https://doi.org/10.48047/AFJBS.6.14.2024.3456-3461

African Journal of Biological Sciences

Journal homepage: http://www.afjbs.com

Research Paper Open Access

INTERIOR DOMINATION POLYNOMIALS OF SOME DIFFERENT OPERATIONS IN GRAPHS

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Volume 6, Issue 14, Aug 2024

Received: 09 June 2024

Accepted: 19 July 2024

Published: 12 Aug 2024

doi: 10.48047/AFJBS.6.14.2024.3456-3461

Abstract:
The Interior Domination Polynomials $D_{Id}(G,x)$ of G is defined as $D_{Id}(G,x) = \sum_{i=\gamma_{Id}(G)}^{|V(G)|} d_{Id}(G,i)x^i$. Where $d_{Id}(G,i)$ is the number of interior dominating sets of G of cardinality i and $\gamma_{Id}(G)$ is cardinality of minimum interior domination number.

Keywords: interior dominating sets, interior domination polynomial.

Introduction

Let $G = (V, E\ )$ be a undirected graph, without loop and multiple edges. A non empty set $D \subseteq V$ is a dominating set of G, if every vertex $V - D$ is adjacent to minimum one vertex in D. The cardinality of minimum dominating set is named as the domination number and is denoted by $\gamma(G)$[4]. A vertex v is an interior vertex of G if for every vertex u distinct from v, there exists a vertex w such that v lies between u and w. A set $D \subseteq V(G)$ is an interior dominating set if D is a dominating set of G and every vertex v interior vertex of G. Any pendent vertices will not be a member in interior set of a graph. $\gamma_{Id}(G)$ denoted as the cardinality of minimum interior dominating set[4]. Let $D_{Id}(G,i)$ be the family of all interior dominating sets of a graph with cardinality i and Let $d_{Id}(G,i) =$

$|D_{Id}(G,i)|$. Then interior domination polynomial $D_{Id}(G,x)$ of G is defined as $D_{Id}(G,x) = \sum_{i=\gamma_{Id}(G)}^{|V(G)|} d_{Id}(G,i)x^i$.

2. Interior Dominating Sets and Polynomials of some different operations in Graphs

Definition : 2.1

Let $D_{Id}(G,i)$ be the family of all interior dominating sets of a graph with cardinality i. Then the interior domination number of G is defined as the minimum cardinality taken over all interior dominating sets of vertices in G and it is denoted by $\gamma_{Id}(G)$.

Theorem 2.2

For any connected graph G with n vertices, $D_{Id}(G \circ K_1, x) = x^n$.

Proof

Let G be a connected graph with n vertices. Since $G \circ K_1$ has $2n$ vertices.

Clearly, $\{v_1, v_2, v_3, \ldots, v_n\}$ is the minimum interior dominating set of $G \circ K_1$.

Therefore, $\gamma_{Id}(G \circ K_1) = n$ and $d_{Id}(G \circ K_1, n) = 1$. Hence, $D_{Id}(G \circ K_1, x) = x^n$.

Theorem 2.3

For any connected graph G with n vertices, $D_{Id}(G \circ K_2, x) = x^n$.

Proof

Let G be a connected graph with n vertices. Since $G \circ K_2$ has $3n$ vertices.

Clearly, $\{v_1, v_2, v_3, \ldots, v_n\}$ is the minimum interior dominating set of $G \circ K_2$.

Therefore, $\gamma_{Id}(G \circ K_2) = n$ and $d_{Id}(G \circ K_2, n) = 1$. Hence, $D_{Id}(G \circ K_2, x) = x^n$.

Theorem 2.4

For any connected graph G with n vertices, $D_{Id}(G \circ K_m, x) = x^n$.

Proof

Let G be a connected graph with n vertices. Since $G \circ K_m$ has $(m+1)n$ vertices.

Clearly, $\{v_1, v_2, v_3, \ldots, v_n\}$ is the minimum interior dominating set of $G \circ K_m$.

Therefore, $\gamma_{Id}(G \circ K_m) = n$ and $d_{Id}(G \circ K_m, n) = 1$. Hence, $D_{Id}(G \circ K_m, x) = x^n$.

Theorem 2.5

The interior domination polynomial of $K_m \oplus K_n$ is $D_{Id}(K_m \oplus K_n, x) = x^2$.

Proof

Label the vertices of K_m and K_n as $\{u_1, u_2, u_3, \dots, u_{m-1}, u\}$ and $\{v_1, v_2, v_3, \dots, v_{n-1}, v\}$.

Therefore, $\{u_1, u_2, u_3, \dots, u_{m-1}, u, v_1, v_2, v_3, \dots, v_{n-1}, v\}$ is the vertex set of $K_m \oplus K_n$.

Let $D = \{u, v\}$. The vertex u and v dominate all the vertices and also u and v are interior vertex of $K_m \oplus K_n$. Therefore, D is a minimum interior dominating set of $K_m \oplus K_n$.

Therefore, $\gamma_{Id}(K_m \oplus K_n) = 2$ and $d_{Id}(K_m \oplus K_n, 2) = 1$. Hence, $D_{Id}(K_m \oplus K_n, x) = x^2$.

Theorem 2.6

The interior domination polynomial of $K_m \odot K_n$ is $D_{Id}(K_m \odot K_n, x) = x$.

Proof

Label the vertices of K_m and K_n as $\{u_1, u_2, u_3, \dots, u_{m-1}, u\}$ and $\{v_1, v_2, v_3, \dots, v_{n-1}, v\}$.

Therefore, $\{u_1, u_2, u_3, \dots, u_{m-1}, u = v, v_1, v_2, v_3, \dots, v_{n-1}\}$ is the vertex set of $K_m \odot K_n$.

Let $D = \{u\}$. The vertex u dominate all the vertices and also u is a interior vertex of $K_m \odot K_n$.

Therefore, D is a minimum interior dominating set of $K_m \odot K_n$.

Therefore, $\gamma_{Id}(K_m \odot K_n) = 1$ and $d_{Id}(K_m \odot K_n, 1) = 1$. Hence, $D_{Id}(K_m \odot K_n, x) = x$.

Theorem 2.7

Let T_n be a chain triangular cactus with $2n + 1$ vertices. Then the interior domination polynomial of T_n is $D_{Id}(T_n, x) = x^{n-1}$.

Proof

Let T_n be a chain triangular cactus with $2n + 1$ vertices.

Assume the vertices of T_n as $\{v_1, v_2, v_3, \dots, v_{2n-1}, v_{2n}, v_{2n+1}\}$.

Clearly, every interior dominating set of T_n must be contain the vertex set $\{v_3, v_5, v_7, \dots, v_{2n-1}\}$.

Therefore, the a minimum interior dominating set of T_n is $\{v_3, v_5, v_7, \dots, v_{2n-1}\}$.

Therefore, $\gamma_{Id}(T_n) = n - 1$ and $d_{Id}(T_n, n - 1) = 1$. Hence, $D_{Id}(T_n, x) = x^{n-1}$.

Theorem 2.8

Let B_n be a Barbell graph with $2n$ vertices. Then the interior domination polynomial of B_n is $D_{Id}(B_n, x) = x^2$.

Proof

Let B_n be a Barbell graph with $2n$ vertices.

Assume the vertices of B_n as $\{v_1, v_2, v_3, \dots, v_{n+1}, \dots, v_{2n-1}, v_{2n}\}$ as given in Figure 2.1

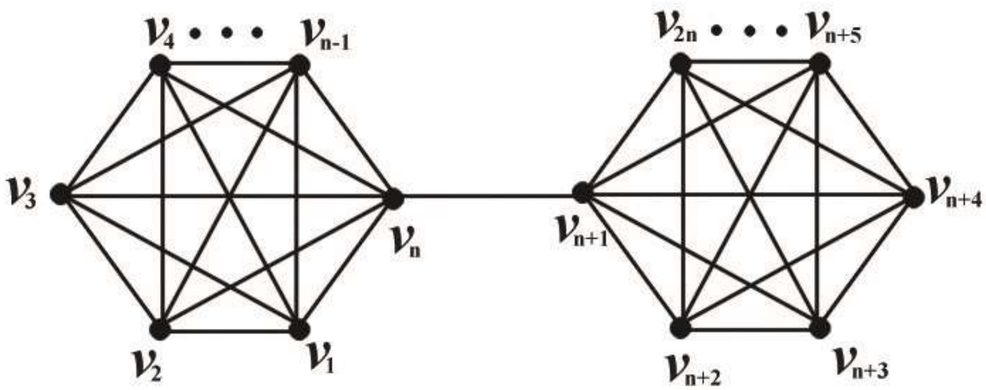

Figure 2.1

Let $D = \{v_n, v_{n+1}\}$. The vertex v_n and v_{n+1} dominate all the vertices and also v_n and v_{n+1} are interior vertex of B_n . Therefore, D is a minimum interior dominating set of B_n .

Therefore, $\gamma_{Id}(B_n) = 2$ and $d_{Id}(B_n, 2) = 1$. Hence, $D_{Id}(B_n, x) = x^2$

Theorem 2.9

Let $L_{n,1}$ be a Lollipop graph with $n + 1$ vertices. Then the interior domination polynomial of $L_{n,1}$ is $D_{Id}(L_{n,1}, x) = x$.

Proof

Let $L_{n,1}$ be a Lollipop graph with $n + 1$ vertices.

Assume the vertices of $L_{n,1}$ as $\{v_1, v_2, v_3, \dots, v_{n+1}\}$ as given in Figure 2.2

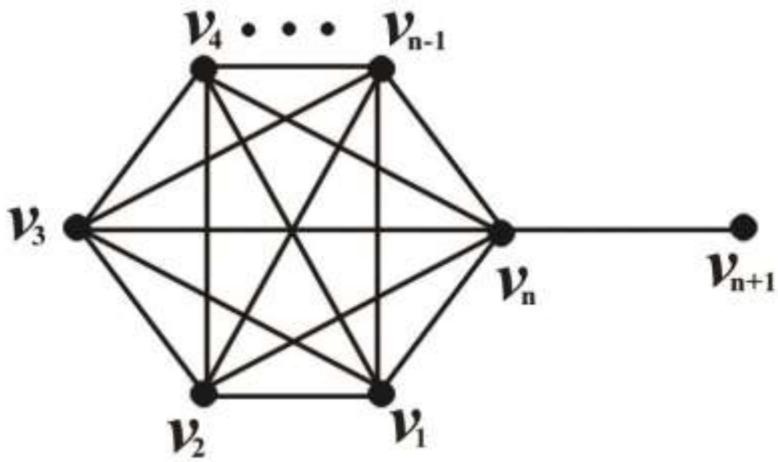

Figure 2.2

Let $D = \{v_n\}$. The vertex v_n dominate all the vertices and also v_n is a interior vertex of B_n .

Therefore, D is a minimum interior dominating set of $L_{n,1}$.

Therefore, $\gamma_{Id}(L_{n,1}) = 1$ and $d_{Id}(L_{n,1}, 1) = 1$.

Hence, $D_{Id}(L_{n,1}, x) = x^2$.

3. Conclusion

In this paper we have described interior domination polynomials of some different operations in graphs.

References

[1]. *S. Alikhani and Y. H. Peng, "Dominating Sets and Domination Polynomials of Paths", International journal of Mathematics and mathematical sciences, 2009.*
[2]. *S. Alikhani and Y. H. Peng, "Introduction to Domination Polynomial of Graph", arXiv : 0905.225 [v] [math.co] 14 may, 2009.*
[3]. *S. Alikhani and Y. H. Peng, "Dominating Sets of Centipedes", Journal of discreate Mathematical Sciences and Cryptography, August 2009.*
[4]. *A. Vijayan and K. Lal Gipson, "Domination Sets and Domination Polynomials Square of Paths", Open Journal of Discrete Mathematics, 3, 60-69, January- 2013, USA.*
[5]. *A. Anto Kinsely and C. Caroline Selvaraj, "A study on Interior Domination in Graphs", IOSR Journal of Mathematics (IOSR-JM), 12(2-VI), 55-59, (2016).*
[6]. *R. Ajitha and K. Lal Gipson, "Interior Domination Sets and Interior Domination Polynomials of Paths", AIP Conference proceedings 2261, 030095(2020); https://doi.org/10.1063/5.0016838 ,October 05 2020.*
[7]. *R. Ajitha and K. Lal Gipson, "Interior Domination Sets and Interior Domination Polynomials of Cycle* C_{n+3} *", Sambodhi,Vol-44, N0.-03 January-March (2021)*