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Blood Flow through Unilateral Stenosis: A Mathematical Model Utilizing the Interpenetration Model

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Abstract

The present investigation deals with the numerical simulation of Newtonian fluid flow employing the control volume method, which simulates blood flow in a stenotic blood artery in the presence of magnetic effects. The induced magnetic effects are neglected. The process is described by use of an interpenetrating model of two-phase media. An artery side becomes lessened and an aneurysm is caused by the tangent stress pressure produced by a stenotic blood vessel. Under appropriate simplifying assumptions, we obtain from this model the Navier-Stokes equations in the free zone (which typically is, outside the stenosis region) and a system of equations generalizing the filtration equation in the stenotic region. Stenosis has been considered to be a porous material. The impact of both the Reynolds number and the degree of stenosis on the blood's flow patterns investigated. The result obtained is very sensitive to the values of the initial conditions and this helps to explain the condition of hypertension.

Keywords: Stenosis, Navier-Stokes equation, Mathematical Model, Interpenetration Model, blood artery.

2020 Mathematics Subject Classification: 92C10, 90C20.

1. Introduction

Stenosis is the narrowing of an artery due to unusual tissue development or arteriosclerotic deposits. Blood flow is hindered as the tumor expands into the artery's lumen. Stenosis movement could result from the obstruction injuring the divider's interior cells. Because each of them affects the other, the growth of stricture and the blood splattering over the hallway are consequently linked. Vein stenosis movement will cause real problems and disrupt the normal functioning of the vessel plan. A number of the leading causes of death worldwide are heart-related conditions. A momentary decrease in the oxygen or blood supply to the heart is the main cause of cardiac problems. The lack of certain lipids like cholesterol, calcium, cellular waste products, and so forth may be the consequence of a constriction or limitation in the blood supply to that location. Stenosis is the term we use for describing this deposit. Atherosclerosis may be an underlying cause in heart attacks. Arterial stenosis occurs due to the deposition of cholesterol on its walls. Stenoses restrict blood flow. This change in blood flow caused by stenosis can lead to the development of arterial disease and arterial deformity.

Many studies have been investigated to understand the nature of blood flow disturbances in stenosis. Both experimental and numerical methods have been used by various researchers to determine the development of stenosis and its magnitude [1-14]. It is assumed here that the movement in the vessels is laminar. To simulate the flow of blood in vessels with stenosis, we use an interpenetrating model. We consider stenosis as the second phase in an interpenetrating model for describing heterogeneous media with a fixed zero velocity. The area free from stenosis, the blood flow is described by the Navier-Stokes equations. We represent stenosis as a porous medium. Kumar and Agarwal [15] profound a computer modeling and analysis of the hydrodynamic parameters of blood flow through stenotic artery. Kumar [16] made a mathematical model of blood flow in arteries with porous effects.

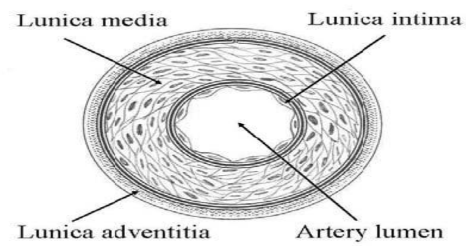

Figure 1. A cross section of the stenosed arterial vessel.

2. Formulation of problem and Mathematical model

Consider an interpenetrating model describing the flows of two-phase media in a Cartesian coordinate system (two-dimensional problem) [6-7]:

$$
\begin{aligned}
&\rho_i \frac{\partial u_i^k}{\partial t} + \sum_{k=1}^{2} \rho_i\, u_i^k = -f_i \frac{\partial p}{\partial x_j} \\
&+ \mu_i \sum_{k=1}^{2} \frac{1}{3}(\delta_i^k + 1) \\
&+ \mu_i \sum_{k=1}^{2} \Big(1 \\
&- \frac{5}{3}\delta_i^k\Big) \frac{\partial}{\partial x_k}\left(f_i \frac{\partial u_i^{3-j}}{\partial x_{3-k}}\right) \\
&+ K\left(u_{3-i}^j - u_i^j\right) + \rho_i\, g_i^j - Mui
\end{aligned}
\qquad (1)
$$

$$
\frac{\partial \rho_i}{\partial t} + \sum_{k=1}^{2} \frac{\partial\left(\rho_i u_i^k\right)}{\partial x_k} = 0, \qquad (2)
$$

$$
f_1 + f_2 = 1. \qquad (3)
$$

Here, u_i^j– the j-th velocity component ,p–pressure,f_i–volumetric concentration of the i–phase, μ_i–the viscosity of the i–phase, K–the phase interaction coefficient, ρ_i,ρ_{0i}–are the reduced and true density of the i–phase (ρ_{0i}–const)

g^j– the j-th component of the i–phase mass force. $_i$

If we represent in the equations that the second phase is stationary, we obtain a system of equations describing the flow of a liquid through a stationary porous layer:

$$\rho\frac{\partial u^j}{\partial t}+\sum_{k=1}^{2}\rho u^k\frac{\partial u^j}{\partial x_k}=-f\frac{\partial p}{\partial x_j}+\mu\sum_{k=1}^{2}\left(\frac{1}{3}\delta_j^k+1\right)\frac{\partial}{\partial x_k}\left(f\frac{\partial u^j}{\partial x_k}\right)+$$
$$+\mu\sum_{k=1}^{2}\left(1-\frac{5}{3}\delta_j^k\right)\frac{\partial}{\partial x_k}\left(f\frac{\partial u^{3-j}}{\partial x_{3-k}}\right)-Ku_j+\rho g^j, \tag{4}$$

$$\sum_{k=1}^{2}\frac{\partial\left(\rho u^k\right)}{\partial x_k}=0. \tag{5}$$

At accepted in porous media as given below:

$$K=\frac{150(1-f)^2}{d^2f^2}.$$

$$fu\frac{\partial u}{\partial x}+fv\frac{\partial u}{\partial x}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial x}+\frac{4}{3\,\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial u}{\partial x}\right)+$$
$$+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)-\frac{2}{3\,\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-f)^2}{f^3}u, \tag{6}$$

$$fu\frac{\partial v}{\partial x}+fv\frac{\partial v}{\partial x}=-\frac{f}{\mathrm{Re}}\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial x}\right)+$$
$$+\frac{4}{3\,\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial y}\right)+\frac{1}{\mathrm{Re}}\frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial y}\right)-\frac{2}{3\,\mathrm{Re}}\frac{\partial}{\partial y}\left(f\frac{\partial v}{\partial x}\right)-\frac{D^2}{\mathrm{Re}}\frac{(1-f)^2}{f^3}v, \tag{7}$$

$$\frac{\partial(f\,u)}{\partial x}+\frac{\partial(f\,v)}{\partial x}=0. \tag{8}$$

Where u,v—are the longitudinal and transverse components of the velocity ,x, y—are the Cartesian coordinates, Re=$hU\square/\mu$—is The Reynolds number, $\boldsymbol{U}$—is the characteristic velocity, h—is the characteristic size, $D=\sqrt{\alpha}h/d$ is the characteristic size of the porous medium, α—coefficient of proportionality. By using suitable generalizations, the control volume approach [8] was utilize for the numerical resolution of the system of equations (6) - (8). Thickness close to the stenosis provides an irregular coordinated mesh.

3. Boundary conditions

The vessel's entrance presents a parabolic velocity distribution law; the soft condition is at the output, and the no-slip condition is on the soft vessel wall:

$N\ x=0,\ u=6y(1-y),\ v=0, \qquad p=p^0.$

Assuming there is sufficient space, we generate the soft condition.

$$x = L, \qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = 0.$$

4. **Modeling of Stenosis :** Based on the model (6–8), consider estimating the contribution of stenosis in blood vessels. Let's use the form seen in [9] as an example of stenosis. The stenosis appears to be asymmetrical (Fig. 1). The formula determines the stenosis's form.

$$y(x) = \frac{h_0}{2}\left[1 + \cos\frac{2\pi}{L_0}\left(x - d - \frac{L_0}{2}\right)\right].$$

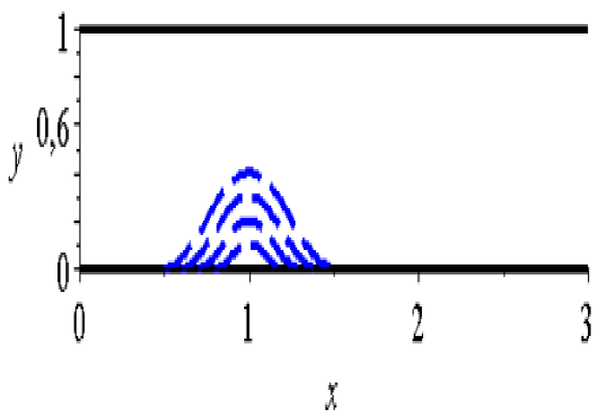

Figure: 2. Representation of area of flow with stenosis

5. Results and Discussion

Using the computation to developed, a series of numerical experiments have been carried out to study the flow behaviour of blood through a porous medium stenosed artery with magnetic effects and to investigate the significance of effect on blood flow for various cases of stenosis severity. Limit obstacles play a crucial role in computing solutions for real-world problems that are replicated. Blood particles follow the inward surface of the blood vessel portion under discussion, so the superficial pivotal quickness (u) of blood atoms, which relates to a one-layered stream, could be compared to the bearing speed of vascular divider material. In the computation, the parameters of stenosis were taken: *d (*0,5); Out in the area [0,1]x[0,*L*](*L* 4).Number of nodes 50x30.

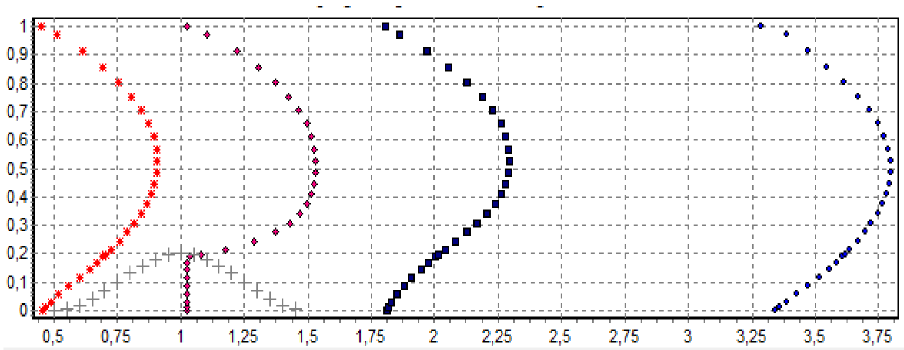

Figure: 3 Velocity profile over the sections of the vessel, the fullness of the vessel is 20% and M=1

In fig. 3 depicts the distribution of the velocity in different sections: the asterisks represent the distribution of the velocity in the section: diamond – in cross section x =0,45; rectangles-x =1,025and circles-x =3,337.Thecrosses represent the border of the stenosis.

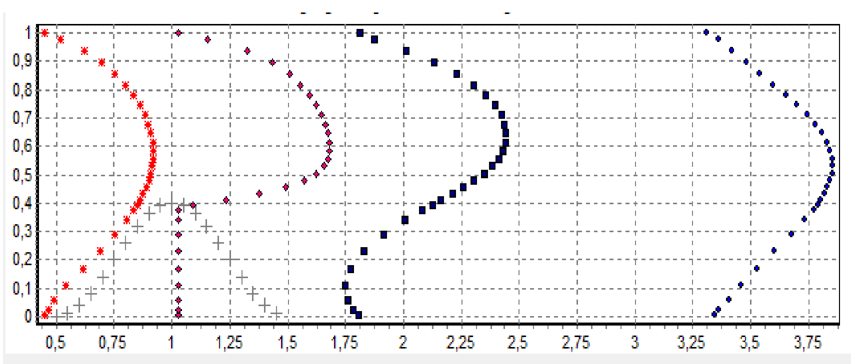

Figure: 4 Velocity profile over the sections of the vessel, the fullness of the vessel is 40% and M=1

Figure3depicts the calculation results for the same parameter values. The difference lies in the fullness of the vessel. In this case, there is a circulating flow behind the stenosis in the area $x \in [1,3;2,56]$. For example, in Fig.3 the graph of the velocity Distribution corresponding to the cross section x=1,809 in the lower part is observed reverse flow. However, with the same parameters, but with porosity f=0,6, there is no circulation flow.

With an increase in the filling of the vessel (Fig.5),an increase in the zone of reverse flows is observed. The secondary flow covers the zone $x \in [1,25;3,8]$.

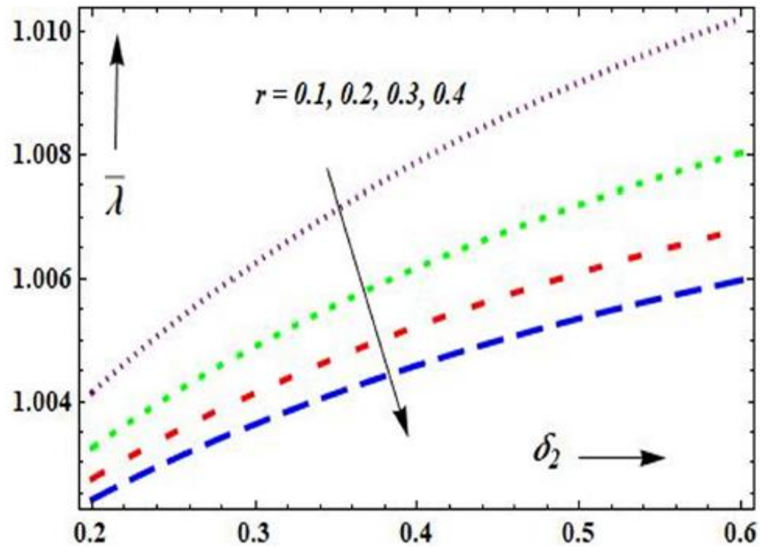

Figure:5 Velocity profile over the cross-sections of the vessel, the fullness of the vessel 50 percentage and M=1/

6. Conclusion

Through this study, we have created a mathematical model that accounts for the of the stenosis in order to simulate magneto hydrodynamic (MHD) blood flow in a stenoses artery under porous media. On the basis of an interpenetrating model of the flow of heterogeneous media, a mathematical description of the flow of blood with porous stenosis is proposed. For two-dimensional problems, a discrete model of the equation is constructed using the control volume method. The results of calculations and the influence of grid parameters, reflecting the legitimacy of the application of the model and the proposed numerical approach, are presented. We hope that our investigation may be helpful for the medical practitioners and other persons in the area of bio fluid dynamics to understand the flow of blood in the presence of magnetic field. The effects of a magnetic field have been used to control the flow, which may be useful in certain hypertension cases, etc.

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