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Throughput Maximization in aMultiple vacation Markovian Queueing System withBalking, Encouraged Arrival,Vacation intervention, Waiting Server and double threshold policy

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Abstract

A multiple vacation single server Markovian queueing system with balking, encouraged arrival, vacation intervention, waiting Server and double threshold policyare analyzed in this research. We increased the system size and system throughput by applying encouraged arrival. This model could be applied to simulate the power-saving mechanisms in the real-world application like WiMAX. After sending all the users in the awake mode, the server switches to the rest mode for nearly arbitrary period span η_1. After the end of the period η_1, If there are no users waiting for service, the server goes into monitoring mode and initiates a timer η_2. Both the rest (sleep) mode and the monitoring mode can be intervened when the system size achieves two predetermined threshold levels of t_1 and t_2. This model yields an explicit description of a steady-state solution. We solve the normalization condition. Performance measures like the system throughput and system size probabilities are calculated. Finally, simulation results, numerical illustrations and profit-Revenue analysis with illustrations are offered to investigate the performance of the system.

Keywords Performance measures, Steady-state solution, Vacation intervention, Threshold policies.

1.Introduction

Queuing systems with vacation have captured the attention of investigators and system developers in recent years. Due to their widespread use in tracking and monitoring systems, manufacturing systems, DRX, WiMAX, LTE systems, and communication network systems. The idea of vacation is added to the queueing mechanism to efficiently utilize the server's idleness. Comparable to sleep or energy-saving methods in computer and telecom systems is the idea of vacation in the queueing system.[16] developed the WiMAX power saving mechanism in a multiple vacation M/M/1/DV Queue with a Vacation Interruption, two threshold policy. Increasing the number of existing consumers can increase sales, which frequently results in higher profitability. Users will pay more attention if they are encouraged by certain special offers, which will increase revenue and earnings for the business. By using encouraged arrival [31,18] maximized the system-size in a single server queueing model. Balking happens when a consumer decides to skip a queue or service opportunity often because it seems excessively long or expensive and it affects business profit. [19] developed the optimal pricing strategy to avoid balking loss and reneging loss by using consumer discounts. In this paper, Multiple server vacationsof Markovian queue with, balking, encouraged arrival of users, vacation intervention, waiting Server and double threshold policyare analyzed. AnM/M/1 queue with working vacation was introduced by [1].Fundamental and basic concepts of queueing models have been discussed in [3].An M/M/1 queueing model with working vacations and customer impatience have been developed by [4].Multiple vacation M/M/1 queue with differentiated vacations developed in [6]. A generating function technique is used in order to analyze the steady-state behavior of an M/M/1/DV queueing model with working vacation and customer impatience were done by [7]. Transient state analysis of an M/M/1/DV queue have been obtained in [9]. An M/M/1/N queue with vacation, balking, reneging analyzed in [10]. Customer Reneging behavior of an M/M/1 queueing model with differentiated vacations are presented by [11]. An M/M/1/N queue with encouraged customer arrival, balking and maintaining the impatient customers under quality control environment have been developed by [17].AnM/M/1 systemwithseveral working vacation, multiple stage random atmosphere reneging, balking and a waiting service provider were done by [18]. The balking and reneging are introduced and developed with discount as key parameters on increasing the profit in [19]. An M/M/1 queue with differentiated non-working vacations in which the service provider takes two types of vacationshave been considered in [20]. An M/M/1 queue with reneging clients and dual types of working server vacations have been developed by [21]. M/M/1 queueing system with working vacations and *N*-policy were done by [22]. There are several ways that the server vacation could be interrupted. An M/M/1 queue with vacation interruptions have been derived in [2]. An M/M/1 multiple vacation queue with dual types of server vacations, partial and complete interruptions are developed in [5].One server Markovian queues with multiple working server vacations and interruptions, jobs equilibrium, joining-balking behavior have been developed by [8]. Limited space queue with differentiated WS (working vacation), Bernoulli vacation interruption, balking, reneging and cost analysis were done by [12]. Time independentsolution of an M/M/1/WVqueue with vacation interruption and impatient jobs have been developed in [13]. Transient state probabilities of an M/M/1 queue with differentiated vacations, partial vacation interruption and predefined threshold policy have been obtained by [14]. Unlimited space M/M/1 queue model with vacation interruption and impatience of users were done by [15]. Multiple vacation M/M/1/DV Queuing model with a Vacation Interruption, threshold policies and a Waiting Server for both the steady-state and transient state have been derived in [16]. The equilibrium customer linkingtactics in an M/M/1 queueing model with working service processor vacations and vacation interruptions have been developed in [23]. Renewal input continuous time and discrete time queueing models with vacation, interruptions and balkingweredone by [24].A finite space *N*-policy queue with vacation interruption were done by [25]. A retrial queue system with various working vacations and interruptions have been developed by [26]. GI/M/1 queueing systemrunning in a multi-stage service with working vacations and v interruption have been developed in [27]. An M/G/1 orbit queueing model with general retrial times and vacation and interruption is considered by [28].*M/G/*1 G-queueing system with breakdown, working vacations and interruption is analyzed in [29]. The bulk input and general service retrial queueing model with customer balking, feedback customers, multiple working vacation and interruption has been investigated in [30].The paper is organized as follows: Introduction is provided in section 1. Description of the model is discussed in section 2.Steady-state probabilities of the queueing model is formulated in section 3. Performance measures of the queueing model is derived in section 4. Numerical illustrations are provided in section 5. Simulation results have been provided in section 6. Results of system throughput in section 7. Profit-revenue analysiswith exampleiscalculated in section 8.Resultsand discussions are discussed in section 9. Finally, this paper is concluded in section 10.

2.Model description

This section presents a model description of a multiple vacation Markovian queueing system with a balking, encouraged arrival, vacation intervention, Waiting Server and double threshold policy. One after one, the users arriveaccording to a Poisson process encouraged user arrival with mean $\lambda(\xi+1)$, where "ξ" represents the rate of offer.Users are encouraged by data offers, price discounts etc. The service times 'μ' are exponentially distributed.Balking happens when a user decides to skip a queue or service opportunity often because it seems excessively long or expensive. As you might have guessed, it's bad for business and it affects business profit. when a consumer refuses to use your services or offerings. On arrival, the user may balk with certain balking probability. User balk with chance β and not balk with probability α (= 1 – β).If all users are sent in theworking mode, then the processor shifts to the rest mode (type-Iserver vacation) V_1 for aarbitrarytime period. If one of the two circumstances occurs, the server will return to its working busy state from the type-I server vacation.

Case (i) While the typeI server vacation duration, if the levelachieves a threshold rate of t_1, theserverstops the vacation and come back to the normal working state.

Case (ii) when the type1 vacation is over, if any consumer is holding for service.

Later the I-vacation, if nobody remains on hold for facility, the processor shifts to a monitoring mode referred to as type II vacation $V_2$with a mean□$_1$. The server's vacation periods in types I, II service processor vacations obey an exponential distribution with the mean values□$_1$ and □$_2$.If no user exists to wait for service at the completion of the type II vacation duration, the server shifts in to rest mode and remains hold for the user to arrive.The holding timedistribution of the service processor in the restmode follows an exponential distribution witha rate of □$_0$. If no user arrives during freeperiod, the serverchanges to typeIvacation.After the completion of typeII server vacation period, if any user is holding for getting facility(i.e., if the size of the system is fewer than t_2), the service providerturns to anworkingmode and restart service. The typeII vacation is intervened if the size of the systemattains athreshold value t_2 where $t_1>t_2$. The users are served on F-I-F-Opolicy.

Let $\{S(t), t \geq 0\}$ denotes the condition of the system at period of time "t" and$N(t)$represents the number of users in the system

$$S(t) = \begin{cases} b - \text{ the server is in awake mode (busy) and providing service with the rate "}\mu\text{"} \\ V_1 - \textit{the server is in type I vacation} \\ V_2 - \textit{the server is in type II vacation} \end{cases}$$

Then, $\{S(t), N(t), t \geq 0\}$ denotes the state space of Markov process

$S = \{(b, n): n = 0, 1, 2, \ldots,\} \cup \{(V_1, n) : n = 0, 1, 2, \ldots, t_1 - 1\} \cup \{(V_2, n): n = 0, 1, 2, \ldots t_2 - 1\}$.

$P_{j,n}(t) = P\{S(t) = j, N(t) = n\}$, $j= b$ where $n = 0, 1, 2, \ldots$,
$P_{j,n}(t)= P\{S(t) = j, N(t) = n\}$, $j=V_1$ where $n = 0, 1, 2, \ldots, t_1 - 1$
$P_{j,n}(t) = P\{S(t) = j, N(t) = n\}$, $j= V_2$ where$n = 0, 1, 2, \ldots, t_2 - 1$

Then, $P_{j,n}(t)$ satisfies the following forward Kolmogorov equations

$$p'_{b,0}(t) = -(\lambda(\xi + 1) + \delta_0)p_{b,0}(t) + \delta_2 p_{V_2,0}(t) \quad (1.1)$$

$$p'_{b,t_2}(t) = -(\alpha\lambda(\xi + 1) + ⍰)p_{b,t2}(t) + \alpha(\lambda(\xi + 1))p_{b,t_2-1}(t) + \mu p_{b,t_2+1}(t) + \alpha(\lambda(\xi + 1))p_{V_2,t_2-1}(t) + \delta_1 p_{V_1,t_2}(t) \quad (1.2)$$

$$p'_{b,n}(t) = -(\alpha(\lambda(\xi + 1)) + ⍰)p_{b,n}(t) + \alpha(\lambda(\xi + 1))p_{b,n-1}(t) + \mu p_{b,n+1}(t) + \delta_1 p_{V_1,n}(t) + \delta_2 p_{V_2,n}(t)$$

$$; n = 1,2,3 \ldots, t_2 - 1 \quad (1.3)$$

$$p'_{b,t_1}(t) = -(\alpha(\lambda(\xi + 1)) + ⍰)p_{b,t_1}(t) + \alpha(\lambda(\xi + 1))p_{b,t_1-1}(t) + \mu p_{b,t_1+1}(t) + \alpha(\lambda(\xi + 1))p_{V_1,t_1-1}(t) (1.4)$$

$$p'_{b,n}(t) = -(\alpha\lambda(\xi + 1) + ⍰)p_{b,n}(t) + \alpha\lambda(\xi + 1) + p_{b,n-1}(t) + \mu p_{b,n+1}(t) + \delta_1 p_{V_1,n}(t)$$

$$; n = t_2 + 1, t_2, t_2 - 1, \ldots, t_1 - 1 \quad (1.5)$$

$$p'_{b,n}(t) = -\alpha\lambda(\xi + 1) + ⍰)p_{b,n}(t) + \alpha\lambda(\xi + 1)p_{b,n-1}(t) + \mu p_{b,n+1}(t); n = t_1 + 1, t_1 + 2, \ldots \quad (1.6)$$

$$p'_{V_1,0}(t) = -(\lambda(\xi + 1) + \delta_1)p_{V_1,0}(t) + \mu p_{b,1}(t) + \delta_0 p_{b,0}(t) (1.7)$$

$$p'_{V_1,n}(t) = -(\alpha\lambda(\xi + 1) + \delta_1)p_{V_1,n}(t) + \alpha(\lambda(1 + \xi))p_{V_1,n-1}(t) \; ; n = 1,2,3 \ldots, t_1 - 1 (1.8)$$

$p'_{V_2,0}(t) =-(\lambda(\xi+1)+\delta_2)p_{V_2,0}(t)+\delta_1 p_{V_1,0}(t)$ (1.9)

$p'_{V_2,n}(t)=-(\alpha\lambda(\xi+1)+\delta_2)p_{V_2,n}(t)+\alpha\lambda(\xi+1)p_{V_2,n-1}(t)\ ; n=1,2,3\ldots,t_2-1$(2.10)

To construct the beginningcircumstances, it is supposed that the service provider is in the vacation typeI originally. Hence,$p_{V_1,n}(0)=1, p_{b,n}(0)=0$ for n = 0, 1, 2, . . .;$p_{V_1,j}(0)=0$ for j = 1, 2, 3, . . ., t_1-1 and $p_{V_2,j}(0)=0$ for j = 0, 1, 2, . . ., t_2-1.

3.Steady-state probabilities

In this section, we present the steady-state probabilities

$(\lambda(\xi+1)+\delta_0)p_{b,0}=\delta_2 p_{V_2,0}$ (3.1)

$(\alpha\lambda(\xi+1)+\square)p_{b,t2}=\alpha\lambda(\xi+1)p_{b,t_2-1}+\mu p_{b,t_2+1}+\alpha\lambda(\xi+1)p_{V_2,t_2-1}+\delta_1 p_{V_1,t_2}$ (3.2)

$(\alpha\lambda(\xi+1)+\square)p_{b,n}=\alpha\lambda(\xi+1)p_{b,n-1}+\mu p_{b,n+1}+\delta_1 p_{V_1,n}+\delta_2 p_{V_2,n}$

$; n=1,2,3\ldots,t_2-1$ (3.3)

$(\alpha\lambda(\xi+1)+\square)p_{b,t_1}=\alpha\lambda(\xi+1)p_{b,t_1-1}+\mu p_{b,t_1+1}+\alpha\lambda(\xi+1)p_{V_1,t_1-1}$(3.4)

$(\alpha\lambda(\xi+1)+\square)p_{b,n}=\lambda(\xi+1)p_{b,n-1}+\mu p_{b,n+1}+\delta_1 p_{V_1,n}$

$;n=t_2+1,t_2,t_2-1,\ldots,t_1-1$ (3.5)

$(\alpha\lambda(\xi+1)+\square)p_{b,n}=\alpha\lambda(\xi+1)p_{b,n-1}+\mu p_{b,n+1}\ ; n=t_1+1,t_1+2,\ldots$ (3.6)

$(\lambda(\xi+1)+\delta_1)p_{V_1,0}=\mu p_{b,1}+\delta_0 p_{b,0}$(3.7)

$(\alpha\lambda(\xi+1)+\delta_1)p_{V_1,n}=\alpha(\lambda(\xi+1))p_{V_1,n-1}\ ;n=1,2,3\ldots,t_1-1$(3.8)

$(\lambda(\xi+1)+\delta_2)p_{V_2,0}=\delta_1 p_{V_1,0}$(3.9)

$(\alpha\lambda(\xi+1)+\delta_2)p_{V_2,n}=\alpha\lambda(\xi+1)p_{V_2,n-1}$where$n=1,2,3\ldots,t_2-1$(3.10)

Using Equations (3.1), (3.7)–(3.10), after algebraic manipulation, we get

$p_{V_1,n}=\frac{(\alpha\lambda(\xi+1))^n}{(\alpha\lambda(\xi+1)+\delta_1)^{n+1}}\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$(3.11)

$p_{V_2,n}=\frac{\delta_1(\alpha\lambda(\xi+1))^n}{(\alpha\lambda(\xi+1)+\delta_1)(\alpha\lambda(\xi+1)+\delta_2)^{n+1}}\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$(3.12)

$p_{b,0}=\delta_0^{-1}\left(\sum_{i=0}^{\infty}\varphi^{1+i}\mu p_{b,1}\right)$(1.13)

Where;

$\varphi=\delta_0\delta_1\delta_2\left((\lambda(\xi+1)+\delta_0)+(\lambda(\xi+1)+\delta_1)+(\lambda(\xi+1)+\delta_2)\right)^{-1}$(3.14)

Therefore, the server vacation mode probabilities $p_{V_1,n},\ p_{V_2,n}$and the freemode$p_{b,0}$ are stated in $p_{b,1}$. To obtain an expression for $p_{b,1}$.

$G_1(z)=\sum_{n=0}^{t_1-1}p_{V_1,n}z^n$(3.15)

$G_2(z)=\sum_{n=0}^{t_2-1}p_{V_2,n}z^n$(3.16)

Multiplying Equations (3.8) and (3.10) by appropriate powers of thez, totaling over n =1, 2, 3, . . . , t_1, n and

$n = 1, 2, 3, \ldots, t_2, n$, respectively, we obtain

$$G_1(z) = \frac{(\alpha\lambda\ (\xi+1)+\delta_1)}{(\alpha\lambda\ (\xi+1)(1-z)+\delta_1)} p_{V_1,0} \quad (3.17)$$

$$G_2(z) = \frac{(\alpha\lambda\ (\xi+1)+\delta_2)}{(\alpha\lambda\ (\xi+1)(1-z)+\delta_2)} p_{V_2,0} \quad (3.18)$$

Let

$$G_3(z) = \sum_{n=1}^{\infty} p_{b,n} z^n \quad (3.19)$$

Multiplying Equations (3.2)–(3.6) by proper powers of z and totaling over $n = 1, 2, 3, \ldots$ we get

$$G_3(z) = \frac{-\lambda(\xi+1)\alpha z\left((\lambda(\xi+1)zp_{b,0} - \mu p_{b,1} + \delta_1 \sum_{n=1}^{t_1-1} p_{V_1,n} z^n + \delta_2 \sum_{n=1}^{t_2-1} p_{V_2,n} z^n + \alpha\lambda(\xi+1)p_{V_1,t_1-1} z^{t_1} + \alpha\lambda(\xi+1)p_{V_2,t_2-1} z^{t_2}\right)}{(z-1)(z-\mu)}$$

(3.20)

Using Equations (3.16) and (3.17) in Eq. (3.19), we get

$$G_3(z) = \frac{-\lambda(\xi+1)\alpha z\left((\lambda(\xi+1)zp_{b,0} - \mu p_{b,1} + \delta_1(G_1(z) - p_{V_1,0}) + \delta_2(G_2(z) - p_{V_2,0}) + \alpha\lambda(\xi+1)p_{V_1,t_1-1} z^{t_1} + \alpha\lambda(\xi+1)p_{V_2,t_2-1} z^{t_2})\right)}{(z-1)(z-\mu)} \quad (3.21)$$

Putting $z = 1$ in Equation (3.21), we get

$$\mu p_{b,1} = \left(\lambda(\xi+1)p_{b,0} + \lambda(\xi+1)p_{V_1,0} + \lambda(\xi+1)p_{V_2,0} + \alpha\lambda(\xi+1)p_{V_1,t_1-1} + \alpha\lambda(\xi+1)p_{V_2,t_2-1}\right)$$

(3.22)

Substituting Eq. (3.22) in Eq. (3.20), after some algebraic manipulation, we get

$$G_3(z) = \frac{\alpha(\lambda(\xi+1))^2 z\left((\lambda(\xi+1)zp_{b,0} + (G_1(z)) + (G_2(z)) + \alpha\lambda(\xi+1)p_{V_1,t_1-1}\sum_{i=0}^{t_1-1} z^i + \alpha(\lambda(\xi+1))p_{V_2,t_2-1}\sum_{i=0}^{t_2-1} z^i)\right)}{(\mu - z\alpha\lambda(\xi+1))}$$

(3.23)

Put $z = 1$ in Eq. (3.23) and using Equations. (3.11) – (3.13), we obtain

$$G_3(1) = \frac{\frac{\alpha(\lambda(\xi+1))^2}{\mu}}{\left(1-\frac{\alpha\lambda(\xi+1)}{\mu}\right)}\left(\delta_1^{-1} + \frac{\varphi}{\delta_0} + \frac{\delta_1}{(\alpha\lambda(\xi+1)+\delta_1)\delta_2} + \frac{(\alpha\lambda(\xi+1))^{t_1-1}}{(\alpha\lambda(\xi+1)+\delta_1)^{t_1}} + \frac{\delta_1(\alpha\lambda(\xi+1))^{t_2-1}}{(\alpha\lambda(\xi+1)+\delta_1)(\alpha\lambda(\xi+1)+\delta_2)^{t_2}}\right)\sum_{i=0}^{\infty}\varphi^i \mu p_{b,1} \quad (3.24)$$

By using the normalization condition $G_1(1) + G_2(1) + G_3(1) + p_{b,0} = 1$, we get

$$p_{b,1}^{-1} = \left(\left(\delta_1^{-1} + \frac{\varphi}{\delta_0} + \frac{\delta_1}{(\alpha\lambda(\xi+1)+\delta_1)\delta_2}\right)\left(\frac{\frac{\alpha(\lambda(\xi+1))^2}{\mu}}{\left(1-\frac{\alpha\lambda(\xi+1)}{\mu}\right)}\right) + \right.$$

$$\left(\frac{\frac{\alpha\,(\lambda\,(\xi+1))^2}{\mu}}{\left(1-\frac{\alpha\,(\lambda\,(\xi+1))}{\mu}\right)}\right)\left(\frac{(\alpha\,(\lambda\,(\xi+1)))^{t1-1}}{(\alpha\,\lambda\,(\xi+1)+\delta_1)^{t1}}+\frac{\delta_1(\alpha\,\lambda\,(\xi+1))^{t2-1}}{(\alpha\,\lambda\,(\xi+1)+\delta_1)(\alpha\,\lambda\,(\xi+1)+\delta_2)^{t2}}\right)\Bigg)\sum_{i=0}^{\infty}\varphi^i\mu\,p_{b,1}.$$

(3.25)

Hence, we have obtained an explicit expression for $p_{b,1}$, where $\frac{\lambda\,(\xi+1)}{\mu}<1$

4.Performance measures

In this section, we presentthe mean number of users and mean waiting time of users in the system

Mean number of users in type 1 vacation

$$\mathrm{N}\,(V_1)=\sum_{n=0}^{t_1-1}n\,p_{V_{1,n}}$$

$$=\frac{\alpha\lambda\,(\xi+1)}{\delta_1(\alpha\,\lambda\,(\xi+1)+\delta_1)}\left[\frac{(\alpha\,\lambda\,(\xi+1)+\delta_1)}{\delta_1}\left(1-\left(\frac{\alpha\lambda\,(\xi+1)}{(\alpha\,\lambda\,(\xi+1)+\delta_1)}\right)^{t_1-1}\right)-(t_1-1)\left(\frac{\alpha\lambda\,(\xi+1)}{(\alpha\,\lambda\,(\xi+1)+\delta_1)}\right)^{t_1-1}\right]\sum_{i=0}^{\infty}\varphi^i\,\mu p_{b,1}$$

Mean number of users in type 1 vacation

$$\mathrm{N}\,(V_2)=\sum_{n=0}^{t_2-1}n\,p_{V_{2,n}}$$

$$=\frac{\alpha\lambda\,(\xi+1)\delta_1}{\delta_2(\alpha\,\lambda\,(\xi+1)+\delta_1)(\alpha\,\lambda\,(\xi+1)+\delta_2)}\left[\frac{(\alpha\,\lambda\,(\xi+1)+\delta_2)}{\delta_2}\left(1-\left(\frac{\alpha\lambda\,(\xi+1)}{(\alpha\,\lambda\,(\xi+1)+\delta_2)}\right)^{t_2-1}\right)-(t_2-1)\left(\frac{\alpha\lambda\,(\xi+1)}{(\alpha\,\lambda\,(\xi+1)+\delta_2)}\right)^{t_2-1}\right]\sum_{i=0}^{\infty}\varphi^i\,\mu p_{b,1}$$

Now

N(b)=$\lim_{z\to 1}G_3{}'(z)$

Put z=1 and differentiate equation (3.23), we get

Mean number of users during busy state

$$E(Nb)=\Bigg(\left(1+\left(1-\frac{\alpha\,\lambda\,(\xi+1)}{\mu}\right)^{-1}\right)\left(\frac{(\alpha\,\lambda\,(\xi+1))^{t_1-1}}{(\alpha\,\lambda\,(\xi+1)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\,\lambda\,(\xi+1))^{t_2-1}}{(\alpha\,\lambda\,(\xi+1)+\delta_1)(\alpha\,\lambda\,(\xi+1)+\delta_2)^{t_2}}\right)$$
$$+\frac{\delta_1}{(\alpha\,\lambda\,(\xi+1)+\delta_1)\delta_2}\left(\frac{\alpha\,\lambda\,(\xi+1)}{\delta_2}+\left(1-\frac{\alpha\lambda\,(\xi+1)}{\mu}\right)^{-1}\right)$$
$$+\delta_1{}^{-1}\left(\frac{\alpha\,\lambda\,(\xi+1)}{\delta_1}+\left(1-\frac{\alpha\,\lambda\,(\xi+1)}{\mu}\right)^{-1}\right)+\frac{\varphi}{\delta_0}\left(1-\frac{\alpha\,\lambda\,(\xi+1)}{\mu}\right)^{-1}\Bigg)$$

Mean number of users in the system

$$E(N)=\left(\sum_{n=0}^{t_1-1}n\,p_{V_{1,n}}+\sum_{n=0}^{t_2-1}n\,p_{V_{2,n}}+E(Nb)\right)$$

$$E(N)=\left(\left(1+\left(1-\frac{\alpha\,\lambda\,(\xi+\,1)}{\mu}\right)^{-1}\right)\left(\frac{(\alpha\,\lambda\,(\xi+\,1\,))^{t_1-1}}{(\alpha\lambda\,(\xi+\,1\,)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\,\lambda\,(\xi+\,1\,))^{t_2-1}}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)(\alpha\,\lambda\,(\xi+\,1\,)+\delta_2)^{t_2}}\right)+\frac{\delta_1}{(\alpha\,(\lambda\,(\xi+\,1\,))+\delta_1)\delta_2}\left(\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\delta_2}+\left(1-\frac{\alpha\,\lambda\,(\xi+\,1)}{\mu}\right)^{-1}\right)+\delta_1^{-1}\left(\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\delta_1}+\left(1-\frac{\alpha\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)+\frac{\varphi}{\delta_0}\left(1-\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)$$

Mean waiting time of users in the system

$$W(s)=\frac{1}{\lambda\,(\xi+\,1\,)}\left(\left(1+\left(1-\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)\left(\frac{(\alpha\,\lambda\,(\xi+\,1\,))^{t_1-1}}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\,\lambda\,(\xi+\,1\,))^{t_2-1}}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)(\alpha\,\lambda\,(\xi+\,1\,)+\delta_2)^{t_2}}\right)+\frac{\delta_1}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)\delta_2}\left(\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\delta_2}+\left(1-\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)+\delta_1^{-1}\left(\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\delta_1}+\left(1-\frac{\alpha\,\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)+\frac{\varphi}{\delta_0}\left(1-\frac{\lambda\,(\xi+\,1\,)}{\mu}\right)^{-1}\right)$$

Probability of the service processor is in workingmode

$p_{b,\square}=\sum_{n=1}^{\infty}p_{b,n}$

$p_{b,\square}=$

$$\frac{\frac{\alpha\,(\lambda\,(\xi+1\,))^2}{\mu}}{\left(1-\frac{\alpha\,\lambda\,(\xi+1\,)}{\mu}\right)}\left(\delta_1^{-1}+\frac{\varphi}{\delta_0}+\frac{\delta_1}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)\delta_2}+\frac{(\alpha\,\lambda\,(\xi+\,1\,))^{t_1-1}}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\,\lambda\,(\xi+\,1\,))^{t_2-1}}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)(\alpha\,\lambda\,(\xi+\,1\,)+\delta_2)^{t_2}}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$$

Probability that the service processor is on vacation mode

Using Equations (3.11) and (3.12), we get

Let and $p_{V_{1,\square}}$ is the probability of the server is in type-I vacation

$$p_{V_{1,\square}}=\frac{1}{\delta_1}\left(1-\left(\frac{\lambda\,(\xi+\,1\,)}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)}\right)^{t_1}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$$

Let $p_{V_{2,\square}}$ signifies the probability of the service processor is in the vacation of type-II

$$p_{V_{2,\square}}=\frac{\delta_1}{\delta_2(\alpha\,\lambda\,(\xi+\,1\,)+\delta_1)}\left(1-\left(\frac{\alpha\lambda\,(\xi+\,1\,)}{(\alpha\,\lambda\,(\xi+\,1\,)+\delta_2)}\right)^{t_2}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$$

Throughput of the system

Let s_p is thethroughput of the system then,

$$s_p = ⍰\left(1-\left(\sum_{n=0}^{t_1-1} p_{V_{1,n}} + \sum_{n=0}^{t_1-1} p_{V_{2,n}} + p_{b,0}\right)\right)$$

Average rate of balking

$$P_{balk} = \alpha\lambda(\xi+1)\left(\sum_{n=0}^{\infty} p_{V_{1,n}} + \sum_{n=0}^{\infty} p_{V_{2,n}}\right) + \alpha(\lambda+\xi\lambda)$$

$$\left(\frac{\frac{\alpha(\lambda(\xi+1))^2}{\mu}}{\left(1-\frac{\alpha\lambda(\xi+1)}{\mu}\right)}\left(\delta_1^{-1}+\frac{\varphi}{\delta_0}+\frac{\delta_1}{(\alpha\lambda(\xi+1)+\delta_1)\delta_2}+\frac{(\alpha\lambda(\xi+1))^{t_1-1}}{(\alpha\lambda(\xi+1)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\lambda(\xi+1))^{t_2-1}}{(\alpha\lambda(\xi+1)+\delta_1)(\alpha\lambda(1+\xi)+\delta_2)^{t_2}}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}\right)$$

5.Numerical illustrations

In this section, we consider the numerical illustrations in [16] and introducing the consumer encouragement arrival processand balking concept on it. A real-world example is taken into consideration to investigate the behavior of the system parameters. The server operates in three modes in the PSC-II of IEEE 802.16e: awake, sleep, and listening. Data packets are provided with the rate of 1.25/s after being uplinked by the BS at a rate of 1/s. It is assumed that a data packet's arrival process from the network to the BS is a Poisson process. It is assumed that each data packet's service time will be dispersed exponentially. When there is no data transfer between the BS and server, the server is idle for 1.25 s before switching to a 3.3-s sleep window. If there is no data transmission at completion of the rest (sleep) window epoch, server moves to the 1.11-second monitoring window. The length of the listening, sleeping, and idle windows is distributed exponentially. Predefined threshold values of 10 and 5 are placed in the sleeping (rest) and monitoring windows. To control the frequency of switching between the rest window and available state. As a result, throughout the sleep(rest) window, the server waits until the line has accumulated a group of 10 data packets before switching to the active mode. Steady-state analysis are carried out for this issue. The performance measuresreported in the study are simulated using python software.

6.Simulations results

In, this section, By using the above numerical illustrations we demonstrate the simulation results of the outcome of the parameters λ, μ,ξ,β, t_1, t_2, □$_1$and □$_2$on N(V_1), N(V_2),E(N). where μ=1.25

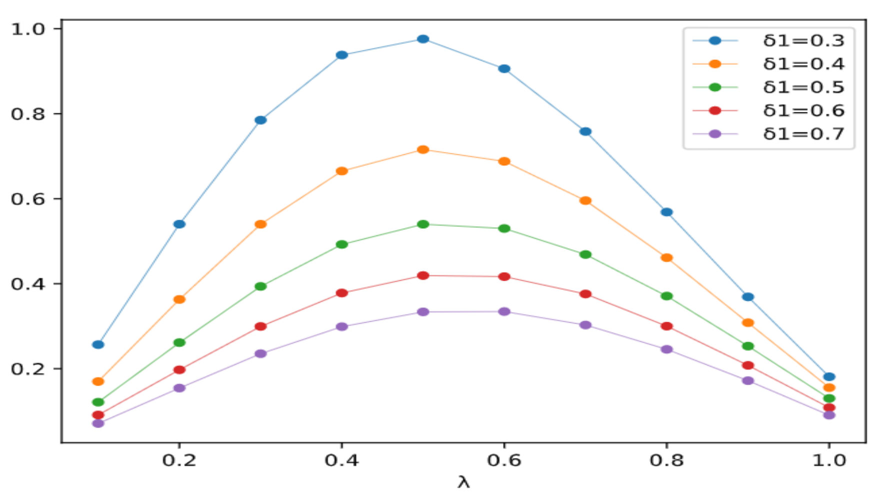

Fig 1: Mean number of usersduring type-I vacation state versus λ

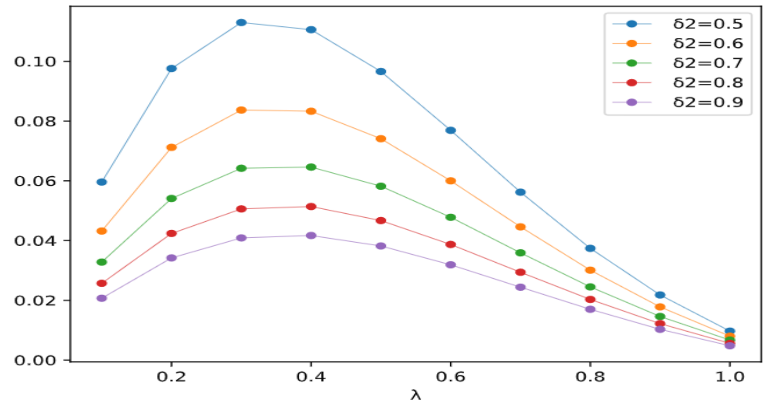

Fig 2: Mean number of consumers in the type-II vacation state versus λ

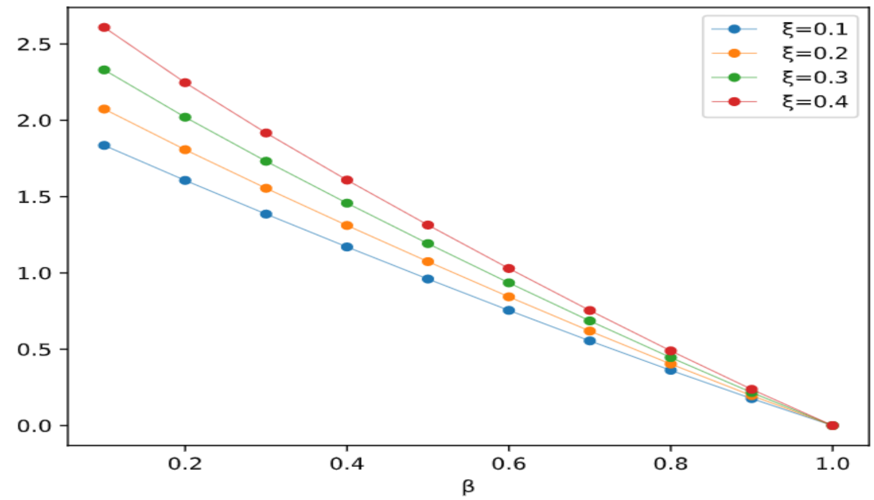

Fig 3: Mean number of users in the system with different rates of ξ versus balking β

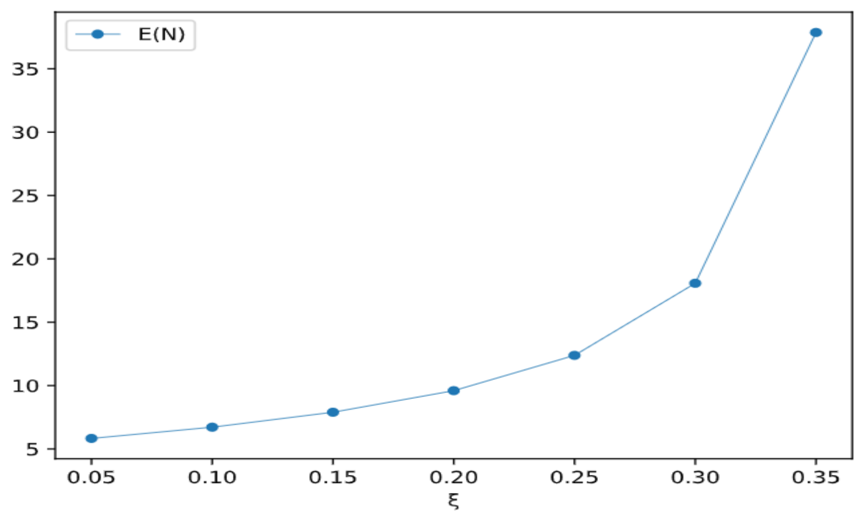

Fig 4: Mean number of users in the system E(N) versus ξ with the fixed value λ=1

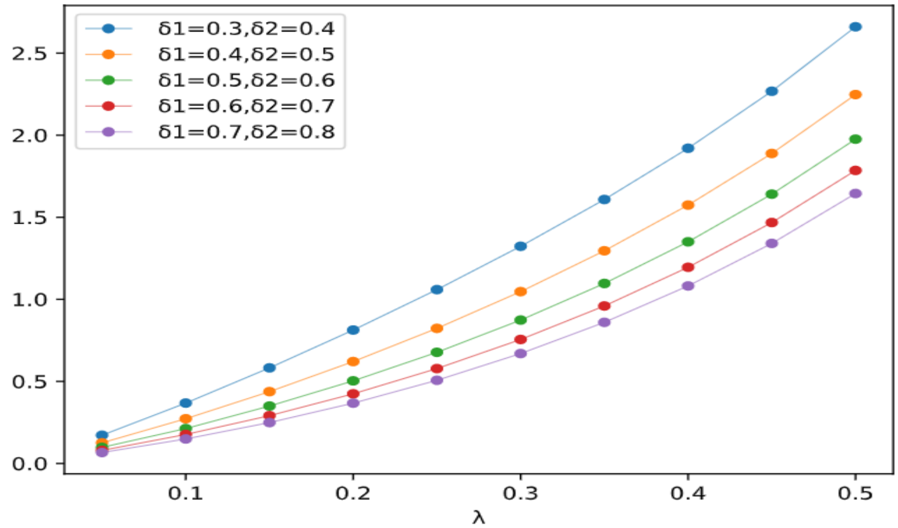

Fig 5: Mean number of users in the system versus λ with ξ=35%

7 Results of the system throughput

Table 1

The impact of λ and μ on systemthroughput when ξ=10%

Let ξ=10 or 0.1 % or 0.1, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1

λ	μ=1.1	μ=1.2	μ=1.3	μ=1.4
0.1	0.0118	0.0117	0.0116	0.0115
0.2	0.0508	0.0501	0.0496	0.0491
0.3	0.1206	0.1185	0.1169	0.1155
0.4	0.2216	0.2175	0.2141	0.2114
0.5	0.3488	0.3426	0.3377	0.3338
0.6	0.4922	0.4852	0.4797	0.4755
0.7	0.6395	0.6339	0.6298	0.6267
0.8	0.7792	0.7779	0.7775	0.777
0.9	0.9033	0.9090	0.9147	0.9203
1.0	1.0078	1.0224	1.0360	1.0486

Table 2

The impact of μ and λ on throughput of the system when ξ=20%

Let ξ=20% or 0.2, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1

λ	μ=1.1	μ=1.2	μ=1.3	μ=1.4
0.1	0.0142	0.0141	0.0140	0.0139
0.2	0.0611	0.0603	0.0596	0.0590
0.3	0.1452	0.1426	0.1405	0.1388
0.4	0.2653	0.2603	0.2564	0.2532
0.5	0.4127	0.4059	0.4006	0.3964
0.6	0.5728	0.5663	0.5613	0.5574
0.7	0.7298	0.7267	0.7246	0.7234
0.8	0.8713	0.8749	0.8787	0.8826
0.9	0.9903	1.0032	1.0152	1.0265
1.0	1.0850	1.0850	1.1305	1.1506

8.Profit-Revenue analysis with examples

Here in thispart, An economic study is offered by generating a cost model. The total expected cost (T-E-C) Δ, total expected revenue (T-E-R) **ϕ**, and total expected profit (T-E-P) ϴare computed by using MATLABsoftware:

$$\Delta = C_1 G1(1) + C_2 G2(1) + C_3 G3(1) + C_4\mu + C_b P_{balk} + C_5(N(s)).$$

$$\Delta = \sum_{i=0}^{\infty}\varphi^i \mu p_{b,1}\left(\frac{C_1}{\delta_1}\left(1-\left(\frac{\alpha\lambda(\xi+1)}{(\alpha\lambda(\xi+1)+\delta_1)}\right)^{t_1}\right)+\frac{C_2\delta_1}{\delta_2}\left(1-\left(\frac{\alpha\lambda(\xi+1)}{(\alpha\lambda(\xi+1)+\delta_2)}\right)^{t_2}\right)\right.$$
$$+\frac{\frac{C_3\alpha(\lambda(\xi+1))^2}{\mu}}{\left(1-\frac{\alpha\lambda(\xi+1)}{\mu}\right)}\left(\delta_1^{-1}+\frac{\varphi}{\delta_0}+\frac{\delta_1}{(\alpha\lambda(\xi+1)+\delta_1)\delta_2}+\frac{(\alpha\lambda(\xi+1))^{t_1-1}}{(\alpha\lambda(\xi+1)+\delta_1)^{t_1}}\right.$$
$$\left.\left.+\frac{\delta_1(\alpha\lambda(\xi+1))^{t_2-1}}{(\alpha\lambda(\xi+1)+\delta_1)(\alpha\lambda(\xi+1)+\delta_2)^{t_2}}\right)\right)+C_4\mu+C_b\alpha\lambda(\xi+1)\left(\sum_{n=0}^{\infty}p_{V_{1,n}}+\sum_{n=0}^{\infty}p_{V_{2,n}}\right)$$
$$+C_b\alpha(\lambda+\xi\lambda)$$
$$\left(\frac{\frac{\alpha((\lambda(1+\xi))^2}{\mu}}{\left(1-\frac{\alpha\lambda(\xi+1)}{\mu}\right)}\left(\delta_1^{-1}+\frac{\varphi}{\delta_0}+\frac{\delta_1}{(\alpha\lambda(\xi+1)+\delta_1)\delta_2}+\frac{(\alpha\lambda(\xi+1))^{t_1-1}}{(\alpha\lambda(\xi+1)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha\lambda(\xi+1))^{t_2-1}}{(\alpha\lambda(\xi+1)+\delta_1)(\alpha(\lambda(1+\xi)+\delta_2)^{t_2}}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}\right)$$
$$+C_5\left(\sum_{n=0}^{t_1-1}n\,p_{V_{1,n}}+\sum_{n=0}^{t_2-1}n\,p_{V_{2,n}}+N(b)\right)$$

By using Eq. (3.24), we get

$$\phi = \frac{\frac{\alpha((\lambda(1+\xi))^2}{\mu}(R)}{\left(1-\frac{\alpha(\lambda(1+\xi)}{\mu}\right)}\left(\delta_1^{-1}+\frac{\varphi}{\delta_0}+\frac{\delta_1}{(\alpha(\lambda(1+\xi)+\delta_1)\delta_2}+\frac{(\alpha(\lambda(1+\xi))^{t_1-1}}{(\alpha(\lambda(1+\xi)+\delta_1)^{t_1}}+\frac{\delta_1(\alpha(\lambda(1+\xi))^{t_2-1}}{(\alpha(\lambda(1+\xi)+\delta_1)(\alpha(\lambda(1+\xi)+\delta_2)^{t_2}}\right)\sum_{i=0}^{\infty}\varphi^i\mu p_{b,1}$$

$$\Theta = \phi - \Delta$$

Where;
C_1- cost per duration (/U/T) when the processor is on type-I vacation
C_2- cost per (/U/T) when the server is on type-II vacation
C_3- cost per (/U/T) when the server is busy mode
C_4- Cost per users served with rate μ
C_5- Holding cost per (/U/T)
C_b- Cost per balk
R- Earned Revenue by service providing.

Table 3 to table 11 shows the outcome of parameters λ, μ, β, t_1,$t_2$and discount/offer "ξ" onTER, TEP, and TEC. For the profit revenue analysis, the values of the cost are preferred as follows C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

Table 3T-E-C, T-E-R, T-E-Pversus ξ (offer)

Take λ=1, μ=1.25, ξ=10%, 20%, 25%, 30% and 35%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

	ξ =10%	ξ =20%	ξ =25%	ξ =30%	ξ =35%
E(N)	6.7169	9.5992	12.3853	18.0767	37.8690
Δ	495.8312	559.7677	620.0101	740.7116	1153.900
ϕ	5088.7	8381.4	11930.00	19594.00	47239.00

ϴ	4592.8	7821.6	11310.00	18853.00	46085.00

Table 4T-E-C, T-E-R, T-E-Pversus λ withoffer ξ=20%

Take λ=0.6 to 1.0, μ=1.25, ξ=20% (0.2), $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

λ	0.6	0.7	0.8	0.9	1.0
E(N)	2.727	3.5498	4.6571	6.3853	7.2269
Δ	410.4178	425.4476	449.3162	487.2191	559.7677
ϕ	2268	2573	3250.80	4704.50	8381.4
ϴ	1857	2148	2801.500	4217.30	7821.6

Table 5T-E-C, T-E-R, T-E-Pversus λ with offer ξ=25%

Take λ=0.6 to 1.0, μ=1.25, ξ=30%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

λ	0.6	0.7	0.8	0.9	1.0
E(N)	3.1122	4.1492	5.6787	8.4949	18.0767
Δ	416.9541	438.1065	472.3556	535.5043	740.7116
ϕ	2389.2	2907.00	4085.40	7055.2	19,594
ϴ	1970.2	2468.90	3613.00	6519.5	18,853

Table 6T-E-C, T-E-R, T-E-Pversus β

Take λ=1 to 1.0, μ=1.25, ξ=20%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1 to 0.5, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

β	0.1	0.2	0.3	0.4	0.5
E(N)	6.7169	4.9612	3.8686	3.0730	2.4312
Δ	495.8312	599.6706	697.8569	788.8257	872.4182
ϕ	5088.7	3555.4	2770.5	2330.00	2085.1
ϴ	4592.8	2955.8	2072.7	1541.20	1212.7

Table 7T-E-C, T-E-R, T-E-Pversus β with offer ξ=30%

Take λ=1 to 1.0, μ=1.25, ξ=30%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1 to 0.5, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

β	0.1	0.2	0.3	0.4	0.5
E(N)	18.0767	8.1569	5.4863	4.0742	3.113
Δ	740.7116	727.4986	816.7925	912.9	1003.5
ϕ	1959.4	7080.8	4191.6	2967.6	2352.8
ϴ	1885.3	6353.3	3374.8	2054.7	1349.2

Table 8T-E-C, T-E-R, T-E-Pversus μwithξ=10%,

Take λ=1, μ=1.1 to 1.7, ξ=10%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

μ	1.1	1.2	1.3	1.4	1.5
E(N)	11.9031	7.6350	6.0878	5.2754	3.113

Δ	540.5353	496.6903	500.0124	516.1813	1003.5
ϕ	7435.00	5415.80	4926.60	4864.6	2352.8
ϴ	6894.20	4919.10	4426.60	4348.5	4454.2

Table 9T-E-C, T-E-R, T-E-Pversus μ with ξ=20%,

Take λ=1, μ=1.1 to 1.7, ξ=20%, $\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

μ	1.1	1.2	1.3	1.4	1.5
E(N)	56.3414	12.2268	8.1513	6.5865	5.7438
Δ	13767	592.9880	548.3414	550.1067	565.2629
ϕ	44971	10268	74377	66179	63903
ϴ	43594	96755	68883	60678	58251

Table 10T-E-C, T-E-R, T-E-Pversus t_1 and t_2 with ξ=20%

Take λ=1 to 1.0, μ=1.25, ξ= 20%,$\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

	$\boldsymbol{t_1}$=10 and $\boldsymbol{t_2}$=5	$\boldsymbol{t_1}$=15 and $\boldsymbol{t_2}$=10	$\boldsymbol{t_1}$=20 and $\boldsymbol{t_2}$=15	$\boldsymbol{t_1}$=25 and $\boldsymbol{t_2}$=20	$\boldsymbol{t_1}$=30 and $\boldsymbol{t_2}$=25
E(N)	9.599	9.7964	9.8637	9.8863	9.8938
Δ	559.7677	563.4362	564.8563	565.3136	565.4567
ϕ	8381.4	3524.7	2266.7	1905.5	1799.9
ϴ	7821.6	2961.2	1701.9	1340.2	1234.4

Table 11T-E-C, T-E-R, T-E-Pversus t_1 and $t_2$with ξ=30%,

Take λ=1 to 1.0, μ=1.25, ξ=30%,$\delta_0 = 0.8$, δ_1=0.3, δ_1=0.9, t_1=10, t_2=5, β=0.1, C_1=50, C_2=70, C_3=20, C_4=40, C_5=10, C_b=50, *R=300*

	$\boldsymbol{t_1}$=10 and $\boldsymbol{t_2}$=5	$\boldsymbol{t_1}$=15 and $\boldsymbol{t_2}$=10	$\boldsymbol{t_1}$=20 and $\boldsymbol{t_2}$=15	$\boldsymbol{t_1}$=25 and $\boldsymbol{t_2}$=20	$\boldsymbol{t_1}$=30 and $\boldsymbol{t_2}$=25
E(N)	18.0767	18.3839	18.4881	18.5231	18.5348
Δ	740.711	727.622	724.4873	723.5610	723.2739
ϕ	19594.00	6956.90	3376.100	2258.700	1903.300
ϴ	18853.00	62293.00	2651.700	1535.200	1180.00

9.Results and Discussions

Figure 1 presents the mean number users during type I vacation with different rate of λ and Figure 2 demonstrates the mean number of users in type II vacation. Figure 3 demonstrates the mean number of users in busy state. Figure 3 demonstrates the mean number of users in the system 'E(N)'. When ξ=0.35 (35%) there are more number consumers in the system. So, it is evident that there is an increase in the system size when there is an increase in rate useroffer "ξ".Figure 4 unveils the mean number of users in the system with different rates of "ξ". Increase in "β" balking rate reduces the system size. At the same time,The system size is getting maximized when we increasethe rate of ξ (offer). When ξ=0.4 (40%) there are more number users in the system. when companies offer discounts (or) extra resources of their services, users are more interested in joining and staying in the queue for getting services so this will definitely reduce balking loss and getting more user responses.Figure 5 demonstrates the mean number of users in the system (system-size). The system size is getting maximized when we increase the rate of λ with ξ=35%. Table 1 shows the impact of λ and μ on system throughput when ξ=10% and Table 2 it is evident that there is more system throughput than in table 1 due to ξ=20%(0.2). From table 3 it is evident that there is an increase in E(N), **ϕ** (T-E-R) and ϴ (T-E-P) of the system when there is an increase in rate offer "ξ".From table 4 it is evident that there is an increase in E(N),**ϕ** (T-E-R) and Θ (T-E-P) of the system when there is an increase in rate λwith offerξ=20%and from table 5 it is evident that there is an increase in **ϕ** (T-E-R) and Θ (T-E-P) of the system than in table 4 due to the more data offer(ξ=25%) in table 4.In table 6 and table 7 it is clear that there is an increase in T-E-C and decrease in E(N),**ϕ** (T-E-R) and Θ (T-E-P) of the system when there is an increase in rate "β" balking. But table 7 has more (T-E-R) and Θ (T-E-P) than in table 6 because of more consumer encouragement (ξ=30%) in table7. From table 9 it is clear that there is an increase in E(N), **ϕ** (T-E-R) and Θ (T-E-P) of the system than in table 8because of more offer(ξ=20%) in table 9.From table 10 that there is an increase in E(N) and decrease in **ϕ** (T-E-R) and Θ (T-E-P) when we increase the threshold values.From table 11 it is apparent that there is an increase in E(N), **ϕ** (T-E-R) and Θ (T-E-P) of the systemthan in table 10 due to the more offer(ξ=30%).As per the above results encouraging consumers by certain offers will improve the system profit. users are more likely to join and remain in the line for services when firms give discounts (or extra resources), therefore this will undoubtedly reduce balking losses while improving user responses.

10.Conclusion

A Multiple vacation queueing system with a balking,encouraged arrival, vacation intervention, waiting serverand double threshold policy are investigated in this research. We maximized the system sizeandsystem throughput by maximizing the rate ofoffer "ξ".The model's outcomes enable the system analyst in understanding how to attractusers and power-saving mechanisms. It is logically assumed that when companies offer discounts on the prices (or) extra data of their services, users are more interested in joining and staying in the queue for services.The novelty of this research is the introduction ofencouraged arrival and balking in the presence of M/M/1/DV queuing systems**.** Furthermore, performance measures likethe system size probabilities and systemthroughput arederived. Simulation results and Profit-Revenue analysis with examplesare provided to investigate the outcome of system parameters. Also, the analytical conclusions that are proven by numerical examples may be helpful in a variety of oriented real-world applications to generate the results.

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