



MEASURING DISTANCE BETWEEN INTUITIONISTIC FUZZY BINARY SOFT SETS – TWO TERM APPROACH

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ABSTRACT

Maji et al. pioneered a basic inquiry into the field of uncertainty handling by initiating a study involving fuzzy sets and soft sets. In 2016, Acikgoz and Nikal Tas [2] laid the framework for later advances by defining the fundamental structures of binary soft sets across two initial universal sets, U_1 and U_2 . In 2020, J. Subhashini and P. Gino Metilda [5] investigated the fundamental structure of fuzzy binary soft sets, gaining valuable insights into the merging of fuzzy and soft set theories. Building on these advances, we introduced an extension namely, Intuitionistic Fuzzy Binary Soft Sets (IFBSS), over two initial universal sets, U_1 and U_2 . We define the distance between Intuitionistic Fuzzy Binary Soft Sets using two term approach (involves both the membership and non-membership function). We present four fundamental distance metrics: Hamming distance, normalized Hamming distance, Euclidean distance, and normalized Euclidean distance.

Keywords: Intuitionistic fuzzy, Soft set, Hamming, Euclidean

1.INTRODUCTION:

In 1965, Zadeh [11] introduced a new method for analysing fuzzy set theory. The Russian researcher Molodtsov [1] first proposed soft set theory in 1999. Soft set theory is a whole new way to model fuzzy, vaguely defined objects. Then fuzzy soft set theory was proposed by Maji et al. [4] and its applications were further explained. In 1983, K. T. Atanassov [3] presented the concept of an Intuitionistic Fuzzy set. Acikgoz and Tas [2] recently presented a Binary Soft set over two universal sets U_1 and U_2 . In 2020, Dr. J. Subhashini and P. Gino Metilda [5] proposed and analyzed the concept of a Fuzzy Binary Soft Set. Here, we present a new definition for distances between Intuitionistic Fuzzy Binary Soft Sets.

In this paper, we would like to measure the distance between two Intuitionistic Fuzzy Binary Soft Sets using two term approach (involves both the membership and non-membership function). We present four fundamental distance metrics: Hamming distance, normalized Hamming distance, Euclidean distance, and normalized Euclidean distance with suitable examples. Also, we define a new algorithm to measure the distance between two Intuitionistic Fuzzy Binary Soft Sets and gave a real-life problem as an example.

2.1 Definitions and Preliminary Results:

Definition 2.1.1 [1] Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E . A pair (F, A) is called a **soft set** over U and is defined by the set

$$(F, A) = \{(e, F(e)): e \in A, F(e) \in P(U)\},$$

where F is a mapping given by $F: A \rightarrow P(U)$ such that $F(e) = \emptyset$ if $e \notin A$

Definition 2.1.2. [4] Let U be a common universe. Let E be a set of parameters and $A \subseteq E$. Then a pair (F, A) is called a **fuzzy soft set** over U , where F is a mapping given by $F: A \rightarrow F(U)$.

Definition 2.1.3. [2] Let U_1, U_2 be two initial universal sets and E be a set of parameters. Let $P(U_1), P(U_2)$ denote the power set of U_1, U_2 respectively. Also, Let $A, B \subseteq E$. A pair (F, A) is said to be a **binary soft set** over U_1, U_2 , where F is defined as below:

$$F: A \rightarrow P(U_1) \times P(U_2)$$

$$F(e) = (X, Y) \text{ for each } e \in A \text{ such that } X \subseteq U_1, Y \subseteq U_2$$

Definition 2.1.4. [3] Let X be a crisp finite non-empty set. An intuitionistic fuzzy subset \tilde{A} of X is defined by,

$$\tilde{A} = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$$

where the functions μ_A and ν_A are defined from X to $[0,1]$ with the condition

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \text{ for all } x \in X$$

The values $\mu_A(x)$ and $v_A(x)$ denote the membership degree and non-membership degree of the $x \in X$, respectively. The value $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is called the degree of uncertainty of the element $x \in X$ to the intuitionistic fuzzy set \tilde{A} .

Moreover, universal and intuitionistic fuzzy empty set are defined by,

$$\tilde{X} = \{(x, 1, 0) : x \in X\} \text{ and } \tilde{\emptyset} = \{(x, 0, 1) : x \in X\}$$

Definition2.1.5. [10] Let $\tilde{F}(U)$ be a set of all intuitionistic fuzzy set over U . Then an **intuitionistic fuzzy soft set** Γ_A over $\tilde{F}(U)$ is a set defined by a function representing a mapping,

$$\gamma_A(x) : E \rightarrow \tilde{F}(U) \text{ such that } \gamma_A(x) = \tilde{\emptyset} \text{ if } x \notin A$$

where, \emptyset is the intuitionistic fuzzy empty set. The function γ_A is called approximate function of the intuitionistic fuzzy soft set Γ_A . The value $\gamma_A(x)$ is an intuitionistic fuzzy set called x -element of Γ_A for all $x \in E$. So, it is defined by

$$\gamma_A(x) = \{(u, \mu_{\gamma_A}(u), v_{\gamma_A}(u)) : u \in U\}$$

for all $x \in E$. Here the functions μ_A and v_A are defined from U to $[0, 1]$ with the condition

$$0 \leq \mu_{\gamma_A(x)}(u) + v_{\gamma_A(x)}(u) \leq 1$$

denote the membership degree and non-membership degree of $u \in U$ to the intuitionistic fuzzy set $\gamma_A(x)$, respectively.

An Intuitionistic fuzzy set over $\tilde{F}(U)$ can also be represented by the set of ordered pairs,

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in X\}$$

2.2. INTUITIONISTIC FUZZY BINARY SOFT SET:

Let U_1, U_2 , be the two initial universal sets and E be a set of parameters. Let $F(U_1)$ and $F(U_2)$ denote the set of all fuzzy sets of U_1 and U_2 , respectively. Let $A, B \subset E$. A pair (F_{IFBSS}, A) is said to be an Intuitionistic fuzzy binary soft set over U_1, U_2 , where F_{IFBSS} is a mapping given by,

$$F_{IFBSS} : F(U_1) \times F(U_2) \rightarrow [0,1]$$

Then, Intuitionistic fuzzy binary soft sets can be written as,

$$(F_{IFBSS}, A) = \{(x, \gamma_A(x), \delta_A(x)) : x \in E\}$$

The value $\gamma_A(x)$ is an Intuitionistic fuzzy soft set called x -element of F_{IFBSS} for all $x \in E$, is defined by,

$$\gamma_A(x) = \{ (u_1, \mu_{\gamma_A}(x)(u_1), \nu_{\gamma_A}(x)(u_1)) : u_1 \in E \}$$

The value $\delta_A(x)$ is an Intuitionistic fuzzy soft set called x-element of F_{IFBSS} for all $x \in E$, is defined by,

$$\delta_A(x) = \{ (u_2, \mu_{\delta_A}(x)(u_2), \nu_{\delta_A}(x)(u_2)) : u_2 \in E \}$$

For all $x \in E$, Here the functions $\mu_{\gamma_A}(x), \mu_{\delta_A}(x)$ and $\nu_{\gamma_A}(x), \nu_{\delta_A}(x)$ are the membership and non-membership functions of defined from U_1, U_2 , to $[0, 1]$ with the conditions, $0 \leq \mu_{\gamma_A}(x) + \nu_{\gamma_A}(x) \leq 1$ & $0 \leq \mu_{\delta_A}(x) + \nu_{\delta_A}(x) \leq 1$

Example 2.2.1. Let $U_1 = \{S_1, S_2, S_3, S_4\}$ is the set of Shirts

Let $U_2 = \{T_1, T_2, T_3, T_4\}$ is the set of T - Shirts

Let $E = \{P_1, P_2, P_3, P_4, P_5\}$ is the set of Parameters, Where, P_1 - Expensive, P_2 - Modern, P_3 - Quality, P_4 - Cheap, P_5 - Size. $A = \{P_1, P_2, P_3\} \subseteq E$.

$$\gamma_A(P_1) = \{(S_1, 0.4,0.6), (S_2, 0.2,0.7), (S_3, 0.7,0.2), (S_4, 0.4,0.4)\}$$

$$\gamma_A(P_2) = \{(S_1, 0.7,0.3), (S_2, 0.3,0.6), (S_3, 0.2,0.8), (S_4, 0.5,0.5)\}$$

$$\gamma_A(P_3) = \{(S_1, 0.4,0.3), (S_2, 0.7,0.2), (S_3, 0.8,0.1), (S_4, 0.6,0.3)\}$$

$$\delta_A(P_1) = \{(T_1, 0.2,0.8), (T_2, 0.5,0.5), (T_3, 0.4,0.6), (T_4, 0.3,0.6)\}$$

$$\delta_A(P_2) = \{(T_1, 0.3,0.7), (T_2, 0.8,0.1), (T_3, 0.6,0.3), (T_4, 0.4,0.4)\}$$

$$\delta_A(P_3) = \{(T_1, 0.5,0.4), (T_2, 0.7,0.2), (T_3, 0.9,0.1), (T_4, 0.6,0.3)\}$$

Then an Intuitionistic fuzzy binary soft set can be written as,

$$(F_{IFBSS}, A) = \{(P_1, (\{(S_1, 0.4,0.6), (S_2, 0.2,0.7), (S_3, 0.7,0.2), (S_4, 0.4,0.4)\} \{(T_1, 0.2,0.8), (T_2,0.5,0.5), (T_3, 0.4,0.6), (T_4, 0.3,0.6)\})), (P_2, (\{(S_1, 0.7,0.3), (S_2, 0.3,0.6), (S_3, 0.2,0.8), (S_4,0.5,0.5)\} \{(T_1, 0.3,0.7), (T_2, 0.8,0.1), (T_3, 0.6,0.3), (T_4, 0.4,0.4)\})), (P_3, (\{(S_1, 0.4,0.3), (S_2,0.7,0.2), (S_3, 0.8,0.1), (S_4, 0.6,0.3)\} \{(T_1, 0.5,0.4), (T_2, 0.7,0.2), (T_3, 0.9,0.1), (T_4,0.6,0.3)\})))\}$$

2.3 DISTANCE MEASURE FOR INTUITIONISTIC FUZZY BINARY SOFT SETS:

Distance measures have gained a lot of attention over the past decades because they can be used in a wide range of applications, including patten recognition, cluster analysis, approximation reasoning, image processing, medical diagnosis and decision-making. Several distance measures have been developed for fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets and so on.

In this section, we present four fundamental distance metrics: Hamming distance, normalized Hamming distance, Euclidean distance, and normalized Euclidean distance with suitable examples.

In this paper, we assume that an Intuitionistic Fuzzy Binary Soft Sets (F_{IFBSS}, A) and (G_{IFBSS}, B) have the same parameter set namely, $A = B$.

Definition 2.3.1

Let (F_{IFBSS}, A) and (G_{IFBSS}, B) be two Intuitionistic Fuzzy Binary Soft Sets over U_1 and U_2 . Let d be a mapping given by $d: IFBSS (U_1) \times IFBSS (U_2) \rightarrow [0, 1]$ is called a distance function between IFBSS if it satisfies the following conditions,

(D1) $d (F_{IFBSS}, G_{IFBSS}) \geq 0$

(D2) $d (F_{IFBSS}, G_{IFBSS}) = d (G_{IFBSS}, F_{IFBSS})$

(D3) $d (F_{IFBSS}, G_{IFBSS}) = 0$ if and only if $F_{IFBSS} = G_{IFBSS}$

(D4) Let (H_{IFBSS}, C) be an Intuitionistic Fuzzy Binary Soft set, if $F_{IFBSS} \subseteq G_{IFBSS} \subseteq H_{IFBSS}$ then $d (F_{IFBSS}, G_{IFBSS}) \leq d (F_{IFBSS}, H_{IFBSS})$ and $d (G_{IFBSS}, H_{IFBSS}) \leq d (F_{IFBSS}, H_{IFBSS})$

Definition 2.3.2

Let U_1 and U_2 be a universal set of elements, $E = \{p_1, p_2, e_3, \dots, p_m\}$ be a universal set of parameters. F_{IFBSS}, G_{IFBSS} be two Intuitionistic fuzzy binary soft sets for all $p \in E$ and d is the distance measure between IFBSS. We define some distance measure between IFBSS for all $p_i \in E$ and $u_j \in U_1, U_2$ which are given below:

i)Hamming Distance:

$$d_{IFBSS}(A, B)_H = \frac{1}{2n} \sum_{i=1}^n \sum_{j=1}^m [|\mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j)| + |\vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j)| + |\mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j)| + |\vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j)|]$$

ii)Euclidean Distance:

$$d_{IFBSS}(A, B)_E = \sqrt{\frac{1}{2n} \sum_{i=1}^n \sum_{j=1}^m [|\mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j)|^2 + |\vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j)|^2 + |\mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j)|^2 + |\vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j)|^2]}$$

iii) *Normalized Hamming Distance:*

$$d_{IFBSS}(A, B)_{nH} = \frac{1}{2nm} \sum_{i=1}^n \sum_{j=1}^m [|\mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j)| + |\vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j)| + |\mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j)| + |\vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j)|]$$

iv) *Normalized Euclidean Distance:*

$$d_{IFBSS}(A, B)_{nE} = \sqrt{\frac{1}{2nm} \sum_{i=1}^n \sum_{j=1}^m [|\mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j)|^2 + |\vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j)|^2 + |\mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j)|^2 + |\vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j)|^2]}$$

Example: 2.3.3

Let $U_1 = \{S_1, S_2, S_3\}$ be a set of Shirts, $U_2 = \{T_1, T_2, T_3\}$ be a set of T-shirts under the consideration of the person who is going to purchase. $E = \{P_1, P_2, P_3\}$ is the set of parameters where P_1 – Expensive, P_2 – Modern, P_3 – Quality. The Intuitionistic Fuzzy Binary soft set (F_{IFBSS}, A) is the choice of the first person, and the Intuitionistic fuzzy binary soft set (G_{IFBSS}, B) is the choice of the second person who is going to purchase the shirts and T-shirts. Let (F_{IFBSS}, A) and (G_{IFBSS}, B) are represented by two tables as follows.

$$(F_{IFBSS}, A) = \{(P_1, (\{(S_1, 0.4,0.6), (S_2, 0.2,0.7), (S_3, 0.7,0.2)\} \{(T_1, 0.2,0.8), (T_2,0.5, 0.5), (T_3, 0.4,0.6)\})), (P_2, (\{(S_1, 0.7,0.3), (S_2, 0.3,0.6), (S_3, 0.2,0.8)\} \{(T_1, 0.3,0.7), (T_2, 0.8,0.1), (T_3, 0.6,0.3)\})), (P_3, (\{(S_1, 0.4,0.3), (S_2,0.7,0.2), (S_3, 0.8,0.1)\} \{(T_1, 0.5,0.4), (T_2, 0.7,0.2), (T_3, 0.9,0.1)\})))\}$$

$$(G_{IFBSS}, B) = \{(P_1, (\{(S_1, 0.3,0.7), (S_2, 0.6,0.4), (S_3, 0.4,0.6)\} \{(T_1, 0.4,0.6), (T_2,0.3, 0.7), (T_3, 0.5,0.5)\})), (P_2, (\{(S_1, 0.4,0.5), (S_2, 0.3,0.7), (S_3, 0.6,0.4)\} \{(T_1, 0.5,0.5), (T_2, 0.6,0.4), (T_3, 0.4,0.5)\})), (P_3, (\{(S_1, 0.5,0.5), (S_2,0.7,0.3), (S_3, 0.5,0.5)\} \{(T_1, 0.8,0.2), (T_2, 0.5,0.5), (T_3, 0.2,0.8)\})))\}$$

i) *Hamming Distance:*

$$d_{IFBSS}(A, B)_H = \frac{1}{2n} \sum_{i=1}^n \sum_{j=1}^m [|\mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j)| + |\vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j)| + |\mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j)| + |\vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j)|]$$

$$d_{IFBSS}(A, B)_H = 1.7666$$

ii)Euclidean Distance:

$$d_{IFBSS}(A, B)_E = \sqrt{\frac{1}{2n} \sum_{i=1}^n \sum_{j=1}^m \left[\left| \mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j) \right|^2 + \left| \vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j) \right|^2 + \left| \mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j) \right|^2 + \left| \vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j) \right|^2 \right]}$$

$d_{IFBSS}(A, B)_E = 0.7078$

iii)Normalized Hamming Distance:

$$d_{IFBSS}(A, B)_{nH} = \frac{1}{2nm} \sum_{i=1}^n \sum_{j=1}^m \left[\left| \mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j) \right| + \left| \vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j) \right| + \left| \mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j) \right| + \left| \vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j) \right| \right]$$

$d_{IFBSS}(A, B)_{nH} = 0.2944$

iv)Normalized Euclidean Distance:

$$d_{IFBSS}(A, B)_{nE} = \sqrt{\frac{1}{2nm} \sum_{i=1}^n \sum_{j=1}^m \left[\left| \mu_{\gamma_{A(x_i)}}(u_j) - \mu_{\gamma_{B(x_i)}}(u_j) \right|^2 + \left| \vartheta_{\gamma_{A(x_i)}}(u_j) - \vartheta_{\gamma_{B(x_i)}}(u_j) \right|^2 + \left| \mu_{\delta_{A(x_i)}}(u_j) - \mu_{\delta_{B(x_i)}}(u_j) \right|^2 + \left| \vartheta_{\delta_{A(x_i)}}(u_j) - \vartheta_{\delta_{B(x_i)}}(u_j) \right|^2 \right]}$$

$d_{IFBSS}(A, B)_{nE} = 0.2890$

Remark: 2.3.4

When considering the distances computed for any pair of Intuitionistic Fuzzy Binary Soft Sets A and B, accounting for both membership and non-membership functions,

(i) $0 \leq d_{IFBSS}(A, B)_H \leq 2n$

(ii) $0 \leq d_{IFBSS}(A, B)_E \leq 2$

(iii) $0 \leq d_{IFBSS}(A, B)_{nH} \leq \sqrt{2n}$

(iv) $0 \leq d_{IFBSS}(A, B)_{nE} \leq \sqrt{2}$

Theorem 2.3.5

The distance measures outlined in the definitions (i) to (iv) represent distance functions applicable to Intuitionistic Fuzzy Binary Soft sets.

Proof:

The Proofs are straightforward and are omitted.

Subsequently, we present certain fundamental algebraic properties associated with the distance function. Since these properties are elementary, we provide statements without including the proofs.

Theorem 2.3.6

The following properties hold for any distance measure between IFBSS,

- i) $d(F_{IFBSS}, G_{IFBSS}) = d(F_{IFBSS} \cup G_{IFBSS}, G_{IFBSS} \cup F_{IFBSS})$
- ii) $d(F_{IFBSS}, F_{IFBSS} \cap G_{IFBSS}) = d(G_{IFBSS}, F_{IFBSS} \cup G_{IFBSS})$
- iii) $d(F_{IFBSS}, F_{IFBSS} \cup G_{IFBSS}) = d(G_{IFBSS}, F_{IFBSS} \cap G_{IFBSS})$
- iv) $d(F_{IFBSS}, G_{IFBSS}) = d(F_{IFBSS}^C, G_{IFBSS}^C)$

Theorem 2.3.7

If d is a distance measure between IFBSS, then for any $F_{IFBSS}, G_{IFBSS}, H_{IFBSS} \in IFBSS(U_1) \times IFBSS(U_2)$ then the following properties holds:

- i) $d(F_{IFBSS}, G_{IFBSS} \cup H_{IFBSS}) \leq d(F_{IFBSS}, G_{IFBSS}) + d(F_{IFBSS}, H_{IFBSS})$
- ii) $d(F_{IFBSS}, G_{IFBSS} \cap H_{IFBSS}) \leq d(F_{IFBSS}, G_{IFBSS}) + d(F_{IFBSS}, H_{IFBSS})$

2.4 APPLICATION**2.4.1 An Application of Intuitionistic Fuzzy Binary Soft Sets in Decision-Making Problem:**

In our daily lives, critical decision-making poses significant challenges. The utilization of Intuitionistic Fuzzy Binary Soft Sets evolves as a crucial tool in addressing these challenges. This paper examines into exploring and understanding the practical applications of Intuitionistic Fuzzy Binary Soft Sets in real-life scenarios. To illustrate, we introduce a decision-making problem and apply Intuitionistic Fuzzy Binary Soft Sets to solve it. The proposed algorithm is defined to systematically address and resolve the difficulties of the decision-making problem.

2.4.2 Algorithm:

Step 1: Input the Intuitionistic Fuzzy Binary Soft sets of A_1, A_2, \dots, A_k

Step 2: Provide the Intuitionistic Fuzzy Binary Soft Sets of B that require recognition.

Step 3: Calculate both normalized Hamming distance and normalized Euclidean distance between each set from step 1 and the set-in step 2.

Step 4: The final decision is to choose A_i with the **minimum** normalized Hamming distance and normalized Euclidean distance.

2.4.3 Problem:

Let us consider the scenario where, Mr. Rajesh wants to choose the best school for his child among the set of schools $U_1 = \{S_1, S_2, S_3\}$ for his child studies. As well as the best course among the set of courses $U_2 = \{C_1= \text{Biology}, C_2= \text{Computer Science}, C_3= \text{Business Mathematics}\}$ under his choice of parameters $E = \{P_1 = \text{Academic Excellence}, P_2 = \text{Teaching}, P_3 = \text{Infrastructure and Facilities}\}$. Also, Mr. Rajesh has the assistance of three counseling members to help in the decision-making problem. He will also provide his personal preferences and opinions to guide the selection of the best school and course for his child.

Now use the algorithm to solve the above decision-making problem.

Step 1: The three counseling members forms the Intuitionistic Fuzzy Binary Soft sets $(F_{IFBSS}, A_1), (G_{IFBSS}, A_2), (H_{IFBSS}, A_3)$ over U_1 and U_2 .

$$\begin{aligned}
 (F_{IFBSS}, A_1) &= \{(P_1, (\{(S_1, 0.2,0.5), (S_2, 0.4,0.5), (S_3, 0.1,0.6)\} \{(C_1, 0.3,0.4), (C_2,0.4, 0.4), \\
 &\quad (C_3, 0.2,0.7)\})), (P_2, (\{(S_1, 0.6,0.4), (S_2, 0.3,0.5), (S_3, 0.4,0.5)\} \{(C_1, 0.2,0.6), \\
 &\quad (C_2, 0.4,0.2), (C_3, 0.9,0.1)\})), (P_3, (\{(S_1, 0.9,0.2), (S_2,0.7,0.5), (S_3, 0.1,0.5)\} \\
 &\quad \{(C_1, 0.4,0.8), (C_2, 0.7,0.1), (C_3, 0.3,0.1)\}))\} \\
 (G_{IFBSS}, A_2) &= \{(P_1, (\{(S_1, 0.6,0.5), (S_2, 0.1,0.8), (S_3, 0.5,0.5)\} \{(C_1, 0.2,0.6), (C_2,0.7, 0.1), \\
 &\quad (C_3, 0.4,0.4)\})), (P_2, (\{(S_1, 0.2,0.3), (S_2, 0.4,0.3), (S_3, 0.8,0.1)\} \{(C_1, 0.3,0.5), \\
 &\quad (C_2, 0.2,0.4), (C_3, 0.5,0.7)\})), (P_3, (\{(S_1, 0.7,0.5), (S_2,0.8,0.1), (S_3, 0.3,0.3)\} \\
 &\quad \{(C_1, 0.9,0.2), (C_2, 0.7,0.3), (C_3, 0.4,0.4)\}))\} \\
 (H_{IFBSS}, A_3) &= \{(P_1, (\{(S_1, 0.9,0.1), (S_2, 0.5,0.4), (S_3, 0.8,0.3)\} \{(C_1, 0.3,0.7), (C_2,0.4, 0.8), \\
 &\quad (C_3, 0.7,0.3)\})), (P_2, (\{(S_1, 0.4,0.5), (S_2, 0.5,0.5), (S_3, 0.7,0.4)\} \{(C_1, 0.2,0.7), \\
 &\quad (C_2, 0.3,0.6), (C_3, 0.4,0.5)\})), (P_3, (\{(S_1, 0.3,0.4), (S_2,0.4,0.6), (S_3, 0.5,0.8)\} \\
 &\quad \{(C_1, 0.7,0.5), (C_2, 0.4,0.4), (C_3, 0.4,0.3)\}))\}
 \end{aligned}$$

Step 2:

Mr. Rajesh personal preferences and opinions to guide the selection of the best school and course for his child is given as (K_{IFBSS}, B) .

$$\begin{aligned}
 (K_{IFBSS}, B) &= \{(P_1, (\{(S_1, 0.9,0.2), (S_2, 0.5,0.1), (S_3, 0.8,0.2)\} \{(C_1, 0.3,0.7), (C_2,0.3, 0.5), \\
 &\quad (C_3, 0.7,0.1)\})), (P_2, (\{(S_1, 0.1,0.5), (S_2, 0.5,0.4), (S_3, 0.7,0.3)\} \{(C_1, 0.3,0.7), \\
 &\quad (C_2, 0.3,0.3), (C_3, 0.4,0.2)\})), (P_3, (\{(S_1, 0.2,0.4), (S_2,0.4,0.4), (S_3, 0.5,0.3)\} \\
 &\quad \{(C_1, 0.1,0.5), (C_2, 0.4,0.5), (C_3, 0.2,0.4)\}))\}
 \end{aligned}$$

Step 3: Calculate both normalized Hamming distance and normalized Euclidean distance between each set from step 1 and the set-in step 2.

i) Normalized Hamming distance:

$$\begin{aligned}
 d_{IFBSS}(A_1, B)_{nH} &= 0.27778 \\
 d_{IFBSS}(A_2, B)_{nH} &= 0.2694 \\
 d_{IFBSS}(A_3, B)_{nH} &= 0.1083
 \end{aligned}$$

ii) Normalized Euclidean distance:

$$\begin{aligned}
 d_{IFBSS}(A, B)_{nE} &= 0.3354 \\
 d_{IFBSS}(A_2, B)_{nE} &= 0.3224 \\
 d_{IFBSS}(A_3, B)_{nE} &= 0.17953
 \end{aligned}$$

Step 4

The normalized hamming distance and the normalized Euclidean distance between A_3 and B is the minimum distance. So, the choice between the council member A_3 and Mr. Rajesh is more similar.

Conclusion of the Problem:

The best decision for Mr. Rajesh based on his choice parameter together with the council members is the school S_3 and the course C_3 that is Business Mathematics.

CONCLUSION:

The combination of binary soft sets and Intuitionistic fuzzy sets provide a tool to modelling the recognition knowledge. Intuitionistic Fuzzy Binary Soft Set is a hybrid model of fuzzy and soft sets. In this paper we define the four fundamental distance metrics: Hamming distance, normalized Hamming distance, Euclidean distance, and normalized Euclidean distance with suitable examples. Also, we develop a new algorithm to find the distance between two Intuitionistic Fuzzy Binary Soft Sets with examples. In this paper, we would like to measure the distance between two Intuitionistic Fuzzy Binary Soft Sets using two term approach (involves both the membership and non-membership function). In future we will develop the distance measure between two Intuitionistic Fuzzy Binary Soft Set using three term approach (involves membership, non-membership and hesitancy value).

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