https://doi.org/10.48047/AFJBS.6.Si4.2024.4701-4716

African Journal of Biological Sciences

Journal homepage: http://www.afjbs.com

Research Paper Open Access

NEW AUM BLOCK DUPLICATION TECHNIQUE FOR PATH GRAPHS WITH APPLICATIONS TO TRANSPORTATION MANAGEMENT

A. Uma Maheswari[1], E. Praveena[2]

[1] Principal, Government Arts and Science college, Perumbakkam, Chennai, Tamil Nadu, India.

umashiva2000@yahoo.com

[2]Research scholar, Quaid - E – Millath Government College for Women(A),
Assistant Professor, Department of mathematics, Soka Ikeda college of arts and science for women, Affiliated to University of Madras, Chennai, Tamil Nadu, India.

praveena171085@gmail.com

Volume 6, Issue Si4, Aug 2024

Received: 09 June 2024

Accepted: 19 July 2024

Published: 08 Aug 2024

doi: 10.48047/AFJBS.6.Si4.2024.4701-4716

Abstract: *In this paper, we present a new innovative technique of duplication of blocks in a graph called as AUM Block Duplication. By AUM Block Duplication of a graph, we mean duplication of a block by appending a new edge and joining the common encompassed vertices with the respective blocks and its neighbors. Duplication of a block by an edge and l – block duplications are presented in this work. This AUM Block Duplication and 2 – block duplication is applied in path graphs and it is applicable for enhanced transportation network management.*

Keywords: AUM Block Duplication, *l – block duplication*, 2 block duplication, path graph, neighborhood of a block.

AMS classification: 05C78.

Introduction

Graph theory is essential to many domains including physics, computer technology, social networks and many real-world applications including traffic management, traveling salesman problem, transportation, google maps and so forth. Rosa originated the idea of labeling [3]. An innovative block labelling technique developed by A. Uma Maheswari in [4][5][7][9][10][11][12] and [13]. Consequently, AUM Block coloring was identified in [7] and [9]. Moreover, AUM Block Sum Prime Distance Labeling has also been introduced by the authors for snake families of graphs in [14]. Vertex duplication and edge duplication has been studied in [15]. This work presents a novel idea of block duplication for path graph and its application. The graph that results from the duplication of blocks is referred as duplicated graph and the process of duplication of a block is called AUM block duplication.

Preliminaries

Block of a graph G [4]: The graph G is said to be separable if it has at least one cut-vertex. Otherwise, G is non-separable. A maximal non separable connected subgraph of graph G is called a **block of graph G.**

Path Graph[1]: A **Path graph P_n** is a graph whose vertices can be listed in the order v_1, v_2, …, v_n such that the edges are $\{v_i v_{i+1}\}$ where $i = 1,2,\dots,n-1$.

1. Duplication of a block by an edge

In this section, we introduce the new concept of AUM Block Duplication by an edge. We apply the block duplication by an edge to the path graph.

Definition 1.1:Neighbourhood of a block :

Let G be any Graph. The neighbourhood of a block B is the set of all blocks that have a common vertex with B and it is denoted by N(B).

Definition 1.2: AUM Block Duplication: Let G be any Graph. **Duplication of a block B_i by an edge e** is the graph, which is obtained by adding the new edge $e = uv$ to G and joining the vertices u and v with the vertex common to B_i and its neighboring blocks B_j .

The graph obtained after the duplication of a graph G is called duplicated graph and it is denoted by $D_G(B)$.

PREPOSITION 1.3: Duplication of a block in P_2 :

Let u_1,u_2 be the vertices and B_1 be the block of the graph P_2.

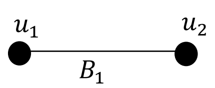

Figure 1: Path Graph P_2

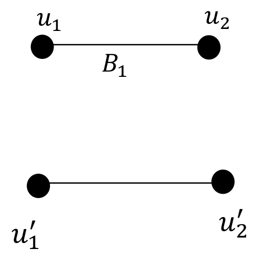

Figure 2: Duplicated Graph $D_{P_2}(B_1)$

Let us duplicate the block B_1 by introducing a new edge u_1' u_2'. Duplication of a block is done by joining u_1' and u_2' with the vertices common to the respective block and its neighbor. But, we have only one block B_1, and it has no neighboring blocks. Hence, there is no common vertex, the duplicated graph will be a disconnected graph as in figure 2.

PREPOSITION 1.4: Duplication of a block in P_3 :

Let u_1,u_2,u_3 be the vertices and B_1 ,B_2 be the blocks of the graph P_3.

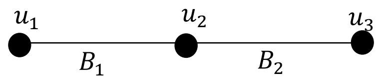

Figure 3: Path Graph P_3

Case (i): Duplication of the block B_1 :

Here B_2 is the neighboring block and u_2 is the common vertex to B_1 and its neighbor B_2. Introduce the new edge $u_1'u_2'$ for duplication of a block B_1. Join the vertices u_1' and u_2' with the vertex u_2.
Then the duplicated graph of P_3 is obtained as follows

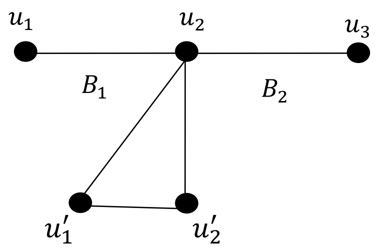

Figure 4: Duplicated Graph $D_{P_3}(B_1)$

Case (ii): Duplication of the block B_2 :

Here B_1 is the neighboring block and u_2 is the common vertex to B_2 and its neighbor B_1. Introduce the new edge $u_2'u_3'$ for duplication of the block B_2. Join the vertices u_2' and u_3' with the vertex u_2.
Then the duplicated graph of P_3 is obtained as follows

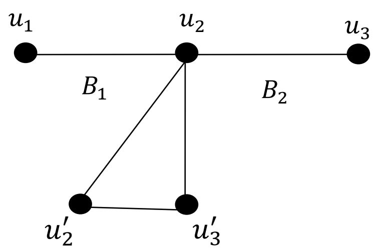

Figure 5: Duplicated Graph $D_{P_3}(B_2)$

Thus in P_3 the duplicated graph is same in both the cases.

PREPOSITION 1.5: Duplication of a block in P_4 :

Let u_1, u_2, u_3, u_4 be the vertices and B_1, B_2, B_3 be the blocks in P_4.

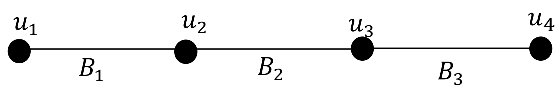

Figure 6: Path Graph P_4

Case (i): Duplication of a block B_1 in P_4 :

Here B_2 is the neighboring block and u_2 is the common vertex to B_1 and its neighbor B_2.
Introduce the new edge $u_1'u_2'$ for duplication of a block B_1. Join the vertices u_1' and u_2' with the vertex u_2.
Then the duplicated graph of P_4 is obtained as follows

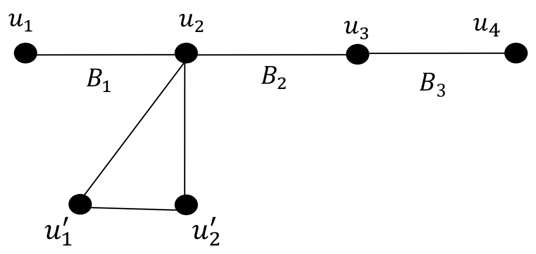

Figure 7: Duplicated graph D_{P_4} (B_1)

Case (ii): Duplication of a block B_2 in P_4 :

Here B_1 and B_3 are the neighboring blocks for B_2.
∴ u_2 and u_3 are the vertices common to the neighboring blocks B_1 and B_3 of B_2 respectively.
Let us introduce the new edge u_2' u_3' with the end vertices u_2' and u_3' for duplication of the block B_2. Join both the vertices u_2' and u_3' with u_2 and u_3.
Then the resulting graph is called the duplicated graph D_{P_4} (B_2).

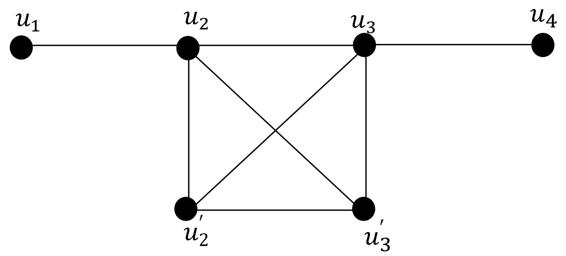

Figure 8: Duplicated graph D_{P_4} (B_2)

Case (iii): Duplication of the block B_3 :

Here B_2 is the only neighboring block of B_3 and u_3 is the common vertex for the block B_3 and its neighboring block B_2.
Let us consider the edge u_3' u_4' with the end vertices in u_3' and u_4' for duplicating the block B_3.
Join the vertices u_3' and u_4' the vertex u_3 for duplication of B_3.
Then the duplicated graph of the P_4 is obtained as follows,

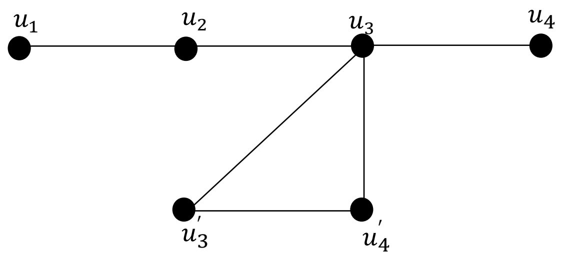

Figure 9: Duplicated graph D_{P_4} (B_3)

PREPOSITION 1.6: Duplication of a block in P_n :

Let $u_1, u_2, \ldots u_n$ be the n vertices and $B_1, B_2, \ldots B_{n-1}$ be the $n-1$ blocks in P_n.

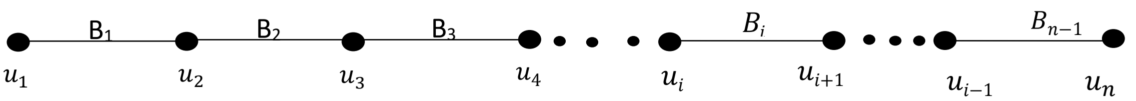

Figure 10: Path Graph P_n

Case (i): Duplication of the block B_1 in P_n .

Here B_2 is the neighboring block and u_2 is the common vertex to B_1 and its neighbor B_2. Introduce the new edge $u_1'u_2'$ for duplication of a block B_1. Join the vertices u_1' and u_2' with the vertex u_2.

Then the duplicated graph of P_n is obtained as follows

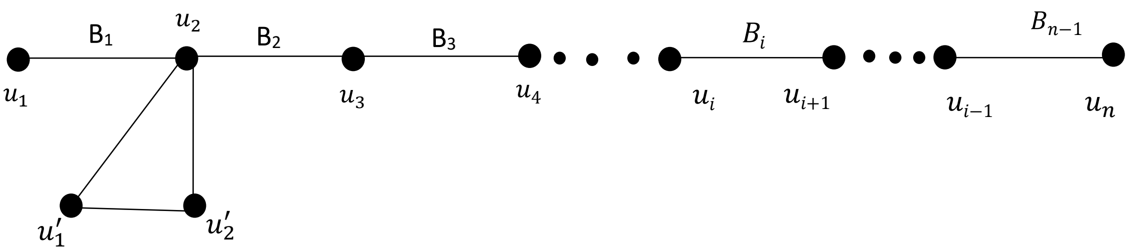

Figure 11: Duplicated graph D_{P_n} (B_1)

Case (ii): Duplication of the block B_i , (2 ≤ i ≤ n – 2) in P_n.

Here B_{i-1}, B_{i+1} are the neighboring blocks for B_i.

∴ u_i , u_{i+1} are the vertices common to the neighboring blocks B_{i-1} and B_{i+1} of B_i respectively.

Let us introduce the new edge u_i' , u_{i+1}' with the end vertices u_i' and u_{i+1}' for duplicating the block B_i. Joint the various u_i' with u_i and u_{i+1} and u_{i+1}' with u_i and u_{i+1}.

Then the duplicated graph is obtained as follows,

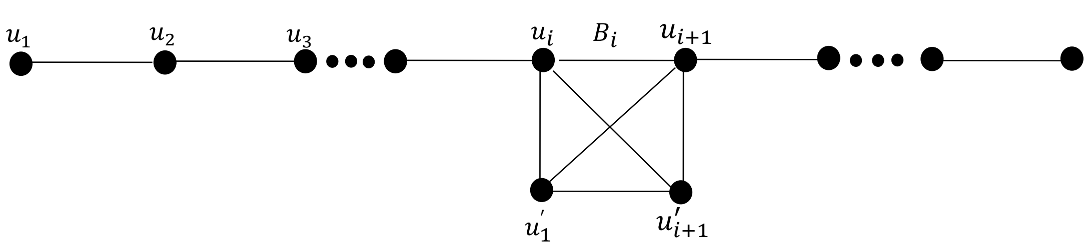

Figure 12: Duplicated graph D_{P_n} (B_i)

Case (iii): Duplication of the block B_{n-1} in P_n.

Here P_{n-2} is the only neighboring block of B_{n-1} and u_{n-1} is the common vertex for the block B_{n-1} and its neighboring block B_{n-2}.
Let us consider the edge $u'_{n-1}\ u'_n$ with the end vertices u'_{n-1} and u'_n for duplicating the block B_{n-1}.
Join the vertices u'_{n-1} and u'_n with the vertex u_{n-1}.
Then the duplicated graph of P_n is obtained as follows,

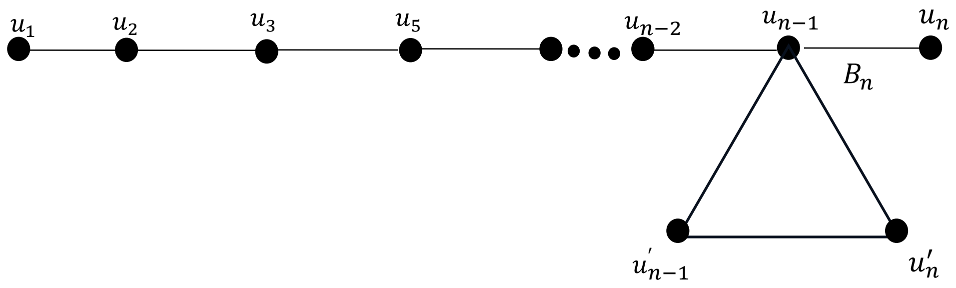

Figure 13: Duplicated graph D_{P_n} (B_n)

2. l - block duplication

Let n be the number of blocks in a graph. Let l be any positive integer.

In this section we shall obtain duplication of l blocks by edges. This process is called l – block duplication. After the duplication the resulting graph is called l - block duplicated graph and it is denoted by $D_G^l(B)$.

We consider different types of duplication of l - blocks by edges.
Type 1: Duplication of l - blocks by same edge.
Type 2: Duplication of l - blocks by distinct edges.
Type 3: Duplication of l - blocks by allowing repeated edges for some blocks.

Duplication of l - blocks by same edge ($l \leq n$):
Let G be any Graph with n blocks. In this type l blocks are duplicated with same edge e.

Duplication of l - blocks by distinct edges ($l \leq n$):
Let G be any Graph with n blocks. In this type l blocks are duplicated by l distinct edges e_1, e_2, e_3, …, e_l.

Duplication of l -blocks by allowing repeated edges for some blocks ($l \leq n$):
Let G be any Graph with n blocks. In this type some blocks are duplicated with distinct edges and some blocks are duplicated with edges that has been already used.

2.1. 2 – block Duplication

In this section we have discussed 2- block duplication of a path graph with same edge of type 1.

PREPOSITION 2.1.1: 2 – Block duplication of P_3 with same edge:

Let u_1,u_2,u_3 be the vertices and B_1, B_2 be the blocks of the path graph P_3.

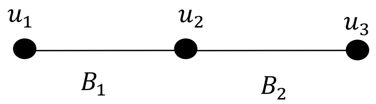

Figure 14: Path Graph P_3

Here the neighbor sets are N (B_1) = {B_2} and N (B_2) = {B_1}.

Let us consider the edge uv with end vertices u and v for duplication of the blocks B_1 and B_2. Since Both the blocks B_1 and B_2 have the same common vertex u_2, the duplicated graph is obtained by joining the vertices u and v with u_2 as follows,

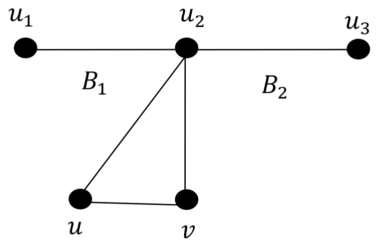

Figure 15: Duplicated graph $D^2_{P_3}$ (B_{12})

Thus 2 - block duplication of P_3 with same edge is same as duplication of a block by an edge in P_3 (refer preposition 1.4)

PREPOSITON 2.1.2: 2 - block duplication of P_4 with same edge

Let u_1, u_2, u_3, u_4 be the vertices and B_1, B_2, B_3 be the blocks of the path graph P_4.

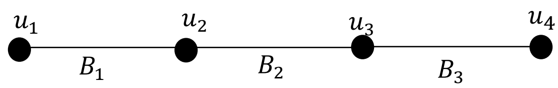

Figure 16: Path Graph P_4

Here the neighbor sets are N (B_1) = {B_2}, N (B_2) = {B_1, B_3} and N (B_3) = {B_2}

Let us consider the edge uv with end vertices u and v for duplication.

And the possible cases for duplication are

Case(i): Duplication of the blocks B_1 and B_2.

Join the vertices u and v with the vertex u_2, which is common to B_1 and its neighbor B_2 for duplicating the block B_1.

Now for duplication of B_2 we have to join the vertices u and v with u_2, which is common to B_2 and its neighbor B_1 and with u_3, which is common to B_2 and its neighbor B_3.

But already u and v have been joined with u_2 while duplicating B_1 , it is enough to join the vertices u and v with u_3.
Thus, the duplicated graph is obtained as follows,

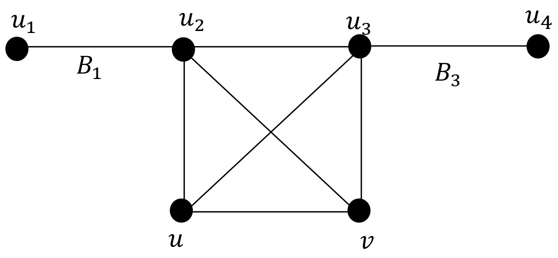

Figure 17: Duplicated graph $D_{P_4}^2$ (B_{12})

Case (ii): Duplication of the blocks B_2 and B_3.

Join the vertices u and v with the vertex u_3, which is common to B_3 and its neighbor B_2 for duplicating the block B_3.
Now duplication of B_2 is done as in case (i).
Thus, the duplicated graph is obtained as in case (i).

Case (iii): Duplication of the blocks B_1 and B_3.

Join the vertices u and v with the vertex u_2, which is common to B_1 and its neighbor B_2 for duplicating the block B_1 and with u_3, which is common to B_3 and its neighbor B_2 for duplicating the block B_3.
Thus, the duplicated graph is again obtained as in case (i).
Hence, 2-block duplication of P_4 with same edge in all the cases is same as duplication of a block B_2 by an edge in P_4.

PREPOSITION 2.1.3: 2 - block duplication of P_5 with the same edge.

Let u_1, u_2, u_3, u_4 ,u_5 be the vertices and B_1, B_2 , B_3, B_4 be the blocks of the path graph P_5.

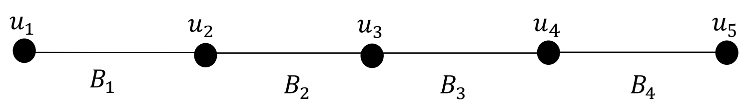

Figure 18: Path Graph P_5

Here the neighbor sets are N $(B_1) = \{B_2\}$, N $(B_2) = \{B_1, B_3\}$, N $(B_3) = \{B_2, B_4\}$and N $(B_4) = \{B_3\}$
Let us consider the edge uv with end vertices u and v for duplication.
And the possible cases for duplications are

Case (i): Duplication of B_1 and B_2:

Duplication of B_1 and B_2 it can be done as in case (i) of preposition 2.1.2.

Thus, the duplicated graph is obtained as follows,

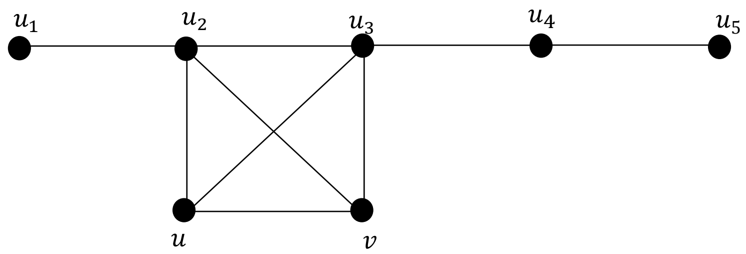

Figure 19: Duplicated graph $D^2_{P_5}$ (B_{12})

Case (ii): Duplication of B_1 and B_3 :

Duplication of B_1 is done by joining u and v with u_2, which is common to B_1 and its neighbor B_2 and duplication of B_3 is done by joining the vertices u and v with u_3 and u_4 which are common to B_3 and its neighbors B_2 and B_4 respectively.
Thus, the duplicated graph is obtained as follows,

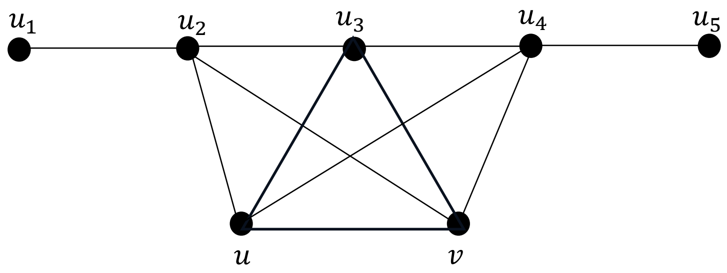

Figure 20: Duplicated graph $D^2_{P_5}$ (B_{13})

Case (iii): Duplication of B_1 and B_4 :

Duplication of B_1 is done by joining u and v with u_2, which is common to B_1 and its neighbor B_2 and duplication of B_4 is done by joining u and v with u_4, which is common to B_4 and its neighbor B_3. Thus, the duplicated graph is obtained as follows,

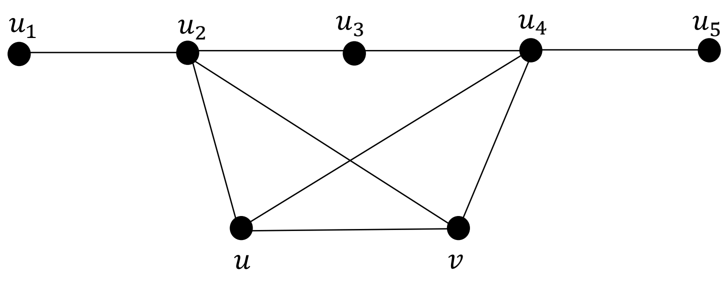

Figure 21: Duplicated graph $D^2_{P_5}$ (B_{14})

Case (iv): Duplication of B_2 and B_3:

Let us join the vertices u and v with u_2 and u_3 for duplication B_2 which are common to B_2 and its neighbors B_1 and B_3 respectively.

For duplication of B_3 , u and v can be joined with u_3 and u_4 which are common to B_3 and its neighbors B_2 and B_4 respectively.
But already u_3 has been joined with u and v while duplicating B_2, it is enough to join u_4 with u and v for duplicating the block B_3.
Thus, the duplicated graph is obtained as follows (which is same as case (ii))

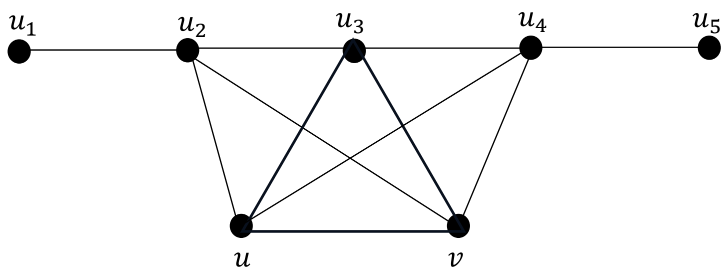

Figure 22: Duplicated graph $D^2_{P_5}$ (B_{23})

Case (v): Duplication of B_2 and B_4 :

N(B_2) = {B_1,B_3}   N (B_4)={B_3}
Let us join the vertices u and v with u_2 and u_3, which are common to B_2 and its neighbors B_1 and B_3 respectively for duplication of B_2 and join with u_4, which is common to B_4 and its neighbor B_3 for duplication of B_4. Thus, the duplicated graph is obtained as follows (again is same as case (iii))

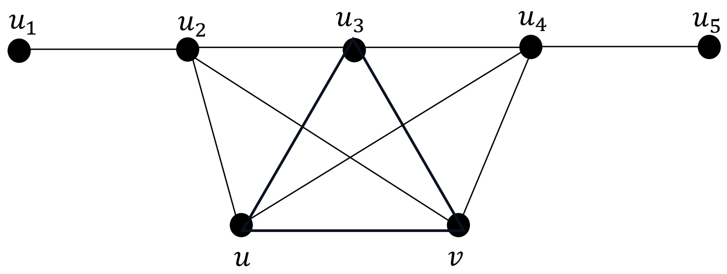

Figure 23: Duplicated graph $D^2_{P_5}$ (B_{24})

Case (vi): Duplication of B_3 and B_4 :

N (B_3)={B_2,B_4}   N (B_4)={B_3}
Let us join the vertices u and v with u_3 which is common to B_3 and its neighbor B_2 and u_4, which is common to B_3 and its neighbor B_4 for duplication of B_3 and with u_4 (which is already joined) for duplication of B_4. Thus, the duplicated graph is obtained as follows,

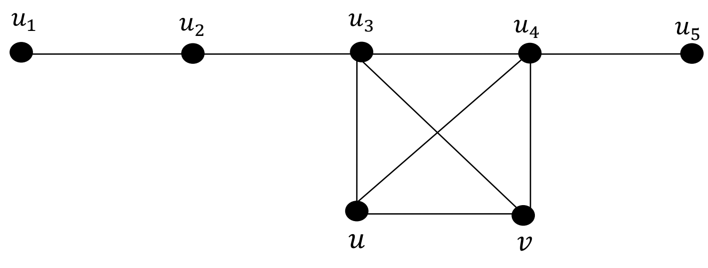

Figure 24: Duplicated graph $D^2_{P_5}$ (B_{34})

PREPOSITION 2.1.4: 2 - block duplication of P_6 with the same edge.

Let u_1, u_2, u_3, u_4 ,u_5, u_6 be the vertices and B_1, B_2 , B_3, B_4, B_5 be the blocks of the path graph P_6.

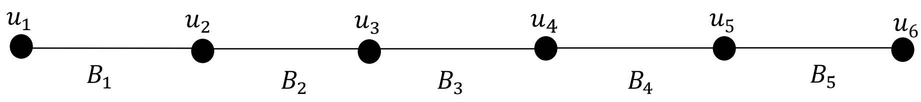

Figure 25: Path Graph P_6

Here the neighbor sets are N $(B_1) = \{B_2\}$, N $(B_2) = \{B_1, B_3\}$, N $(B_3) = \{B_2, B_4\}$
N $(B_4) = \{B_3, B_5\}$ and N $(B_5) = \{B_4\}$
Let us consider an edge uv with vertices u and v for duplication
Then the possible cases for 2 - block duplications are

Case (i): Duplication of B_1 and B_2 :

It can be done as in case (i) of preposition 2.1.2.
Thus, the duplicated graph is as follows

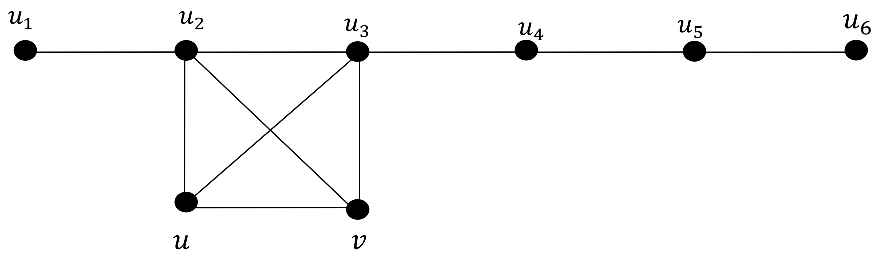

Figure 26: Duplicated graph $D_{P_6}^2$ (B_{12})

Case (ii): Duplication of B_1 and B_3 :

It can be done as in case (ii) of preposition 2.1.3.
Thus, the duplicated graph is as follows

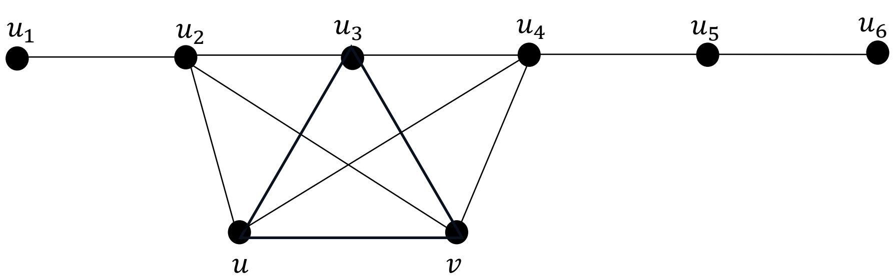

Figure 27: Duplicated graph $D_{P_6}^2$ (B_{13})

Case (iii): Duplication of B_1 and B_4 :

N(B_1) =$\{B_2\}$ N(B_4) = $\{B_3$, $B_5\}$

Let us join u and v with u_2 which is common to B_1 and its neighbor B_2 for duplication of B_1. For duplication of B_4 , Let us join u and v with the vertices u_4 , which is common to B_4 and its neighbor B_3 and u_5 , which is common to B_4 and its neighbor B_5.
Thus, the duplicated graph is obtained as follows,

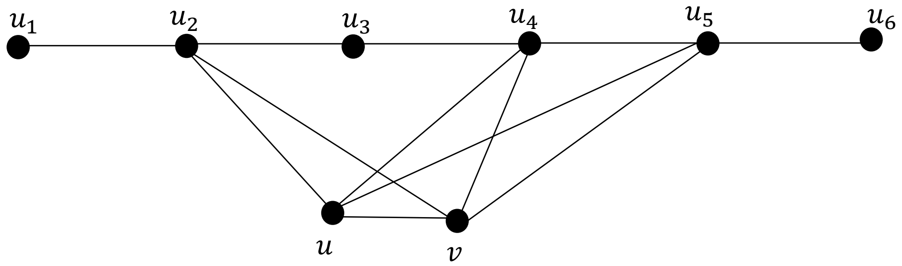

Figure 28: Duplicated graph $D^2_{P_6}$ (B_{14})

Case (iv): Duplication of B_1 and B_5 :

N(B_1) = {B_2} N(B_5) = {B_4}

For duplication B_1 , let us join u and v with u_2 and for duplication of B_5 , u and v can be joined with u_5. Thus, the duplicated graph is obtained as follows,

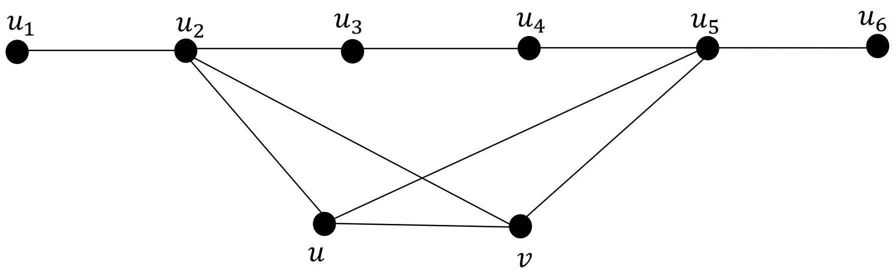

Figure 29: Duplicated graph $D^2_{P_6}$ (B_{15})

Case (v): Duplication of B_2 and B_3 :

N(B_2) = {B_1 ,B_3} N(B_3) = {B_2 ,B_4}

Duplication of B_2 and B_3 can be done by joining u and v with u_2 , u_3 and u_4 as per the definition of duplication of a block by an edge. Thus, the duplicated graph is obtained as follows,

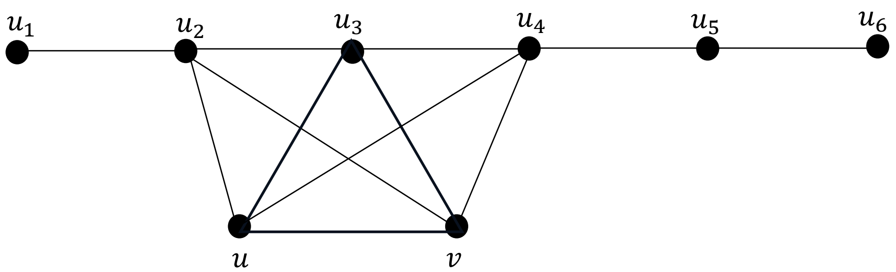

Figure 30: Duplicated graph $D^2_{P_6}$ (B_{23})

Case (vi): Duplication of B_2 and B_4 :

N(B_2) = {B_1 ,B_3} N(B_4) = {B_3 ,B_5}

Let us join u and v with u_2, which is common to B_2 and its neighbor B_1 and u_3, which is common to B_2 its neighbor B_3 for duplication of B_2.
For duplication B_4, u and v can be joined with u_4 and u_5 which are common to B_4 and its neighbors B_3 and B_5 respectively.
Thus, the duplicated graph is obtained as follows,

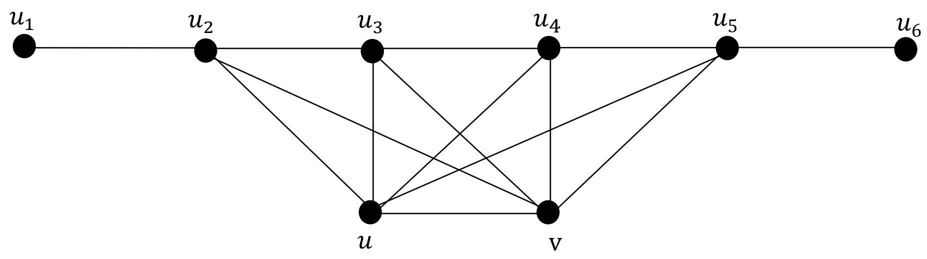

Figure 31: Duplicated graph $D^2_{P_6}$ (B_{24})

Case (vii): Duplication of B_2 and B_5 :

N(B_2) = {B_1 , B_3} N(B_5) = {B_4}

Duplication of B_2 can be done as in case (iv)
Duplication of B_5 can be done as in case (iv). Thus, the duplicated graph is obtained as follows,

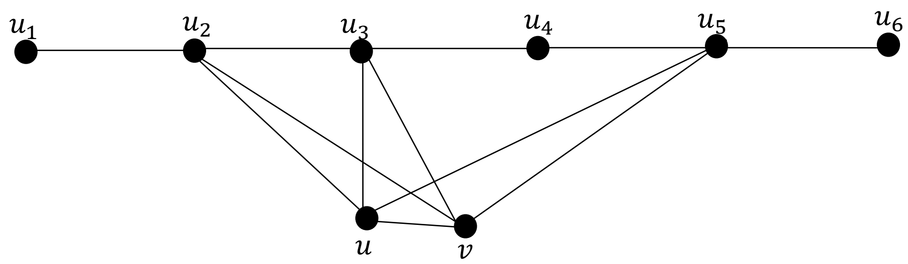

Figure 32: Duplicated graph $D^2_{P_6}$ (B_{25})

Case (viii): Duplication of B_3 and B_4 :

N(B_3) = {B_2 , B_4} N(B_4) = {B_3 , B_5}

Duplication of B_3 and B_4 can be done by joining the vertices u and v with u_3 , u_4 and u_5 as per the definition of duplication of a block by an edge and the repeated edged can be used only once. Thus, the duplicated graph is obtained as follows,

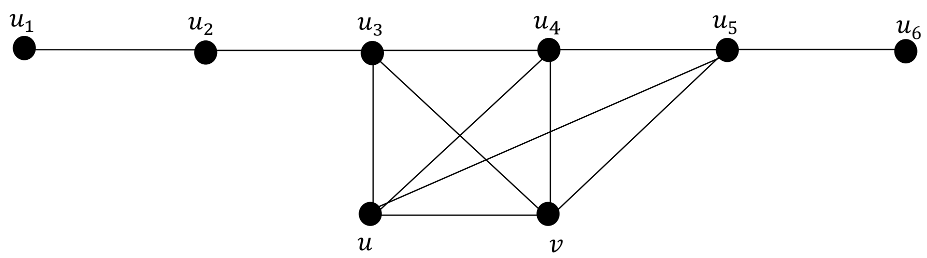

Figure 33: Duplicated graph $D^2_{P_6}$ (B_{34})

Case (ix): Duplication of B_3 and B_5 :

N(B_3) = {B_2 , B_4} N(B_5) = {B_4}

Join the vertices u and v with u_3 and u_4 for the duplication of B_3 and with u_5 for duplication of B_5. Thus, the duplicated graph is obtained as follows,

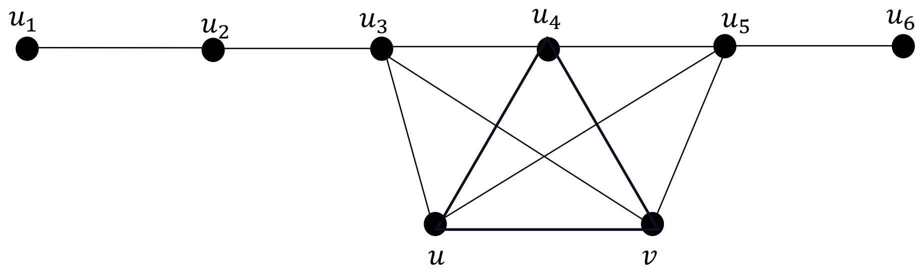

Figure 34: Duplicated graph $D^2_{P_6}$ (B_{35})

Case (x): Duplication of B_4 and B_5 :

N(B_4) = {B_3 , B_5}   N(B_5) = {B_4}

Join the vertices u and v with u_4 and u_5 for the duplication of B_4 and B_5.
Thus, the duplicated graph is obtained as follows,

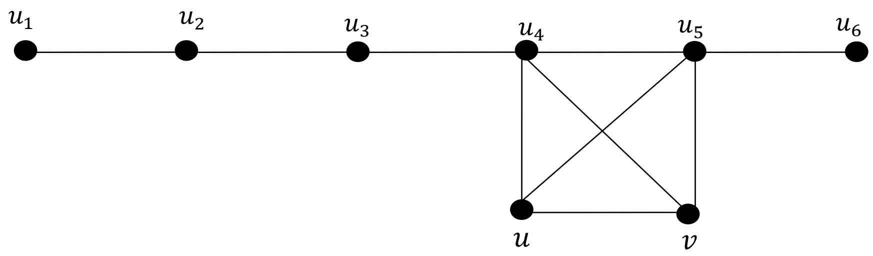

Figure 35: Duplicated graph $D^2_{P_6}$ (B_{45}).

Path Graph	Number of possible cases for duplication	Number of different 2-duplicated graph
P_3	1	1
P_4	3	1
P_5	6	3
P_6	10	5

3. Applications of AUM Block Duplication of path graphs for transportation network management

The duplicated graph will help us to provide alternate routes in transportation network.

This is the one simplest Duplicated graph by using duplication of a block in P_4 will be considered as an example for transportation management.

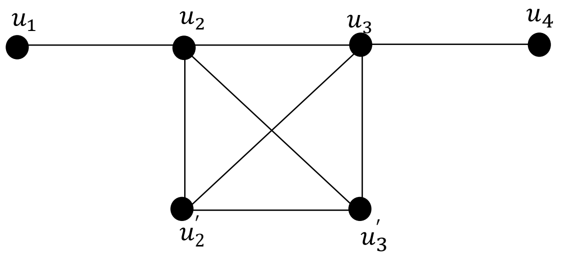

Figure 36: Duplicated graph D_{P_4} (B_2)

We consider one situation. If u_2u_3 is disturbed due to any damage or over traffic, in such cases where u_2u_3 is blocked. The duplicated graph comes in handy in such situations that is $u_2u_3'u_3$ or $u_2u_2'u_3$ are may be feasible alternating. If there is heavy conjunction or obstruction in u_2u_3' or $u_2'u_3$, we can use the other feasible alternative $u_2u_2'u_3'u_3$. This idea can also be used for inaccessible areas during a natural disaster. So, the concept of duplication of graph by an edge will be useful tool in addressing issues related to commutation or transportation.

Conclusion:

In this paper we have defined duplication of a block in a graph, l – block duplication of a graph. Additionally, duplication of a block and 2 – block duplication in a path graph has been discovered. It has several applications in the area of transportation, traffic signal, water supplies and electricity connections. Also, duplication of blocks will be employed for inaccessible areas during a natural disaster. The results can easily be extended to other families of graphs having practical applications.

References:

[1] Diestel, R. Graph theory (5th ed). Springer, (2017).
[2] Lei Li and Baoyindureng Wu " the number of blocks of a graph with given Minimum Degree" Research Article /open access volume 2021/ article ID6691960, https://doi.org/10.1155/2021/6691960.
[3] Rosa. A, "On certain valuations of the vertices of a graph", Theory of graphs (Internat. Symposium Rome, July 1966) Gordon and Breach, N.Y-Dunod Paris (1967), 349-35.
[4] Uma Maheswari . A, S. Azhagarasi, "New Labeling for Graphs-AUM Block Sum Labeling", International Journal of Current Science (IJCSPUB), volume 12, Issue 1, March 2022, pp. 574-584, ISSN: 2250-1770
[5] Uma Maheswari.A, Azhagarasi.S, "AUM Block Sum Labeling for some Special Graphs", International Journal of Mechanical Engineering, Vol.7(Special Issue 5, April 2022), pp.102-110, ISSN:0974-5823.
[6] Uma Maheswari. A, Azhagarasi. S, "A New Algorithm for Encoding and Decoding

Using Aum Block Labeling", Compliance Engineering Journal, ISSN No: 0898- 3577, Volume 13, Issue 4, 2022, pp.264 – 274.

[7] Uma Maheswari. A, Bala Samuvel. J "New Coloring for Blocks – AUM Block Coloring for Standard Graphs", J Compliance Engineering Journal, ISSN No: 0898 – 3577 Volume 13, Issue: 5 Page No: 68 – 77, 2022.

[8] Uma Maheswari.A, S.Azhagarasi, " AUM Block sum Labeling & Aum Block Labeling for Perfect Binary Tree (T3,l) T(4,l) & T(5,l)", Chapter in Edited Book Advances in Graph Labeling, Colouring and Power Domination Theory, NFED Publications, E- ISBN: 78-81-95499-8-5 Vol: 1, Pp: 1-17, May 2022.

[9] Uma Maheswari.A, Bala Samuvel. J "AUM block Coloring for Triangular Snake Graph Family", Chapter in Edited Book Advances in Graph Labeling, Colouring and Power Domination Theory, NFED Publications, E-ISBN: 978-81-95499-8-5 Vol: 1, pp: 72-91, May 2022.

[10] Uma Maheswari.A, A.S. Purnalakshimi," Aum Block Labeling for Friendship, Tadpole and Cactus Graphs", Neuro Quantology, eISSN 1303-5150, June 2022, Volume 20, Issue 6, Page 7876-7884.

[11] Uma Maheswari.A & A.S .Purnalakshimi, "AUM Block labeling for Star, Bi-star and Sunlet graph", Neuro quantology, Vol 20,Issue 9,pg 3967-3975.

[12] Uma Maheswari. A, Ambika. C, "New coding algorithms using AUM block sum labeling", Neuro Quantology, eISSN 1303-5150, August 2022, Volume 20, Issue 9, Page 377-385.

[13] Uma Maheswari.A & A.S .Purnalakshimi, "AUM Block labeling for kite, comb and diamond snake graphs", Journal of North eastern University, Vol 25, Issue 04, ISSN 1005-3026.

[14] Uma Maheswari.A , Praveena.E "AUM Block Sum Prime Distance Labeling" Mukt shabd journal, issn no:2347-3150, July – 2023, volume xii, issue vii, Page no.1771-1783.

[15] Vaidya, S.K., & Dani, N.A.(2011). Cordial and 3 – Equitable Graphs Induced by Duplication of Edge . Mathematics today, 27, 71-82. Scientific Research Publishing. SCIRP .