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On Square Difference Geometric Mean 3-Equitable Graphs

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ABSTRACT:

A Square Difference Geometric Mean (SDGM) 3-Equitable labeling of a graph $G = (V, E)$ is a mapping $f: V(G) \to \{0, 1, 2\}$ such that the induced mapping $g: E(G) \to \{0, 1, 2\}$ is defined by $\left\lceil \sqrt{|(f(u))^2 - (f(v))^2|} \right\rceil, \forall\, uv \in E(G)$ with the condition $|v_f(i) - v_f(j)| \leq 1$ and $|e_g(i) - e_g(j)| \leq 1$ for all $0 \leq i, j \leq 2$. Also, if $|(v_f + e_g)(i) - (v_f + e_g)(j)| \leq 1$ for all $0 \leq i, j \leq 2$ then the labeling is called perfect square difference geometric mean 3-equitable labeling. A graph is called a square difference geometric mean (SDGM) 3-Equitable graph if there exists a SDGM 3-equitable labeling and perfect square difference geometric mean 3-equitable graph if there exists a Perfect SDGM 3-Equitable labeling. In this paper we investigate the SDGM 3-Equitable labeling or Perfect SDGM 3-Equitable labeling of certain cycle related graphs such as alternate triangular cycle graph, flower graph and petersen graph.

Keywords: Alternate Triangular Cycle graph, Flower graph, Petersen graph, Square Difference Geometric Mean 3-Equitable Graph.

1. Introduction

A non-trivial, simple, finite and undirected graphs are considered in this article. An assignment of integers to the vertices or edges, or both subject to certain conditions is called graph labeling [3]. Cahit introduced the concept of cordial and 3-equitable labeling [1]. Ponraj et al., introduced

the concept of mean cordial labeling [5]. Geometric mean cordial labeling was introduced by K. Chitra Lakshmi, K. Nagarajan [2].
Motivated by these definitions, we define the new notion called Square Difference Geometric Mean (SDGM) 3-Equitable labeling. We investigate the SDGM 3-Equitable labeling of certain cycle related graphs such as Alternate Triangular Cycle graph, Flower graph and Petersen graph.

***Definition* 1.1 [4]:** An Alternate Triangular Cycle $A(C_{2n})$ is obtained from an even cycle $C_{2n} = \{u_1, v_1, u_2, v_2, \dots, u_n, v_n\}$ by joining u_i and v_i to a new vertex w_i. That is, every alternate edge of a cycle C_{2n} is replaced by C_3.
***Definition* 1.2 [6]:** A *Flower graph* Fl_n is the graph obtained from a helm by joining each pendant vertex to the central vertex of the helm.

***Definition* 1.3 [7]:** The *generalized Petersen graph* $P(n,k), (n > 2k)$ is defined to be a graph on $2n$ vertices with $V(P(n,k)) = \{v_i, u_i : 1 \le i \le n\}$ and $E\big(P(n,k)\big) = \{v_i v_{i+1}, v_i u_i, u_i u_{i+k} : 1 \le i \le n,\ \text{subscripts modulo } n\}$.

2. Main Results

***Definition* 2.1:**

A Square Difference Geometric Mean (SDGM) 3-Equitable labeling of a graph $G = (V, E)$ is a surjective mapping $f: V(G) \to \{0,1,2\}$ such that the induced mapping $g: E(G) \to \{0,1,2\}$ is defined by $\left\lceil \sqrt{|(f(u))^2 - (f(v))^2|} \right\rceil, \forall\, uv \in E(G)$ with the condition $\left|v_f(i) - v_f(j)\right| \le 1$ and $\left|e_g(i) - e_g(j)\right| \le 1$ for all $0 \le i, j \le 2$. Also, if $\left|(v_f + e_g)(i) - (v_f + e_g)(j)\right| \le 1$ for all $0 \le i, j \le 2$ then the labeling is called Perfect Square Difference Geometric Mean 3-Equitable labeling. A graph is called a Square Difference Geometric Mean (SDGM) 3-Equitable graph if there exists a SDGM 3-Equitable labeling and Perfect Square Difference Geometric Mean 3-Equitable graph if there exists a Perfect SDGM 3-Equitable labeling.
***Remarks 2*.1:** If we consider $f: V(G) \to \{0,1\}$, the definition 3.1 coincides with that of cordial labeling. Hence we consider $f: V(G) \to \{0,1,2\}$.

***Theorem* 2.1:** The Alternate Triangular Cycle graph $A(C_{2n})$ is a Perfect SDGM 3-Equitable graph $\forall\, n$.
Proof: Let G be a Alternate Triangular Cycle graph $A(C_{2n})$ with the vertex set $V(G) = \{u_i, v_i, w_i \,/\, 1 \le i \le n\}$ and the edge set $E(G) = \{u_i v_i, u_i w_i, v_i w_i / 1 \le i \le n\} \cup \{v_i u_{i+1} \,/\, 1 \le i \le n-1\} \cup \{v_n u_1\}$, $|V(G)| = l = 3n$ and $|E(G)| = k = 4n$. The Alternate Triangular Cycle graph $A(C_{2n})$ is shown in the following fig 2.1 (a).

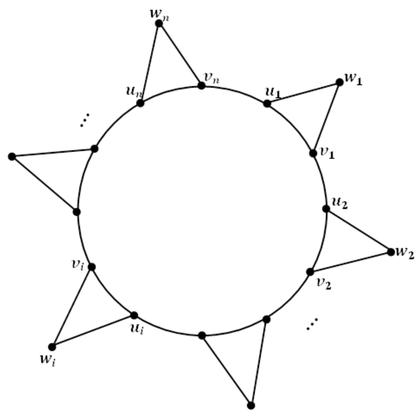

Fig 2.1 (a). Alternate Triangular Cycle graph $A(C_{2n})$

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

Case (i): $n \equiv 0(mod\ 3)$

$$f(u_i) = \begin{cases} 1, & i \equiv 1,2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n$$

$$f(v_i) = \begin{cases} 0, & i \equiv 1,0\ (mod\ 3) \\ 2, & i \equiv 2\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n$$

$$f(w_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n$$

Here $l \equiv 0(mod\ 3)$ i.e. $l = 3t$, so $v_f(0) = v_f(1) = v_f(2) = t$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = (v_f + e_g)(2) = t + s$.

Case (ii): $n \equiv 1(mod\ 3)$

$$f(u_i) = \begin{cases} 1, & i \equiv 1,2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n-1 \quad \text{and } f(u_n) = 0$$

$$f(v_i) = \begin{cases} 0, & i \equiv 1,0\ (mod\ 3) \\ 2, & i \equiv 2\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n-2 \text{ and } f(v_{n-1}) = 1, f(v_n) = 0$$

$$f(w_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n-1 \text{ and } f(w_n) = 2$$

Here $l \equiv 0(mod\ 3)$ i.e. $l = 3t$, so $v_f(0) = v_f(1) = v_f(2) = t$ and $k \equiv 1(mod\ 3)$ i.e. $k = 3s + 1$, so $e_g(0) = e_g(1) = s, e_g(2) = s + 1$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = t + s, (v_f + e_g)(2) = t + s + 1.$

Case (iii): $n \equiv 2(mod\ 3)$

$$f(u_i) = \begin{cases} 1, & i \equiv 1,2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases}$$ for all $1 \le i \le n-2$ and $f(u_{n-1}) = 0, f(u_n) = 2$

$$f(v_i) = \begin{cases} 0, & i \equiv 1,0\ (mod\ 3) \\ 2, & i \equiv 2\ (mod\ 3) \end{cases}$$ for all $1 \le i \le n-2$ and $f(v_{n-1}) = 1, f(v_n) = 0$

$$f(w_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 2\ (mod\ 3) \\ 2, & i \equiv 0\ (mod\ 3) \end{cases}$$ for all $1 \le i \le n-2$ and $f(w_{n-1}) = 1, f(w_n) = 2$

Here $l \equiv 0(mod\ 3)$ i.e. $l = 3t$, so $v_f(0) = v_f(1) = v_f(2) = t$ and $k \equiv 2(mod\ 3)$ i.e. $k = 3s + 2$, so $e_g(0) = e_g(2) = s + 1, e_g(1) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(2) = t + s + 1, (v_f + e_g)(1) = t + s.$

In all the above cases, we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_g(i) - e_g(j)| \le 1$ for all $0 \le i,j \le 2$. Also $|(v_f + e_g)(i) - (v_f + e_g)(j)| \le 1$ for all $0 \le i,j \le 2$.

Hence Alternate Triangular Cycle graph $A(C_{2n})$ is a Perfect SDGM 3-Equitable graph $\forall\ n$.

Illustration 2.1: Perfect SDGM 3-Equitable Labeling of Alternate Triangular Cycle graph $A(C_{12})$ is shown in fig 2.1 (b).

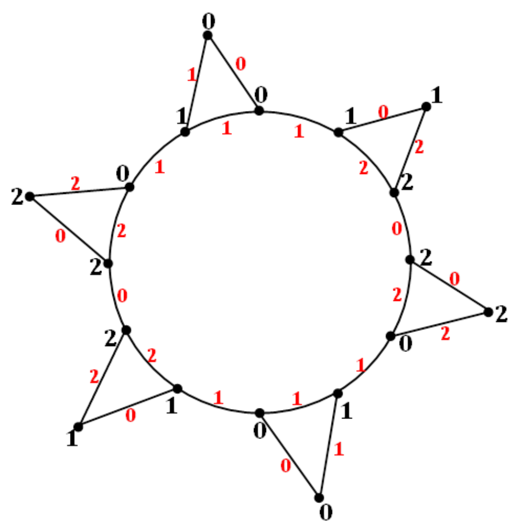

Fig 2.1 (b). Perfect SDGM 3-Equitable Labeling of Alternate Triangular Cycle graph $A(C_{12})$

Here $v_f(0) = v_f(1) = v_f(2) = 6$ and $e_g(0) = e_g(1) = e_g(2) = 8$.
Also $(v_f + e_g)(0) = (v_f + e_g)(1) = (v_f + e_g)(2) = 14$.

Therefore $|v_f(i) - v_f(j)| \leq 1$, $|e_g(i) - e_g(j)| \leq 1$ and $|(v_f + e_g)(i) - (v_f + e_g)(j)| \leq 1$ for all $0 \leq i, j \leq 2$.

***Theorem* 2.2:** The Flower graph Fl_n is a Perfect SDGM 3-Equitable graph when $n \equiv 0,1(mod\ 3)$ and SDGM 3-Equitable graph when $n \equiv 2(mod\ 3)$.

Proof: Let Fl_n be a Flower graph with vertex set $V(Fl_n) = \{u, u_i, v_i: 1 \leq i \leq n\}$ and edge set $E(Fl_n) = \{uu_i, uv_i, u_iv_i : 1 \leq i \leq n\} \cup \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_nu_1\}$. Let $|V(F_n)| = l$ and $|E(F_n)| = k$. Then $l = 2n + 1$ and $k = 4n$. Flower graph Fl_n is shown in the following fig 2.2 (a).

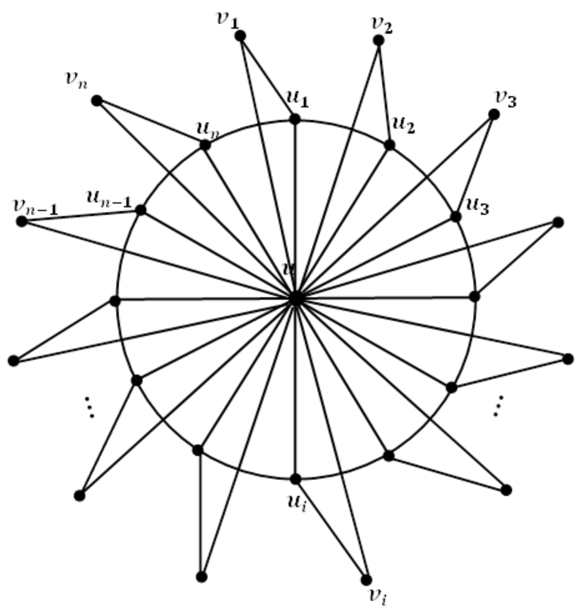

Fig 2.2 (a). Flower graph Fl_n

Define $f : V(G) \to \{0, 1, 2\}$ as follows:

Case (i): $n \equiv 0, 1(mod\ 3)$

$f(u) = 0$

$$f(u_i) = \begin{cases} 1, & i \equiv 1,2\ (mod\ 3) \\ 0, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \leq i \leq n$$

$$f(v_i) = \begin{cases} 2, & i \equiv 1,2\ (mod\ 3) \\ 0, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \leq i \leq n$$

Sub Case (i): $n \equiv 0(mod\ 3)$

Here $\equiv 1(mod\ 3)$ $l = 3t + 1$, so $v_f(0) = t + 1, v_f(1) = v_f(2) = t$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = t + s + 1, (v_f + e_g)(1) = (v_f + e_g)(2) = t + s$.

Sub Case (ii): $n \equiv 1(mod\ 3)$

Here $\equiv 0(mod\ 3)$ $l = 3t$, so $v_f(0) = v_f(1) = v_f(2) = t$ and $k \equiv 1(mod\ 3)$ i.e. $k = 3s + 1$, so $e_g(0) = e_g(1) = s, e_g(2) = s + 1$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = t + s, (v_f + e_g)(2) = t + s + 1$.

Case (ii): $n \equiv 2(mod\ 3)$
$f(u) = 0$

$$f(u_i) = \begin{cases} 1, & i \equiv 1,2\ (mod\ 3) \\ 0, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n$$

$$f(v_i) = \begin{cases} 2, & i \equiv 1,2\ (mod\ 3) \\ 0, & i \equiv 0\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le n-1 \text{ and } f(v_n) = 0$$

Here $\equiv 2(mod\ 3)$ $l = 3t + 2$, so $v_f(0) = v_f(1) = t + 1, v_f(2) = t$ and $k \equiv 2(mod\ 3)$ i.e. $k = 3s + 2$, so $e_g(0) = e_g(1) = s + 1, e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = t + s + 2, (v_f + e_g)(2) = t + s$.

In case (i), we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_g(i) - e_g(j)| \le 1$ for all $0 \le i, j \le 2$. Also $|(v_f + e_g)(i) - (v_f + e_g)(j)| \le 1$ for all $0 \le i, j \le 2$ and in case (ii) we see that $|v_f(i) - v_f(j)| \le 1$, $|e_g(i) - e_g(j)| \le 1$ and $|(v_f + e_g)(i) - (v_f + e_g)(j)| \nleq 1$ for all $0 \le i, j \le 2$.

Hence Flower graph Fl_n is a Perfect SDGM 3-Equitable graph when $n \equiv 0,1(mod\ 3)$ and SDGM 3-Equitable graph when $n \equiv 2(mod\ 3)$.

Illustration 2.2: Perfect SDGM 3-Equitable Labeling of Flower graph Fl_{12} is shown in fig 2.2 (b).

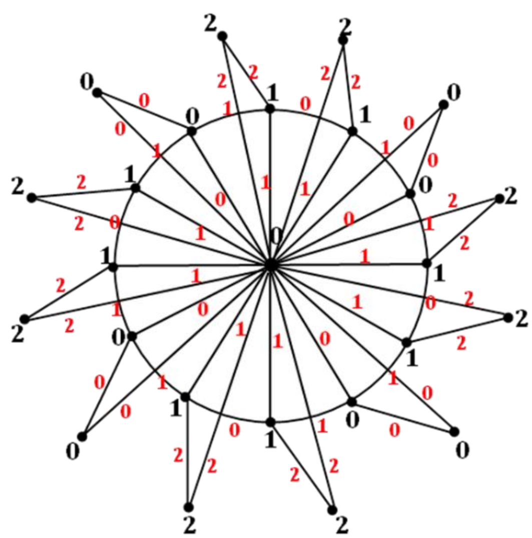

Fig 2.2 (b). Perfect SDGM 3-Equitable Labeling of Flower graph Fl_{12}

Here $v_f(0) = 9, v_f(1) = v_f(2) = 8$ and $e_g(0) = e_g(1) = e_g(2) = 16$.
Also $(v_f + e_g)(0) = 25$, $(v_f + e_g)(1) = (v_f + e_g)(2) = 24$.
Therefore $|v_f(i) - v_f(j)| \leq 1$, $|e_g(i) - e_g(j)| \leq 1$ and $|(v_f + e_g)(i) - (v_f + e_g)(j)| \leq 1$ for all $0 \leq i, j \leq 2$.

***Theorem* 2.3:** Petersen graph $P(n, 2)$ with $n \geq 5$ is a Perfect SDGM 3-Equitable graph when $n \equiv 0,1,2,4,5 (mod\ 6)$.

Proof: Let G be a *Petersen graph* $P(n, 2)$ with the vertex set $V(G) = \{v_i, u_i \,/\, 1 \leq i \leq n\}$, where v_i be the outer vertices and u_i be the inner vertices and the edge set $E(G) = \{v_i v_{i+1} \,/\, 1 \leq i \leq n-1\} \cup \{v_n v_1\} \cup \{v_i u_i \,/\, 1 \leq i \leq n\} \cup \{u_i u_{i+2} \,/\, 1 \leq i \leq n-2\} \cup \{u_{n-1} u_1, u_n u_2\}$, $|V(G)| = 2n$ and $|E(G)| = 3n$. The Petersen graph $P(n, 2)$ is shown in the following fig 2.3 (a).

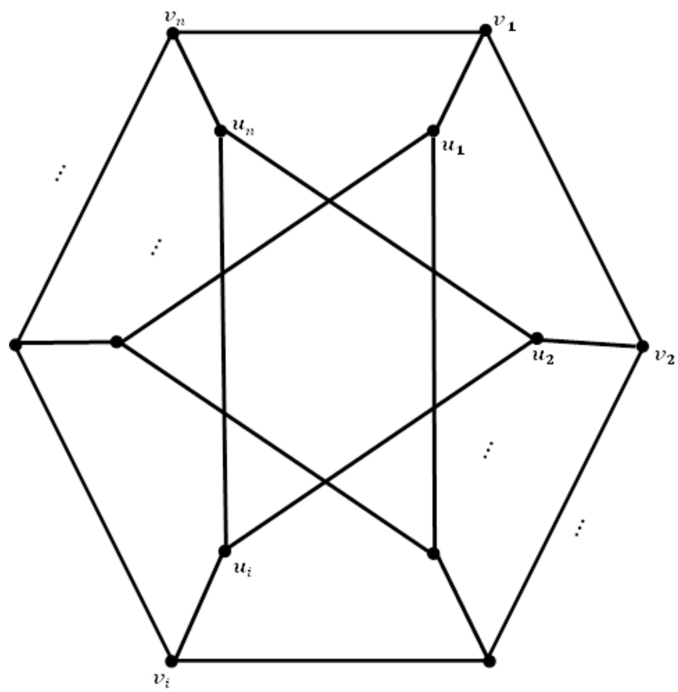

Fig 2.3 (a). Petersen Graph $P(n, 2)$

Define $f : V(G) \rightarrow \{0, 1, 2\}$ as follows:
Case (i): $n \equiv 0 (mod\ 6)$

$$f(v_i) = \begin{cases} 0, & i \equiv 1, 4 \ (mod\ 6) \\ 1, & i \equiv 2, 3 \ (mod\ 6) \\ 2, & i \equiv 0, 5 \ (mod\ 6) \end{cases} \quad \text{for all } 1 \leq i \leq n$$

$$f(u_i) = \begin{cases} 0, & i \equiv 1, 2 \ (mod\ 6) \\ 1, & i \equiv 3, 4 \ (mod\ 6) \\ 2, & i \equiv 0, 5 \ (mod\ 6) \end{cases} \quad \text{for all } 1 \leq i \leq n$$

Here $l \equiv 0(mod\ 3)$ i.e. $l = 3t$, so $v_f(0) = v_f(1) = v_f(2) = t$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = (v_f + e_g)(2) = t + s$.

Case (ii): $\boldsymbol{n \equiv 1(mod\ 6)}$

$$f(v_i) = \begin{cases} 0, & i \equiv 1,4\ (mod\ 6) \\ 1, & i \equiv 2,3\ (mod\ 6) \\ 2, & i \equiv 0,5\ (mod\ 6) \end{cases} \quad \text{for all } 1 \le i \le n-1$$

$$f(v_n) = 2$$

$$f(u_i) = \begin{cases} 0, & i \equiv 1,2\ (mod\ 6) \\ 1, & i \equiv 3,4\ (mod\ 6) \\ 2, & i \equiv 0,5\ (mod\ 6) \end{cases} \quad \text{for all } 1 \le i \le n-1$$

$$f(u_n) = 1$$

Here $l \equiv 2(mod\ 3)$ i.e. $l = 3t + 2$, so $v_f(0) = t, v_f(1) = v_f(2) = t + 1$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = t + s, (v_f + e_g)(1) = (v_f + e_g)(2) = t + s + 1$.

Case (iii): $\boldsymbol{n \equiv 2(mod\ 6)}$

$$f(v_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 0,2\ (mod\ 3) \end{cases} \quad \text{for all } 1 \le i \le \frac{n}{2}$$

$$f(v_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 2, & i \equiv 0,2\ (mod\ 3) \end{cases} \quad \text{for all } \frac{n}{2} + 1 \le i \le n-2$$

$$f(v_{n-1}) = f(v_n) = 2$$

$$f(u_i) = \begin{cases} 0, & i \equiv 2\ (mod\ 3) \\ 1, & i \equiv 0,1\ (mod\ 3) \end{cases} \quad \text{for all } 2 \le i \le \frac{n}{2} + 1$$

$$f(u_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 2, & i \equiv 0,2\ (mod\ 3) \end{cases} \quad \text{for all } \frac{n}{2} + 2 \le i \le n-1$$

$$f(u_1) = 0, f(u_n) = 1$$

Here $l \equiv 1(mod\ 3)$ i.e. $l = 3t + 1$, so $v_f(0) = v_f(1) = t, v_f(2) = t + 1$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = t + s, (v_f + e_g)(2) = t + s + 1$.

Case (iv): $n \equiv 4(mod\ 6)$

$f(v_i) = \begin{cases} 0, & i \equiv 0{,}5\ (mod\ 6) \\ 2, & i \equiv 1, 2{,}3{,}4\ (mod\ 6) \end{cases}$ for all $1 \le i \le n-6$

$f(v_{n-5}) = 1, \quad f(v_{n-4}) = f(v_{n-3}) = f(v_n) = 0, \quad f(v_{n-1}) = f(v_{n-2}) = 2$

$f(u_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 0, 2\ (mod\ 3) \end{cases}$ for all $1 \le i \le n$

Here $l \equiv 2(mod\ 3)$ i.e. $l = 3t + 2$, so $v_f(0) = v_f(1) = t + 1$, $v_f(2) = t$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(1) = t + s + 1$, $(v_f + e_g)(2) = t + s$.

Case (v): $n \equiv 5(mod\ 6)$

$f(v_i) = \begin{cases} 0, & i \equiv 1\ (mod\ 3) \\ 1, & i \equiv 0, 2\ (mod\ 3) \end{cases}$ for all $1 \le i \le \frac{n+1}{2}$

$f(v_i) = \begin{cases} 0, & i \equiv 0\ (mod\ 3) \\ 2, & i \equiv 1, 2\ (mod\ 3) \end{cases}$ for all $\frac{n+3}{2} \le i \le n$

$f(u_1) = 0, f(u_n) = 2$

$f(u_i) = \begin{cases} 0, & i \equiv 2\ (mod\ 3) \\ 1, & i \equiv 0, 1\ (mod\ 3) \end{cases}$ for all $2 \le i \le \frac{n+3}{2}$

$f(u_i) = \begin{cases} 0, & i \equiv 2\ (mod\ 3) \\ 2, & i \equiv 0, 1\ (mod\ 3) \end{cases}$ for all $\frac{n+5}{2} \le i \le n-1$

Here $l \equiv 1(mod\ 3)$ i.e. $l = 3t + 1$, so $v_f(0) = v_f(2) = t$, $v_f(1) = t + 1$ and $k \equiv 0(mod\ 3)$ i.e. $k = 3s$, so $e_g(0) = e_g(1) = e_g(2) = s$.

Also $(v_f + e_g)(0) = (v_f + e_g)(2) = t + s, (v_f + e_g)(1) = t + s + 1$.

In all the above cases, we see that $|v_f(i) - v_f(j)| \le 1$ and $|e_g(i) - e_g(j)| \le 1$ for all $0 \le i,j \le 2$. Also $|(v_f + e_g)(i) - (v_f + e_g)(j)| \le 1$ for all $0 \le i,j \le 2$.

Hence Petersen graph $P\,(n, 2)$ with $n \ge 5$ is a Perfect SDGM 3-Equitable graph when $n \equiv 0{,}1{,}2{,}4{,}5(mod\ 6)$.

Illustration 2.3: Perfect SDGM 3-Equitable Labeling of Petersen graph $P(6{,}2)$ is shown in fig 2.3 (b).

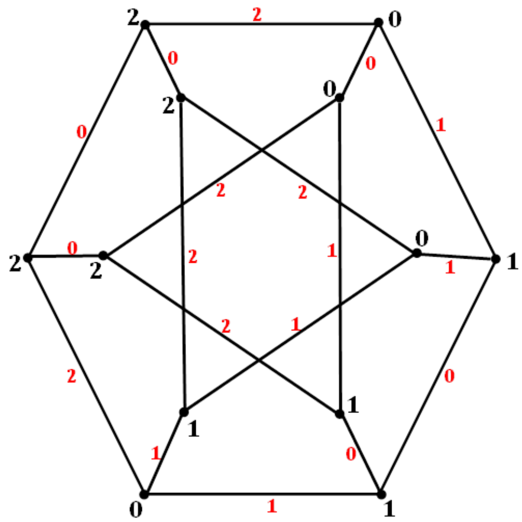

Fig 2.3 (b). Perfect SDGM 3-Equitable Labeling of Petersen Graph $P(6,2)$

Here $v_f(0) = v_f(1) = v_f(2) = 4$ and $e_g(0) = e_g(1) = e_g(2) = 6$.
Also $(v_f + e_g)(0) = (v_f + e_g)(1) = (v_f + e_g)(2) = 10$.
Therefore $|v_f(i) - v_f(j)| \leq 1$, $|e_g(i) - e_g(j)| \leq 1$ and $|(v_f + e_g)(i) - (v_f + e_g)(j)| \leq 1$ for all $0 \leq i, j \leq 2$.

3. Conclusion

In this paper we investigated the SDGM 3-Equitable labeling or Perfect SDGM 3-Equitable labeling of certain cycle related graphs such as Alternate Triangular Cycle graph, Flower graph and Petersen graph. The future work includes SDGM 3-Equitable labeling or Perfect SDGM 3-Equitable labeling of ladder related graphs, tree related graphs and some interconnection networks such as honeycomb network and benes network.

4. References

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