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Poisson-New Linear-Exponential Distribution

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Abstract:

A compound probability distribution with one parameter is the distribution that is suggested. Poisson-New Linear-Exponential Distribution (PNLED) is the result of mixing Poisson distribution with Sah's New Linear-Exponential Distribution (NLED) (2022). The key properties of this distribution have been deduced and described. The parameter of this has been approximated. In order to verify the validity of the theoretical study, some over-dispersed secondary data were fitted using goodness of fit, and the results showed that PNLED gives outperforms than Poisson-Lindley distribution (PLD) of Sankaran (1970).

Keywords: New Linear-Exponential distribution, Poisson-Lindley distribution, Moments, Estimation, Mixing.

Introduction:

Discipline is also important in research work when we compare two or more than two distributions. The proposed distribution is obtained by compounding Poisson distribution with NLED (see [10]) given by its probability density function (p.d.f.)

$$f_1(y;\alpha)=\frac{\alpha^2}{(1+\pi\alpha)}(\pi+y)e^{-\alpha y}\ ;y>0,\alpha>0 \qquad (1)$$

Poisson-Lindley distribution (PLD) was obtained (see [6]) by mixing Poisson distribution with Lindley distribution [5] given by its p.d.f.

$$f_2(y;\alpha)=\frac{\alpha^2}{(1+\alpha)}(1+y)e^{-\alpha y}\ ;y>0,\alpha>0 \qquad (2)$$

The expression (1) and (2) are of the same nature because both distributions are continuous in nature which depend on a single parameter. Hence, it is the appropriate reason to compare the proposed distribution with PLD having Probability mass function (p.m.f.)

$$P_3(Y;\alpha)=\frac{\alpha^2(y+\alpha+2)}{(\alpha+1)^{y+3}}\quad ;y=0,1,2,...\quad ;\alpha>0 \qquad (3)$$

The expression (3) was introduced by Sankaran [6] to model count data. The Probability mass function of Poisson-Mishra distribution (PMD) (see [9]), was given by

$$P_4(Y;\alpha)=\frac{\alpha^3[(1+\alpha)(y+\alpha+2)+(1+y)(2+y)]}{(\alpha^2+\alpha+1)(1+\alpha)^{y+3}}\quad ;y=0,1,2,...\quad ;\alpha>0 \qquad (4)$$

It has been obtained by mixing Poisson distribution with Mishra distribution [8]. The expression (4) is also based on a single parameter. Hence, we may also compare the proposed distribution with PMD (4).

This research is organized into several topics to provide organization. The introduction is placed in the first section. The second portion contains materials and methodology. The third part contains the results of this study, which are organized for simplicity and clarity under the following subheadings.
•The probability mass function of the Poisson-New Linear-Exponential distribution
•PNLED statistical moments and associated metrics
• Determining PNLED's specifications

Under the fourth part, utility and fit of this are discussed. This conclusion is placed in the last section. We also hope to convey through this research that the number of parameters and structure of the distributions we are comparing should be taken into consideration. We have defined and developed probability mass function, probability generating function, and moment generating function of this distribution. The moments regarding the origin and the mean have been presented and explained in a way that makes it easy to examine the variance, shape, and size of this distribution. Both the technique of moments and the likelihood approaches have been used to estimate the parameters of PNLED. This distribution may be used in the fields of biology, ecology, thunderstorms, mistake per page, and accident proneness. Previous studies have employed the goodness of fit method in the over-dispersed data. It has been discovered that PNLED, as opposed to PLD and PMD, provides a superior match to the data utilized in the application part.

Material and Methods:
This paper is based on the concept of compound probability distribution. It is obtained by mixing Poisson distribution with NLED. To conduct applications of this distribution, probability mass function, statistical moments and estimation of parameters have been discussed and derived.

Results:

The results obtained for this distribution are placed under the following sub-headings.

- Poisson-New Linear-exponential Distribution (PNLED) and its characteristics
- Statistical moments and related measures of PNLED
- Estimation of parameters of PNLED

Poisson-New Linear-exponential Distribution (PNLED) and its characteristics:

The Probability mass of function of PNLED can be extracted as follows and given by the expression (5).

$$P(Y;\alpha)=\int_0^\infty\left[\left(\frac{e^{-\lambda}\lambda^y}{y!}\right)\left(\frac{\alpha^2}{1+\pi\alpha}\right)(\pi+\lambda)e^{-\alpha\lambda}\right]d\lambda \ ;y=0,1,2,\ ...;\lambda>0;\alpha>0$$

$$=\left(\frac{\alpha^2}{(1+\pi\alpha)}\frac{1}{y!}\right)\left[\pi\int_0^\infty\lambda^y e^{-(1+\alpha)\lambda}d\lambda+\int_0^\infty\lambda^{y+1}e^{-(1+\alpha)\lambda}d\lambda\right]$$

$$=\left[\frac{\alpha^2}{(1+\pi\alpha}\right]\left[\frac{1+\pi(1+\alpha)+y}{(1+\alpha)^{y+2}}\right] \quad (5)$$

Graphical presentations of p.m.f. of PNLED:

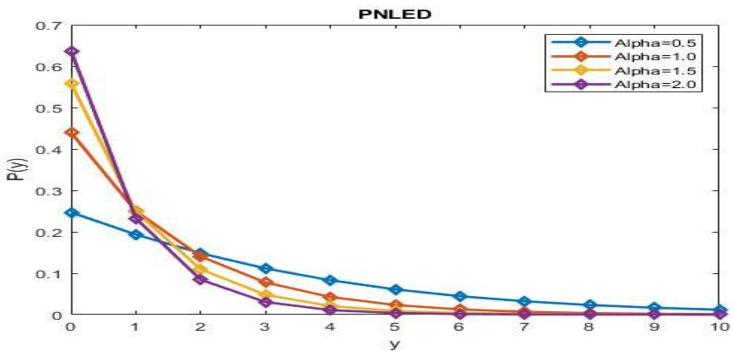

Figure 1. Probability Mass Function of PNLED at $\alpha=1,1.5,2.0,2.5$

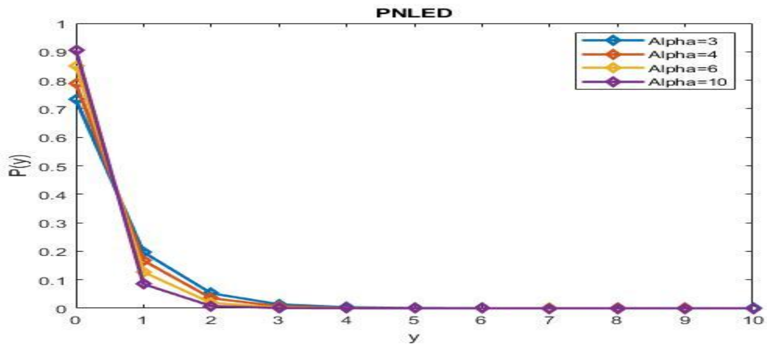

Figure 2. Probability Mass Function of PNLED at $\alpha = 3.0, 4.0, 6.0, 10.0$

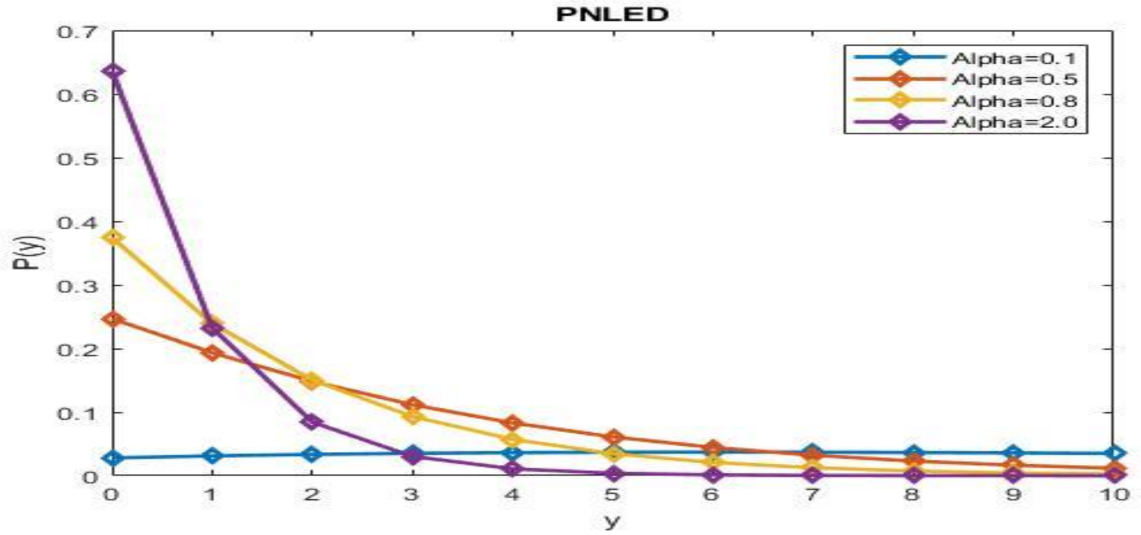

Fig.3 Probability Mass Function of PNLED at $\alpha = 0.1, 0.5, 0.8, 2.0$

Probability Generating Function(p.g.f.) of PNLED: It can be obtained as follows and given by the expression (6).

$$P_Y^{(t)} = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty e^{-\lambda(1-t)}(\pi+\lambda)e^{-\alpha\lambda}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty (\pi+\lambda)e^{-\lambda(1+\alpha-t)}d\lambda$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\left[\pi\int_0^\infty e^{-\lambda(1+\alpha-t)}d\lambda + \int_0^\infty \lambda e^{-\lambda(1+\alpha-t)}d\lambda\right] = \left(\frac{\alpha^2}{(1+\pi\alpha}\right)\left[\frac{1+\pi(1+\alpha-t)}{(1+\alpha-t)^2}\right]; \alpha > 0 \quad (6)$$

Moment Generating Function (M.G.F.) of PNLED:

$$M_Y^{(t)} = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty (\pi+\lambda)e^{-\lambda(1+\alpha-e^t)}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\left[\frac{\pi\Gamma 1}{(1+\alpha-e^t)^1} + \frac{\Gamma 2}{(1+\alpha-e^t)^2}\right]$$

$$= \left(\frac{\alpha^2}{(1+\pi\alpha}\right)\left[\frac{1+\pi(1+\alpha-e^t)}{(1+\alpha-e^t)^2}\right] \quad (7)$$

The first four moments about origin can also be obtained by using the following expression.

$$\mu_r' = \left[\frac{\partial^r[M_Y^{(t)}]}{\partial t^r}\right]_{t=0} \quad (8)$$

-Statistical Moments and Related Measures of PNLED:

$$\mu_r' = E\left[E(Y^r/\lambda\right] = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\sum_{y=0}^\infty \frac{y^r e^{-\lambda}\lambda^y}{y!}\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda \quad (9)$$

The expression (9) is the r^{th} moment about the origin of PNLED and put $r = 1, 2, 3, 4$ in the equation (9), we can obtain the 1^{st} four moments about origin of PNLED as follows.

$$\mu_1' = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\sum_{y=0}^\infty \frac{y^1 e^{-\lambda}\lambda^y}{y!}\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty(\lambda)(\pi+\lambda)e^{-\alpha\lambda}d\lambda$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\left[\int_0^\infty \pi\lambda e^{-\alpha\lambda}d\lambda + \int_0^\infty \lambda^2 e^{-\alpha\lambda}d\lambda\right]$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\left[\frac{\pi}{\alpha^2}+\frac{2}{\alpha^3}\right] = \frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)} \quad (10)$$

Graphical representation of the mean for varying values of alpha is given below.

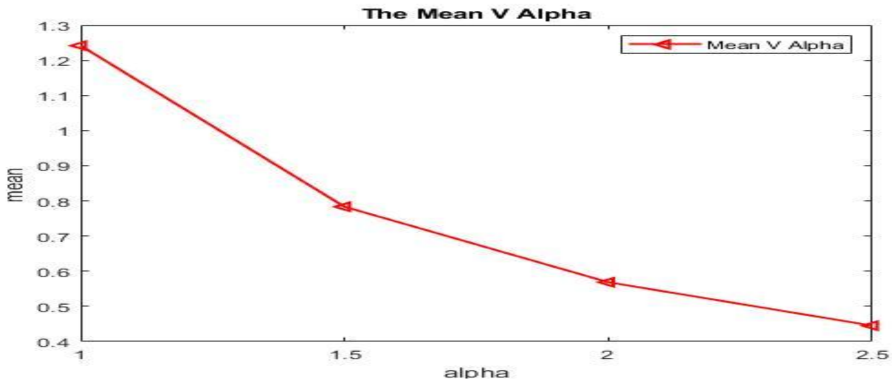

Fig.4 The mean of PNLED for $\alpha = 1.0, 1.5, 2.0, 2.5$

$$\mu_2' = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\sum_{y=0}^\infty \frac{y^2 e^{-\lambda}\lambda^y}{y!}\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\lambda+\lambda^2\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\left[\frac{\pi}{\alpha^2}+\frac{2(1+\pi)}{\alpha^3}+\frac{6}{\alpha^4}\right] = \frac{[\pi\alpha(\alpha+2)+2(\alpha+3)]}{\alpha^2(\pi\alpha+1)} \quad (11)$$

$$\mu_3' = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\sum_{y=0}^\infty \frac{y^3 e^{-\lambda}\lambda^y}{y!}\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\lambda^3+3\lambda^2+\lambda\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\pi\lambda+(1+3\pi)\lambda^2+(3+\pi)3\lambda^3+\lambda^4\right)e^{-\alpha\lambda}d\lambda$$

$$= \frac{1}{(1+\pi\alpha)}\left[\pi+\frac{2(1+3\pi)}{\alpha}+\frac{6(\pi+3)}{\alpha^2}+\frac{24}{\alpha^3}\right] = \frac{1}{(1+\pi\alpha)}\left[\left(\pi+\frac{2}{\alpha}\right)+\left(\frac{6\pi}{\alpha}+\frac{18}{\alpha^2}\right)+\left(\frac{6\pi}{\alpha^2}+\frac{24}{\alpha^3}\right)\right]$$

$$= \frac{\pi\alpha(\pi\alpha+2)}{\alpha(1+\pi\alpha)}+\frac{6(\pi\alpha+3)}{\alpha^2(1+\pi\alpha)}+\frac{6(\pi\alpha+4)}{\alpha^3(1+\pi\alpha)} \quad (12)$$

$$\mu_4' = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\sum_{y=0}^\infty \frac{y^4 e^{-\lambda}\lambda^y}{y!}\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda = \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\lambda^4+6\lambda^3+7\lambda^2+\lambda\right)(\pi+\lambda)e^{-\alpha\lambda}d\lambda$$

$$= \frac{\alpha^2}{(1+\pi\alpha)}\int_0^\infty\left(\pi\lambda+(1+7\pi)\lambda^2+(7+6\pi)\lambda^3+(\pi+6)\lambda^4+\lambda^5\right)e^{-\alpha\lambda}d\lambda$$

$$= \frac{1}{(1+\pi\alpha)}\left[\pi+\frac{(2+14\pi)}{\alpha}+\frac{6(7+6\pi)}{\alpha^2}+\frac{24(\pi+6)}{\alpha^3}+\frac{120}{\alpha 4}\right]$$

$$= \frac{1}{(1+\pi\alpha)}\left[\left(\pi+\frac{2}{\alpha}\right)+\left(\frac{14\pi}{\alpha}+\frac{42}{\alpha^2}\right)+\left(\frac{36\pi}{\alpha^2}+\frac{144}{\alpha^3}\right)+\left(\frac{24\pi}{\alpha^3}+\frac{120}{\alpha^4}\right)\right]$$

$$= \frac{\pi\alpha(\pi\alpha+2)}{\alpha(1+\pi\alpha)}+\frac{14(\pi\alpha+3)}{\alpha^2(1+\pi\alpha)}+\frac{36(\pi\alpha+4)}{\alpha^3(1+\pi\alpha)}+\frac{24(\pi\alpha+5)}{\alpha^4(1+\pi\alpha)} \quad (13)$$

Central Moments of PNLED:

The 1st four central moments of PNLED have been obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = \frac{[\pi\alpha(\alpha+2)+2(\alpha+3)]}{\alpha^2(\pi\alpha+1)} - \left(\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right)^2$$

$$= \frac{[\pi^2\alpha^3+(\pi^2+3\pi)\alpha^2+(4\pi+2)\alpha+2]}{[\alpha(\pi\alpha+1)]^2} \quad (14)$$

Theorem (1): *For PNLED*, Variance > Mean
Proof: We have to show

Variance > Mean

Or, $\frac{[\pi^2\alpha^3+(\pi^2+3\pi)\alpha^2+(4\pi+2)\alpha+2]}{[\alpha(\pi\alpha+1)]^2} > \frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}$

Or, $(\pi^2\alpha^3+\pi^2\alpha^2+3\pi\alpha^2+4\pi\alpha+2\alpha+2) > (\pi^2\alpha^3+3\pi\alpha^2+2\alpha)$

Or, $(\pi^2\alpha^2+4\pi\alpha+2) > 0$

Which is true because $\alpha > 0$.

Graphical Representation of μ_2 verses α :

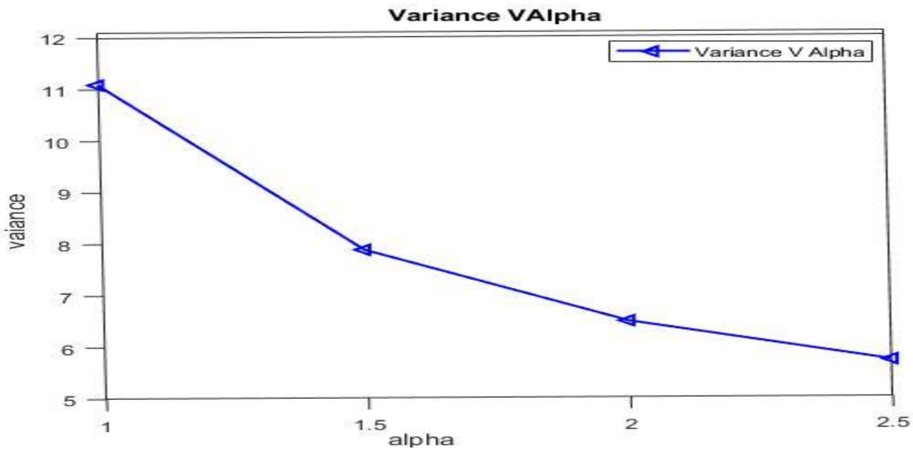

Fig.5 The Variance of PNLED for $\alpha = 1.0, 1.5, 2.0, 2.5$

The 3rd central moment can be obtained as follows

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= \left[\frac{\pi\alpha(\pi\alpha+2)}{\alpha(1+\pi\alpha)} + \frac{6(\pi\alpha+3)}{\alpha^2(1+\pi\alpha)} + \frac{6(\pi\alpha+4)}{\alpha^3(1+\pi\alpha)}\right] - 3\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)} + \frac{(\pi\alpha+6)}{\alpha^2(\pi\alpha+1)}\right]\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right] + 2\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right]^3$$

$$= \frac{[(\pi^3\alpha^3+2)(\alpha^2+3\alpha+2)+(\pi^2\alpha^2)(4\alpha^2+15\alpha+12)+(\pi\alpha)(5\alpha^2+18\alpha+12)]}{[\alpha(1+\pi\alpha)]^3} \quad (15)$$

The expression (15) is the 3rd central moment of PNLED.
The 4th central moment of PNLED can be obtained as

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 + 3(\mu_1')^4$$

$$= \left[\frac{\pi\alpha(\pi\alpha+2)}{\alpha(1+\pi\alpha)} + \frac{14(\pi\alpha+3)}{\alpha^2(1+\pi\alpha)} + \frac{36(\pi\alpha+4)}{\alpha^3(1+\pi\alpha)} + \frac{24(\pi\alpha+5)}{\alpha^4(1+\pi\alpha)}\right]$$

$$-4\left[\frac{\pi\alpha(\pi\alpha+2)}{\alpha(1+\pi\alpha)} + \frac{6(\pi\alpha+3)}{\alpha^2(1+\pi\alpha)} + \frac{6(\pi\alpha+4)}{\alpha^3(1+\pi\alpha)}\right]\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right]$$

$$+6\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)} + \frac{(\pi\alpha+6)}{\alpha^2(\pi\alpha+1)}\right]\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right]^2 - 3\left[\frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}\right]^4$$

$$= \left[\frac{\begin{array}{l}[(24+48\alpha+26\alpha^2+2\alpha^3)+(\pi\alpha)(96+180\alpha+92\alpha^2+7\alpha^3)\\ +(\pi\alpha)^2(132+240\alpha+116\alpha^2+9\alpha^3)\\ +(\pi\alpha)^3(72+126\alpha+60\alpha^2+5\alpha^3)+(\pi\alpha)^4(9+18\alpha+10\alpha^2+\alpha^3)]\end{array}}{[\alpha(1+\pi\alpha)]^4} \right] \quad (16)$$

The expression (16) is the 4th central moment of PNLED.

The nature of PNLED according to shape and size:

$$\gamma_1 = \frac{\mu_3}{(\mu_2)^{3/2}} = \frac{[(\pi^3\alpha^3+2)(\alpha^2+3\alpha+2)+(\pi^2\alpha^2)(4\alpha^2+15\alpha+12)+(\pi\alpha)(5\alpha^2+18\alpha+12)]}{[\pi^2\alpha^3+(\pi^2+3\pi)\alpha^2+(4\pi+2)\alpha+2]^{3/2}} \quad (17)$$

The graphical representation of γ_1 and α :

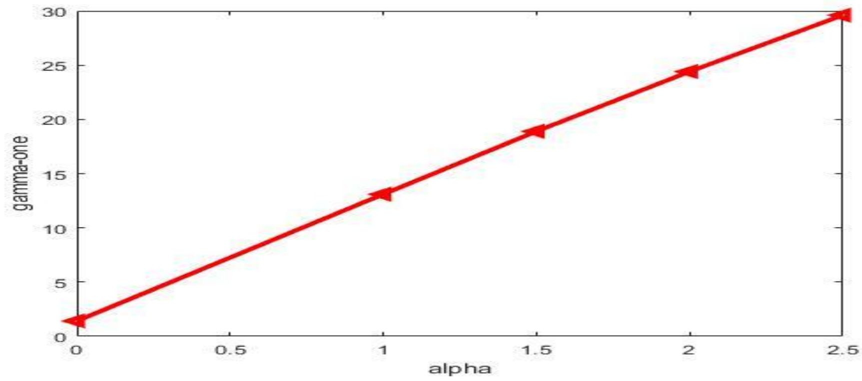

Figure (6): γ_1 of PNLED for $\alpha = 1.0, 1.5, 2.0, 2.5$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2}$$

$$= \left[\frac{\begin{array}{l}[(24+48\alpha+26\alpha^2+2\alpha^3)+(\pi\alpha)(96+180\alpha+92\alpha^2+7\alpha^3)\\ +(\pi\alpha)^2(132+240\alpha+116\alpha^2+9\alpha^3)\\ +(\pi\alpha)^3(72+126\alpha+60\alpha^2+5\alpha^3)+(\pi\alpha)^4(9+18\alpha+10\alpha^2+\alpha^3)]\end{array}}{[\pi^2\alpha^3+(\pi^2+3\pi)\alpha^2+(4\pi+2)\alpha+2]^2} \right] \quad (18)$$

The expression (17) and (18) are the co-efficient of skewness and kurtosis based on moments and it has been found that $(\sqrt{2}) < \gamma_1 < \infty$ and $6 < \beta_2 < \infty$ .Hence, this distribution is Positively skewed by shape and leptokurtic by size.

The graphical representation of β_2 :

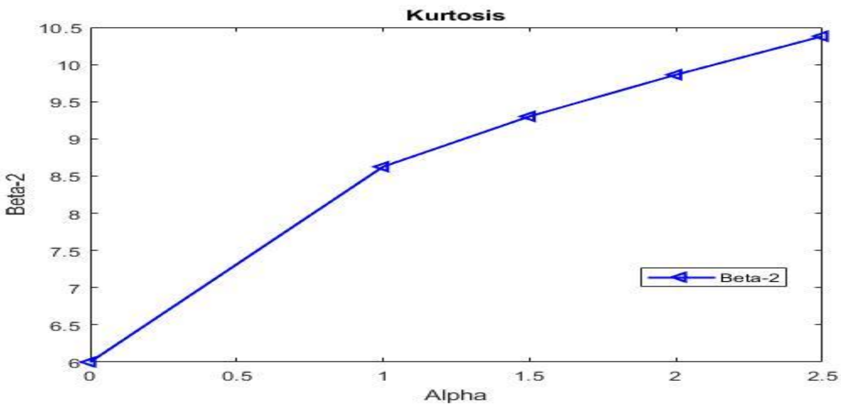

Figure (7): Kurtosis of PNLED for $\alpha = 1.0, 1.5, 2.0, 2.5$

Estimation of Parameters:

Under this sub-section, the value α has been estimated using the following two methods
(a)*Method of moments*:

Since there is only one parameter with this distribution, the estimator of the parameter has been obtained using the first moment about the origin of PNLED.

$$\mu_1' = \frac{(\pi\alpha+2)}{\alpha(\pi\alpha+1)}$$

Or, $(\pi\mu_1')\alpha^2 + (\mu_1' - \pi)\alpha - 2 = 0$

Or, $$\hat{\alpha} = \frac{-(\mu_1' - \pi) \pm \sqrt{[(\mu_1' - \pi)^2 + 8\mu_1'\pi]}}{2\pi\mu_1'} \quad (19)$$

The expression (19) is the point estimator of α .
(b) *Method of maximum likelihood:*

The MLE of this distribution has been obtained as

$$L = \left(\frac{\alpha^2}{1+\pi\alpha}\right)^n (1+\alpha)^{-\sum_{i=1}^{k}(y_i+2)f_i} \prod_{i=1}^{k}(1+y_i+\pi(1+\alpha))^{f_i} \quad (20)$$

Or, $$log(L) = 2n\log a - n\log(1+\pi\alpha) - \left(\sum_{i=1}^{k}(y_i+2)f_i\right)\left(\log(1+\alpha)\right) + \sum_{i=1}^{k} f_i \log\left((1+y_i+\pi(1+\alpha))\right)$$

Or, $$\frac{\partial(log(L))}{\partial\alpha} = \frac{2n}{\alpha} - \frac{\pi n}{(1+\pi\alpha)} - \frac{\left(\sum_{i=1}^{k}(y_i+2)f_i\right)}{(1+\alpha)} + \sum_{i=1}^{k}\frac{\pi f_i}{(1+y_i+\pi(1+\alpha))} \quad (21)$$

The expression (21) gives an estimated value of α .

Applications of PNLED:

Applications and goodness of fit have been conducted to this distribution with the support of secondary data used by other researchers. To test goodness of fit the following data are used.

Example (1): Let Y and O denotes Number of errors per group and observed frequency respectively.

Y	0	1	2	3	4^+
O	35	11	8	4	2

Example (2) Let Y and O denotes Number of insects per leaf and observed frequency respectively.

Y	0	1	2	3	4	5
O	33	12	6	3	1	1

Example (3) Let Y and O denotes Class / Exposure ($\mu g / kg$)and observed frequency respectively.

Y	0	1	2	3	4	5	6
O	200	57	30	7	4	0	2

Example (4): Let Y and O denotes Number of red mites per leaf and observed frequency respectively

Y	0	1	2	3	4	5	6	7
O	38	17	10	9	3	2	1	0

Table I: PLD Verses PNLED of Example (1)

Y	O	Expected frequency	
		PLD	PNLED
0	35	33.0	33.5
1	11	15.3	14.9
2	8	6.8	6.6
3	4	2.9	2.9
4	2	2.0	2.1
Total	60	60	60
μ'_1	0.78333333		
μ'_2	1.8500		
$\hat{\alpha}$		1.7434	1.500066066
d.f.		2	2
χ^2		1.78	1.58
P-value		0.61	0.65

Table II: PLD Verses PNLED of Example (2)

Y	O	Expected frequency	
		PLD	PNLED
0	33	31.51	31.9
1	12	14.2	13.8
2	6	6.1	5.9
3	3	2.5	2.5
4	1	1.0	1.1
5	1	0.7	0.8
Total	60	60	60
μ'_1	0.75		
μ'_2	1.8571		
$\hat{\alpha}$		1.8081	1.559364619
d.f.		2	2
χ^2		0.53	0.32
P-value		0.83	0.88

Table III: PLD Verses PNLED of Example (3)

Y	O	Expected frequency	
		PLD	PNLED
0	200	191.8	192.8
1	57	70.3	69.2
2	30	24.9	24.6
3	7	8.6	8.7
4	4	2.9	3.1
5	0	1.0	1.1
6	2	0.5	0.5
Total	300	300	300
μ'_1	0.553333333		
μ'_2	1.253333333		
$\hat{\alpha}$		2.353339	2.050115531
d.f.		2	2
χ^2		3.91	3.62
P-value		0.1415	0.1634

Table IV: PLD Verses PNLED of Example (4)

Y	O	Expected frequency	
		PLD	PNLED
0	38	35.8	36.8
1	17	20.7	20.1
2	10	11.4	10.9
3	9	6.0	5.8
4	3	3.1	3.1
5	2	1.6	1.7
6	1	0.8	0.9
7^+	0	0.6	0.7
Total	80	80	60
μ'_1	1.15		
μ'_2	3.4		
$\hat{\alpha}$		1.255891	1.069071439
d.f.		3	3
χ^2		2.47	1.23
P-value		0.48	0.808

Kemp & Kemp are the source of the first example [4]. Beall is credited with the second example [1]. Catcheside et al. [2] is responsible for the third case, while Garman [3] is responsible for the fourth. The theoretical frequency that was determined using PNLED and PLD has also been reported in the mentioned tables alongside the observed frequency, allowing for a comparison of the two distributions. B. K. Sah [7] utilized the first three examples in his doctoral thesis.

Conclusion:

Table-V shows that, for each table, the P-value derived using PNLED is higher than the P-value extracted using PLD. In order to determine the nature of the suggested distribution, we have also examined the statistical moments concerning the origin and the mean.

Table- V

PLD Verses PNLED

Table	PLD			PNLED		
	$d.f.$	$\chi^2_{d.f.}$	$P-Value$	$d.f.$	$\chi^2_{d.f.}$	$P-Value$
I	2	1.78	0.61	2	1.58	0.65
II	2	0.53	0.83	2	0.32	0.88
III	2	3.91	0.1415	2	3.62	0.1634
IV	3	2.47	0.48	3	1.23	0.88

Hence, we have identified the following concluding points for PNLED.

- Because the P-value achieved by PNLED is bigger than the P-value obtained by PLD, it is a better option to PLD for statistical modeling in the majority of over-dispersed data-sets (see [6]).

-It is over-dispersed, positively skewed by shape because $(\sqrt{2})<\gamma_1<\infty$, and Leptokurtic by size because $6<\beta_2<\infty$.

CONFLICT OF INTEREST

We have only tried to contribute to probability mixture distributions with selfless spirit.

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