+k\overline{N_1})}{L_2\alpha_{21}}\left(N_2 - \overline{N_2}\right)^2 \quad (2.\ 6.4)$$

$V^1(t) < 0$, hence the system is globally stable at positive equilibrium point $E_4\left(\overline{N_1}, \overline{N_2}\right)$

## 3. Results and Discussion

### 3.1 Numerical Simulation:

**Example 3.1:** Let $a_1$=0.3, $a_2$=0.4, k =10, $\alpha_{12}$=0.04, $\alpha_{21}$=0.6666, $L_1$=10, $L_2$=10, $N_1$ =7, $N_2$ =1.



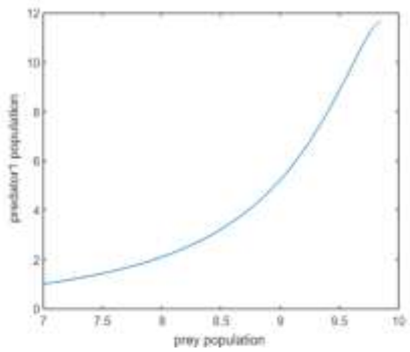
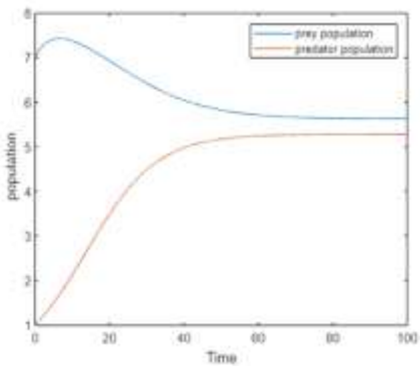




***Figure 3.1(A)***
*Represents time series plot*

***Figure 3.1(B)***
*Represents phase portrait*

The converging solutions of prey and predator populations to fixed equilibrium point [9,11].

**Example 3.2:** Let $a_1$=0.3, $a_2$=0.04, k =10, $\alpha_{12}$=0.96851, $\alpha_{21}$=0.6666, $L_1$=8, $L_2$=2, $N_1$ =7, $N_2$ =1.



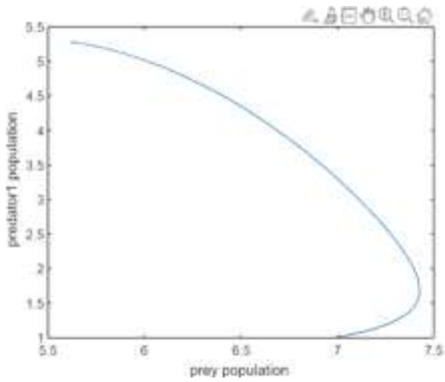




***Figure 3.2.(A)***
*Represents time series plot*

***Figure 3.2(B)***
*Represents phase portrait*

The converging solutions of prey and predator populations to fixed equilibrium point [6,7].

## 4. CONCLUSION

We consider a two species ecological model based on prey- predators' interactions with prey holling type-II response is taken for investigation. The mathematical model with prey predator dynamics was studied and prove that the eco-system is stable. The properties of the model were studied like positivity, boundedness and permeance of the system. The stability analysis of the model was discussed at possible equilibrium points. The global stability analysis of co-existing state is also addressed by choosing proper Lyapunov's function. Numerical simulation is performed in support of analytical results. The stability analysis at four equilibrium points and its nature with three different harvesting efforts are placed below

| **Equilibrium points** | **Nature of the system** |
|---|---|
| $E_1: \overline{N}_1 = 0, \overline{N}_2 = 0$ | Un stable |
| $E_2: \overline{N}_1 \neq 0, \overline{N}_2 = 0$ | Neutrally stable |
| $E_3: \overline{N}_1 = 0, \overline{N}_2 \neq 0$ | Neutrally stable |
| $E_4: \overline{N}_1 \neq 0, \overline{N}_2 \neq 0$ | Asymptotically stable if $\frac{a_1 N_1}{L_1} + \frac{a_2 N_2}{L_2} > \frac{k\alpha_{12} N_1 N_2}{(1+kN_1)^2}$ and $\frac{a_1 a_2 N_1 N_2}{L_1 L_2} + \frac{\alpha_{12}\alpha_{21} N_1 N_2}{(1+kN_1)^3} > \frac{k a_2 \alpha_{12} N_1 {N_2}^2}{L_2 (1+kN_1)^2}$ |

**Table 4.1**

The global stability analysis of co-existing state is also addressed by choosing proper Lyapunov's function and prove that the system is globally asymptotically stable. Further Numerical simulation is performed in support of analytical results shows that system is globally asymptotically stable.

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