



Differential Equation-Based Mathematical Model to Assess the Effect of a Flood Disaster on a Population

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Abstract: In our opinion, this paper have devote a lot of attention to the subject of flood management. Natural and man-made catastrophes are the two main categories of disasters, and both have a catastrophic impact on human life. human-caused natural disasters, like power outages, terrorism, fires, pollution, and hailstorms. Natural disasters include things like hailstorms, floods, and earthquakes. The main objective of this study is to develop an appropriate mathematical model for calculating the effects of a flood disaster on a population. One nation that has experienced numerous man-made and natural disasters is India. An appropriate plan needs to be in place to reduce the loss of human life and infrastructure. This paper demonstrates how to utilize ordinary differential equations in an area of mathematics to relate the value of a function for an unknown function involving one or more variables.

Keywords- Mathematical Model, Population size, Limits, Integration, Disaster management, Convergence.

2020 Mathematics Subject Classification: 92C10, 90C20, 34XX, OOA3.

Introduction

The last several decades have seen significant changes in hydrodynamic parameters due to climate change brought on by global warming; occasionally, these changes have resulted in lengthy droughts and other times in heavy precipitation followed by floods [1,2]. Floods still impose tremendous amounts of damage and occasionally even human casualties in many nations, flooding entire communities, demolishing homes and infrastructure, or creating banks that bury farmland and make it nearly impassable for years [3, 4]. Because of the contamination of water supplies, potential for epidemics, and other factors, floods have an impact on the socioeconomic status of the local population as well as their health. Just last year, 30 lakh people lived in three state Uttarkhand, Bihar and Bengal. These populations have been impacted by different diseases and affected by floods [5, 20,21,22].

Flood propagation modeling is the systematic approach to quantitatively delineating the dynamics and progression of water flow when large volumes inundate terrain uncontrollably. In the context of climate change and natural disasters, floods represent a pivotal challenge of the 21st century. India, due to its unique geo-climatic conditions, stands particularly vulnerable to these impacts. The nation has witnessed notable deviations in temperature and precipitation patterns, contributing to an uptick in the frequency and severity of disasters in recent decades.

A mathematical process for forecasting how a flow wave will change in size, velocity, and shape over time (i.e., the flow hydro graph) at one or more locations along a watercourse is known as deterministic modeling, also known as flood routing. An estuary, river, stream, canal, discharge ditch, storm sewer, or reservoir can all be considered streams and rivers. Earthquakes into reservoirs, reservoir releases, precipitation runoff, and tides can all produce flow hydro graphs. Hydro logic (lumped), hydraulic (distributed), and hybrid flood routing are the three categories under which it falls. This study aims to establish key parameters for assessing how floods propagate within a region and their consequential impact on the local population.

Proposed Mathematical Model

There are a number of different approaches that may be used to assess how a disaster has affected a particular area, and each one has its own advantages. We have used an approach to study that combines theories of limits and convergence with differential equations. This methodology comprises a range of strategies that are intended to thoroughly examine the relationship between variables, rather than being a single strategy.

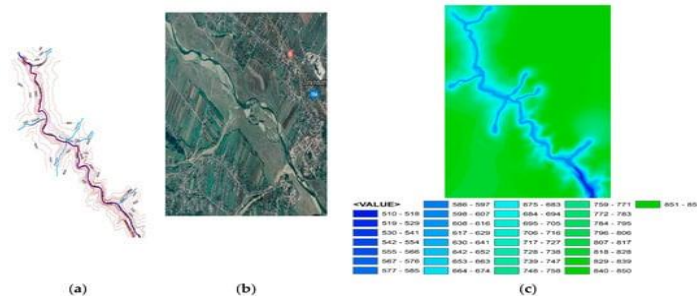


Figure 1. Designated area: (a) Orbit; (b) Google Earth aspect; (c) Data for computational modeling.

The research methodology for this proposed study unfolds through several structured phases: Initial identification of the problem and precise definition of variables under investigation. Comprehensive review and synthesis of existing literature pertinent to the study area. Formulation of the problem into a mathematical framework, leveraging differential equations to model the dynamics. Execution of mathematical solutions within the established model framework. Thorough interpretation of results obtained from simulations and analyses. Derivation of conclusive insights and implications drawn from the study findings. This Mathematical model methodological approach ensures a robust examination of how variables interact during disaster events, providing nuanced understanding and actionable conclusions for disaster management and mitigation strategies.



Figure 2. Flooding before (left) and after (right) the implementation

3. Deterministic Model

The variables $S(t)$, $I(t)$, and $R(t)$ in a population at a given time t indicate different groups within the community impacted by flood catastrophes. $S(t)$ in this case indicates the number of vulnerable people who could be impacted by the flood. The variable $I(t)$ represents the total number of people who are presently impacted by the flood disaster, including those who are feeling its immediate effects.

Meanwhile, $R(t)$ refers to individuals who have been removed from the population due to various outcomes such as recovery, loss, death, hospitalization, or other relevant factors, we have

$$S(t) + I(t) + R(t) = N(t) = B(\text{Constant}) \quad (1)$$

Initially, we focus on simple deterministic models where there are no removals considered. Let n denote the initial count of susceptible individuals affected by flood disasters in the population. In this scenario, we introduce a single flood disaster affected person into the population to initiate the modeling process.

$$S(t) + I(t) = n + 1, \quad S(0) = S_0 = n, \quad I(0) = I_0 = 1 \quad (2)$$

Due to the flood disaster, the population of susceptible individuals $S(t)$ decreases while the number of affected individuals $I(t)$ increases. We assume that the rate of change in $S(t)$, or equivalently the rate of change in $I(t)$, is proportional to the product of $S(t)$ and $I(t)$. This assumption forms the basis of our model, ensuring that it reflects the dynamics where the decrease in susceptible individuals and the increase in affected individuals are interdependent and proportional to their respective counts.

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - I(0) \quad (3)$$

where, β is constant of proportionality.

From equation (2) and (3), we get

$$\frac{dS}{dt} = -\beta S(n+1-S) \quad (4)$$

Solving equation (4), we get

$$\frac{dS}{S(n+1-S)} = -\beta dt$$

$$\frac{1}{S(n+1-S)} = \frac{A}{S} + \frac{B}{S-(n+1)}$$

$$\frac{1}{S(n+1-S)} = \frac{A}{S} + \frac{B}{S-(n+1)} \quad (5)$$

Using partial fraction, from equation (5) we get

$$\frac{1}{S(n+1-S)} = \frac{A}{S} + \frac{B}{S-(n+1)}$$

$$\frac{1}{S(n+1-S)} = \frac{A}{S} + \frac{B}{S-(n+1)} \quad (6)$$

$$\frac{1}{S} - \frac{1}{S-(n+1)} = \frac{A\{S-(n+1)\}}{S} + \frac{B}{S} \frac{A\{S-(n+1)\}}{S} + \frac{B}{S} \quad (7)$$

Now substituting $S=0$ in equation (7) we get

$$1 = A \left\{ -(n+1) \right\} + 0$$

$$\text{Thus } A = \frac{-1}{(n+1)(n+1)}$$

Substituting $S=n+1$ in equation (7) we get

$$1 = 0 + B(n+1)$$

$$\text{This gives } B = \frac{1}{(n+1)(n+1)}.$$

Thus from equation (6) we have

$$\frac{1}{S} \frac{1}{S-(n+1)} = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\frac{1}{\{S-(n+1)\}} - \frac{1}{S} \frac{1}{\{S-(n+1)\}} - \frac{1}{S} \right] \right].$$

Thus from equation (5) we have

$$\beta dt \beta dt = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\frac{ds}{\{S-(n+1)\}} - \frac{ds}{S} \frac{ds}{\{S-(n+1)\}} - \frac{ds}{S} \right] \right]$$

On integrating both sides

$$\beta t \beta t = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\log \{S-(n+1)\} - \log s \right] \right] + C \quad (8)$$

where, C is a constant.

$$\text{Since } S(0) = S_0 = n$$

From equation (8) we get

$$0 = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\log \{n-(n+1)\} - \log n \right] \right] + C$$

$$0 = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\log (-1/n) \right] \right] + C$$

$$\text{we find } C = - \left[- \left[\log (-1/n) \right] \right].$$

Now from equation (8), we get

$$\beta t \beta t = \frac{1}{(n+1)} \left[\frac{1}{(n+1)} \left[\log \{S-(n+1)\} - \log s \right] \right] - \left[- \left[\log (-1/n) \right] \right]$$

$$\beta t \beta t = \frac{1}{(n+1)} \log \left[\frac{1}{(n+1)} \log \left[\frac{\{S-(n+1)\} \{S-(n+1)\}}{S} \right] \right] - \left[- \left[\log (-1/n) \right] \right]$$

$$\beta t \beta t = \frac{n}{(n+1)} \log \left[\frac{n}{(n+1)} \log \left[\frac{\{S-(n+1)\} \{S-(n+1)\}}{S} \right] \right]$$

$$(n+1) \beta t \beta t = n \log \left[n \log \left[\frac{\{S-(n+1)\} \{S-(n+1)\}}{S} \right] \right]$$

$$e^{(n+1) \beta t} e^{(n+1) \beta t} = n \left[n \left[\frac{\{S-(n+1)\} \{S-(n+1)\}}{S} \right] \right]$$

$$e^{(n+1) \beta t} e^{(n+1) \beta t} = n \left[n \left[1 - \frac{(n+1)}{S} \right] 1 - \frac{(n+1)}{S} \right]$$

$$e^{(n+1) \beta t} e^{(n+1) \beta t} = n + \frac{n(n+1)}{S} n + \frac{n(n+1)}{S}$$

$$\frac{n(n+1)n(n+1)}{S} = n + e^{(n+1) \beta t} e^{(n+1) \beta t}$$

$$S(t) = \frac{n(n+1)}{n + e^{(n+1) \beta t}} n + e^{(n+1) \beta t} \quad \text{and } I(0)=0.$$

Since $S(t) + I(t) = n + 1$

So, $I(t) = (n + 1) - S(t)$

$$I(t) = (n + 1) - \frac{n(n+1)}{n + e^{(n+1)\beta t}} = \frac{n(n+1)e^{(n+1)\beta t}}{n + e^{(n+1)\beta t}}$$

$$I(t) = \frac{n(n+1)e^{(n+1)\beta t}}{n + e^{(n+1)\beta t}}$$

4. Results

We are investigation of this paper, disaster management to be a very significant topic for discussion. The natural or man-made disasters are the two main types, and both have an immensely destructive impact on human life. Man-made disasters including war,

terrorism, pollution, fires, blackouts of power, and hailstorms etc. $\lim_{t \rightarrow \infty} S(t) = 0$

$\lim_{t \rightarrow \infty} S(t) = 0$, $\lim_{t \rightarrow \infty} I(t) = n + 1$ and ultimately, all persons of a certain region will be affected by the flood disaster.

Conclusion

The present communication of the paper to develop an appropriate mathematical model for calculating the effects of a flood disaster on a population. Since they facilitate a thorough cost-benefit analysis of a specific proposal for flood protection measures, mathematical models are helpful instruments for the establishment of effective flood protection strategies and ideal allies for decision makers. This techniques primary strengths are its streamlined computational efficacy, clear interpret ability, and methodological simplicity. Time-dependent functions are used to define the variables $S(t)$, $I(t)$, and $R(t)$ in this framework. Flood disasters are among the deadliest sorts of disasters, and our mathematical model reflects that claim. It highlights the enormous challenge that results from the uncontrolled proliferation of flood disasters, which has a significant impact on a significant population in the affected regions. The country's planning and policy-making efforts in the area of disaster management have significantly increased recently. However, few would contest that over this time, the nation's vulnerability to losses from natural disasters has decreased. It is evident from the aforementioned items that the country as a whole will need to stand in solidarity with the victims; it is not a one-man show. It goes beyond simply giving away cash, clothing, food, and other necessities. Until and unless we distinguish between sympathy and empathy we will be unable to combat the tragedies. To aid in the country's earliest recovery, empathy is necessary.

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