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Square Sum Labeling of Super Subdivision of Certain Classes of Graphs

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ABSTRACT:

Ajitha, Arumugam, and Germina defined the square sum graph. A graph $G = (V, E)$ with order p and size q is said to be a square sum graph, if there exists a bijection mapping $f : V(G) \rightarrow \{0, 1, 2, \ldots, (p - 1)\}$ such that the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(uv) = \left(f(u)\right)^2 + \left(f(v)\right)^2$, for every $uv \in E(G)$ is injective. The graph which admits square sum labeling is called a square sum graph. This paper establishes Square sum labeling on the super subdivision of crown and coconut tree.

Keywords: Square sum labeling, super subdivision, crown, coconut tree.
Ams Subject Classification: 05c78

1. Introduction

An assignment of integers to the vertices, edges, or both, under particular conditions, is known as graph labeling[3]. [1] Ajitha, Arumugam, and Germina defined the square sum graph and they proved that the square sum labelings of the following graphs are valid: trees, cycles, $k_2 + mk_1$,k_n if and only if $n < 5$, $c_n^{(t)}$(the one-point union of t copies of C_n), grids $p_m \times p_n$, and $k_{m,n}$if $m \le 4$. Elumalai[2] examined the square sum labeling of the middle graph and the total graph of path, and cycle. In [4], [5] JebaJesintha showed the existence of a super subdivision of graphs on odd graceful and cordial labeling. Sethuraman [7] introduced the concept of super subdivision and showed that every path's super subdivision is graceful and every cycle contains a graceful super subdivision. Patel *et.al* [6]examined some graph operations in the Square sum labeling. The occurrence of Gaussian Prime Labelling of Super Subdivision of Star Graphs has been determined by Rajesh Kumar [8]. A significant quantity of research has been published in the last three decades, with more than 1,000 papers covering various forms of graph labeling. For more details, one can refer to Gallian[3].Square sum labeling can be applied in designing and optimizing communication networks. Assigning labels to vertices and edges to preserve the square sum property can help improve the efficiency and reliability of communication in networks. The super subdivision will highlight connectivity and failure risks.Super subdivisions contribute to developing codes that can detect and correct errors in data transmission. This paper establishes the existence of square sum labeling of super subdivision of crown and coconut tree.A brief overview of definitions that are relevant to the current studies.

Definition 1.1[7]
The super subdivision graph of a $S^*(G)$ is generated from G by replacing every edge uv with a complete bipartite graph $K_{2,m}$.

Definition 1.2[3]
The crown $C_n \odot k_1$ is obtained by joining a pendant edge to each vertex of the cycle C_n.

Definition 1.3[3]
A coconut tree $CT\ (m, n)$ is the graph formed by appending m new pendant edges at an end vertex of the path p_n.

2. Main Results

In this section, we provea few theorems on the super subdivision of certain classes of graphs on square sum labeling.

THEOREM 2.1:
The super subdivision of a crown graph admits square sumlabeling.
Proof: Let graph G represent the super subdivision of a crown. Let $a_i(1 \le i \le n)$ be the vertices of the cycle. Let $b_i(1 \le i \le n)$ be the pendent vertices attached to the cycle vertices a_i.Let $y_j^i(1 \le i \le n,\ 1 \le j \le m)$ be the super subdivision of the vertices of the cycle and let $x_j^i(1 \le i \le n,\ 1 \le j \le m)$ be the super subdivision of the pendent vertices of the cycle. The generalized graph for the super subdivision of a crown is depicted in Figure 1.
Let the vertex and the edge set be defined as follows:
$|V(G)| = 2n + 2nm$and $|E(G)| = 4mn$.

The vertices are labeled as follows:

$f(a_i) = i-1; 1 \leq i \leq n$
$f(b_i) = n+i-1; 1 \leq i \leq n$
$f\left(x_j^i\right) = 2n+(j-1)+m(i-1); 1 \leq i \leq n, 1 \leq j \leq m$
$f\left(y_j^i\right) = mn+2n-1+j+m(i-1); 1 \leq i \leq n, 1 \leq j \leq m$

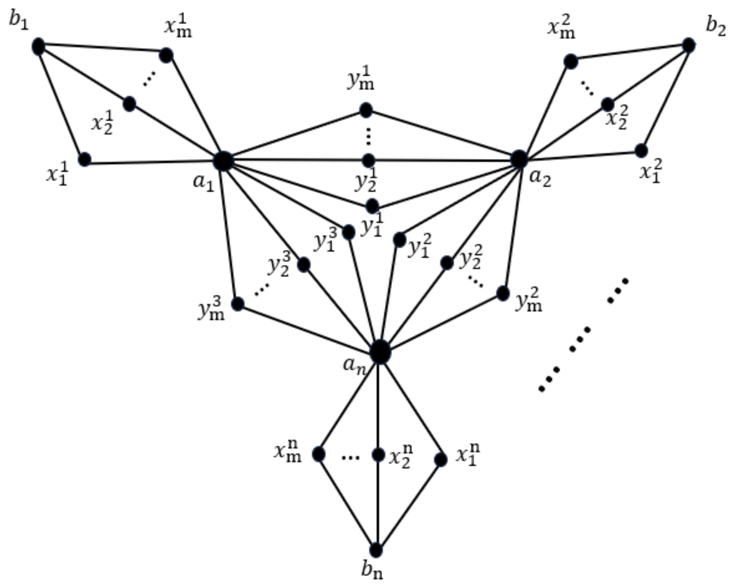

Figure 1: The graph G.

The induced edges are labeled as follows:
$f^*\left(a_i y_j^i\right) = (i-1)^2 + \left(mn+2n-1+j+m(i-1)\right)^2; \; 1 \leq i \leq n, 1 \leq j \leq m$
$f^*\left(y_j^i \, a_i\right) = \left(mn+2n-1+j+m(i-1)\right)^2 + (i-1)^2; \; 1 \leq i \leq n, 1 \leq j \leq m$
$f^*\left(a_i x_j^i\right) = (i-1)^2 + (2n+(j-1)+m(i-1))^2; \; 1 \leq i \leq n, 1 \leq j \leq m$
$f^*\left(x_j^i \, a_i\right) = (2n+(j-1)+m(i-1))^2 + (i-1)^2; \; 1 \leq i \leq n, 1 \leq j \leq m$

Hence, the super subdivision of a crown graph admits square sum labeling. The theorem is illustrated in Figure 2

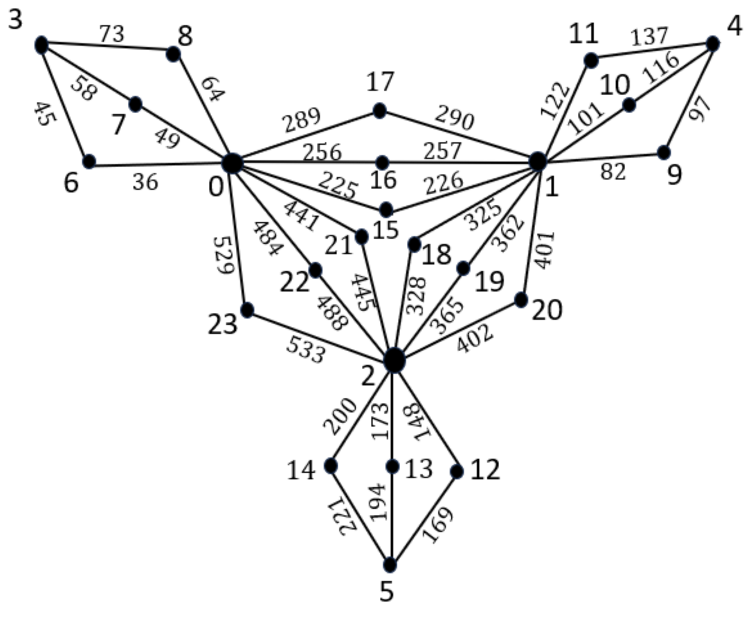

Figure 2. Super Subdivision of a Crown Graph for $n = 3, m = 3$

THEOREM 2.2:
The super subdivision of a coconut tree graph admits square sum labeling.
Proof: Let G be the super subdivision of a coconut tree. G is obtained from the path p_k by attaching a star $k_{1,n}$at P_1. The following is the description of graph G. Let $y_i (1 \leq i \leq n)$ be the pendant vertices of the star, Lety_j^i $(1 \leq i \leq n,\ 1 \leq j \leq m)$ be the vertices of the super subdivision of the star. Let $P_k(1 \leq k \leq s)$ be the vertices of the path andlet $x_j^k(1 \leq k \leq r - 1,\ 1 \leq j \leq m)$ be the vertices of the super subdivision of the path. The generalized graph for the super subdivision of a coconut tree is depicted in Figure 3.

Let the vertex and the edge set be defined as follows:

$$|V(G)| = (n + s)m + n + r - m \ and$$
$$|E(G)| = 2m(n + s - 1)$$

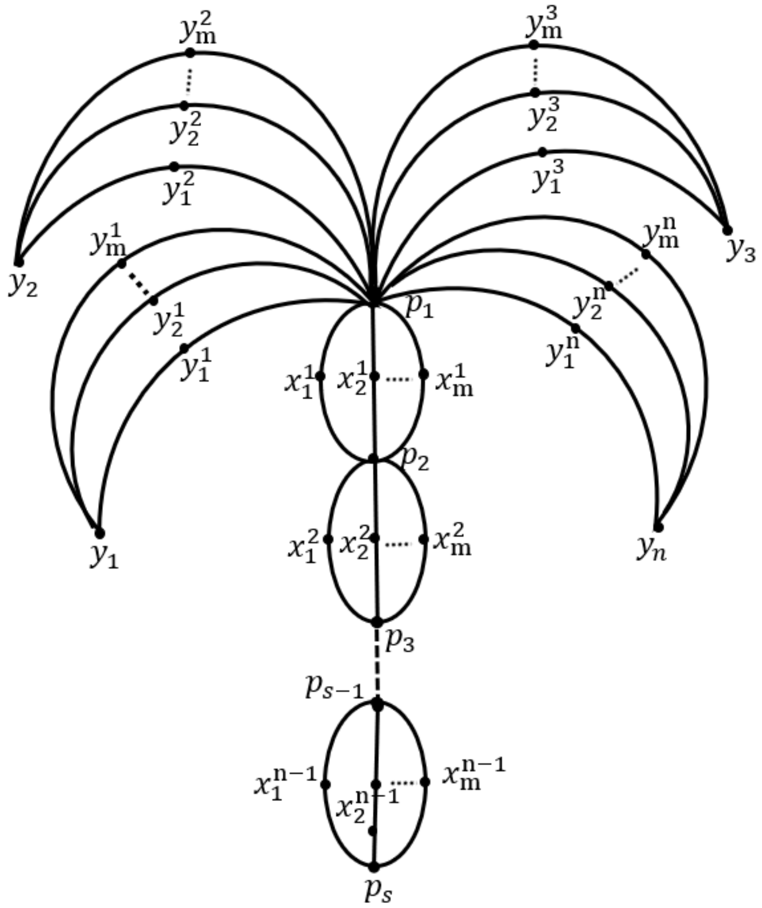

Figure 3:The graph G (SSD of a coconut tree)

The vertices are labeled as follows:

$f(y_i) = i - 1; 1 \leq i \leq n$

$f(p_k)= n + k - 1; 1 \leq k \leq s$

$f\left(y_j^i\right) = n + s - 1 + j + m(i - 1); 1 \leq i \leq n, 1 \leq j \leq m$

$f\left(x_j^i\right) = n + s - 1 + mn + j + m(i - 1); 1 \leq i \leq n, 1 \leq j \leq m$

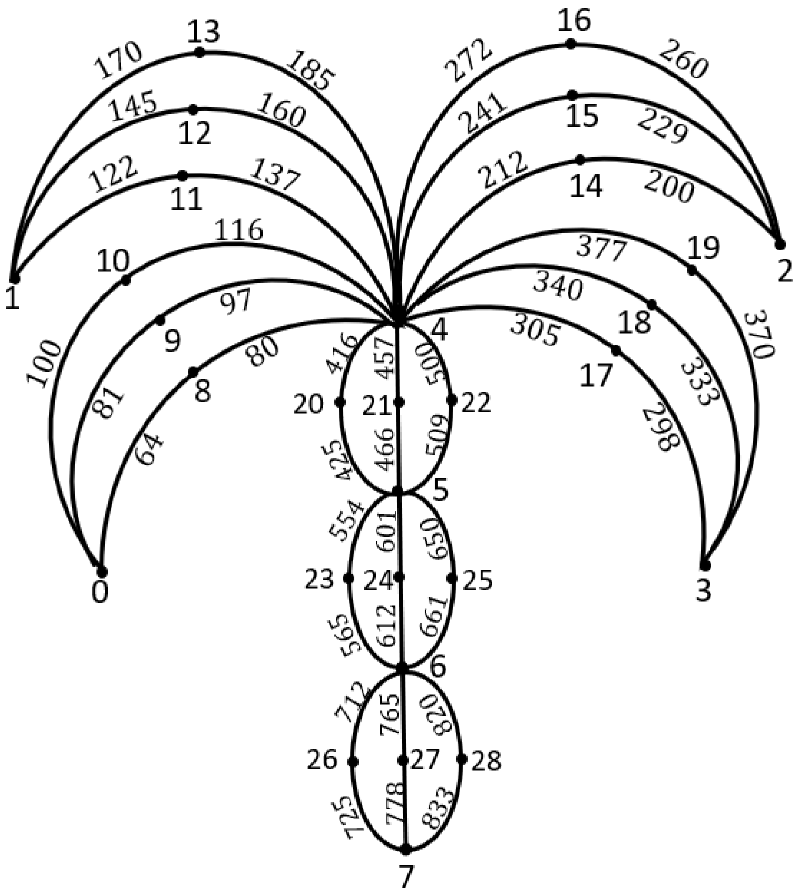

Figure 4. Super Subdivision of a Coconut Tree for $n = 4, m = 3, s = 4$

The induced edges are labeled as follows:

$f^*(y_i y_j^i) = (i-1)^2 + (mn + 2n - 1 + j + m(i-1))^2;\ \ 1 \le i \le n, 1 \le j \le m$

$f^*(y_j^i\, p_k) = (mn + 2n - 1 + j + m(i-1))^2 +\ (i-1)^2;\ \ 1 \le i \le n, 1 \le j \le m$

$f^*(p_k x_j^i) = (i-1)^2 + (2n + (j-1) + m(i-1))^2\ ;\ \ 1 \le i \le n, 1 \le j \le m$

$f^*(x_j^i\, p_k) = (2n + (j-1) + m(i-1))^2 + (i-1)^2;\ \ 1 \le i \le n, 1 \le j \le m$

Hence, the super subdivision of a coconut tree admits square sum labeling. The theorem is illustrated in Figure 2.

3. Conclusion

In this paper, we have proved that the super subdivision of a crown graph and coconut tree graph admits square sum labeling. We plan to demonstrate the existence of characteristics of the concept of super subdivision on the square sum labeling of graphs in the future.

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