https://doi.org/ 10.33472/AFJBS.6.Si2.2024.544-552

African Journal of Biological Sciences

Journal homepage: http://www.afjbs.com

Research Paper Open Access

Split Domination number and its Applications in Diverse Domains

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Article History

Volume 6,Issue Si2, 2024

Received:09 Mar 2024

Accepted : 07 Apr 2024

42doi: 10.33472/AFJBS.6.Si2.2024.544-552

Abstract

The focus of this research is on the possible utilization of split domination in graphs across different domains. A dominating set is a set of vertices in a graph where every vertex in the graph is either a member of this set or is connected to at least one vertex in the set. A graph's dominating set is deemed split dominant if removing it from the graph causes the graph to become disconnected. This article examines the various applications of split domination number in the fields of Computer Network and Public Health.

Keywords

Dominating set, Domination number, Split domination number, network topology,

I. Introduction

Recent research in Mathematics has been focusing on exploring the concept of dominance in graph theory. A dominating set of $G=(V,E)$ is a vertex subset D of G such that each vertex of $V \setminus D$ is adjacent to at least one vertex of D. A minimal dominating set is the smallest dominating set that doesn't contain any other dominating set as a proper subset. The cardinality of the smallest dominating set is called the domination number and is denoted by $\gamma(G)$.

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The graph's dominating set D is splits dominant if on removal of D the resultant graph $V \setminus D$ is disconnected. The split dominant number $\gamma_s(G)$ is the cardinality of the smallest dominating set of vertices that, when removed, causes the graph to be disconnected.

Consider a web Graph $W(2,3)$ [6] as shown in figure 1.

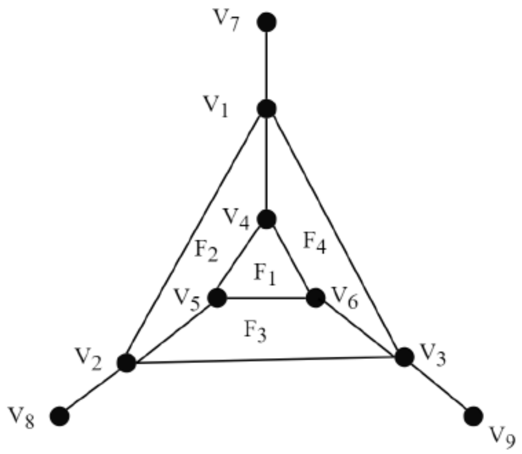


Figure 1

The split domination number of this graph is 3, with $\{v_1, v_2, v_3\}$ as split dominating set.

Dominance can lead to the development of various elements, and it's been a topic of increased interest due to its fundamental problems related to NP-completeness and its connections to other NP-completeness problems [1].

Back in the 1850s, chess posed a challenge regarding dominance. C.F. de Jaenisch, in 1862, sought to determine the minimum number of queens needed on a chessboard to ensure that every square either has a queen or is under attack by one.

Graph theory, particularly the study of dominance sets, has seen significant advancements since the 1960s. In 1962, Ore and Berge [2] introduced the concept of dominant sets in graphs. Following this, Ore further explored the ideas of dominance set and dominance number. Cockayne and Hedetniemi, in their research, focused on dominance and independence numbers, introducing notation for a graph's dominance number [8].

In 1977, Mitchell and Hedetniemi extended the concept to edge domination [4]. V Kulli and Janakiram brought forth the idea of split domination in graphs [9]. T. Chelvam and S. Chellathurai discovered specific values for split dominance number and characteristics of the split dominating set [7]. Detail study of split domination find in [7, 9].

In the study of split dominance, we obtained line split dominance and dual graphs of different graphs in a general form, which is useful in many real-world situations.

Some of the results of our work are quoted as [5-6]:

1. If W_n is Wheel graph of n vertices. Then

a. $\gamma_s\left(G^*(W_4)\right)$ is not defined.

b. $\gamma_s\left(G^*\left(W_n\right)\right)=3$, where $n \geq 5$

c. $\gamma_s(L(W_n))=4$ for $n<6$.

d. $\gamma_s(L(W_n))=n-2$ for $n \geq 6$

where $G^*\left(W_n\right)$ and $L(W_n)$ is physical dual graph and line graph of W_n respectively.

2. $\gamma_s\left(G^*\left(P_n \times P_n\right)\right)=\begin{cases} Not\, defined & , where\ n=2 \\ n & , where\, n \geq 3 \end{cases}$ where $P_n \times P_n$ is Cartesian product of path.

3. $\gamma_s\left(G^*\left(H_n\right)\right)=3$, for $\boldsymbol{n} \geq 5$ where H_n is Helm graph.

4. $$\gamma_s\left(G^*\left(W\left(t,n\right)\right)\right)=\begin{cases} n & : t=2 & n \geq 3 \\ n+1 & : t=3\ or\ 4 & n \geq 3 \\ n+2 & : t=5 & n \geq 3 \\ n+2+\left\lceil \dfrac{n}{3} \right\rceil *(t-6) & : t \geq 6 & n \geq 3 \end{cases}$$

Where, $W\left(t,n\right)$ is the generalized web graph that includes $n-$cycles in multiple of t.

5. If G is any Bi Star Graph then Line graph of Bi star graph is two copies of complete Graph.

6) Let $B(n,n)$ be Bi Star Graph and L($B(n,n)$) is Line graph of Bi Star Graph then

a) $\gamma_s\left(L\left(B\left(n,n\right)\right)\right)=1$

b) $\gamma_s\left(B\left(n,n\right)\right)>\gamma_s\left(L\left(B\left(n,n\right)\right)\right)$

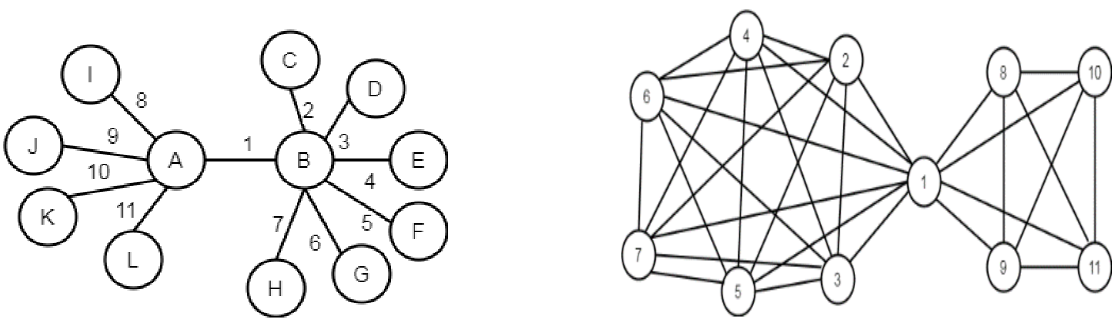


Figure 2

(A) Bi-star graph $B_{(4,6)}$ (B) Line graph of Bi-star graph $B_{(4,6)}$

7) Let $T(m,n)$ is Tadpole graph $m \geq 3$ vertices and a path graph on n vertices then split domination number of line graph of $T(m,n)$ is given by

a) $\gamma_s(L(T(3,1))) = 2$

b) $\gamma_s(L(T(3,n))) = 1 + \left\lceil \frac{n-1}{3} \right\rceil, n > 1$

c) $\gamma_s(L(T(m,1))) = 1 + \left\lceil \frac{m-3}{3} \right\rceil, m > 3$

d) $\gamma_s(L(T(m,n))) = \begin{cases} 1 + \left\lceil \frac{m-2}{3} \right\rceil + \left\lceil \frac{n-2}{3} \right\rceil, m \neq 3k \\ 1 + \left\lceil \frac{m-3}{3} \right\rceil + \left\lceil \frac{n-1}{3} \right\rceil, m = 3k \end{cases}$ for $m \geq 4$ and $k \geq 2$

In this article, we will be discussing possible applications of split domination numbers in the fields of computer networking and public health.

II. Applications of Split Domination in graph

The concept of split domination finds applications in diverse disciplines such as chemical, fluid network, security network, computer network, network analysis, and VLSI designing. However, its implementation has not been fully realized. Here, we only focus on two specific applications of split domination in the fields of computer networking and public health.

1) Computer Network

Split domination can be utilized in computer networks and server management to ensure fault tolerance and network resilience.

A computer network is combination of computer system along with LANs hub, servers, bridges routers switches, gateway etc. each has their use. Hub is a repeater used to interconnect the system in a LAN, Bridge is used to connect two LAN, switch is bridge with more than two ports, used to connects more than two networks, each having individual servers. The whole network can be represented by a graph.

In computer networks computers are connected using star, ring, bus, hybrid, mesh, tree topology etc. The above mentioned graph like tadpole, bistar, wheel, their line graph and dual graph etc. studied by us to find the split dominating number in a generalized form, can be used in different topology.

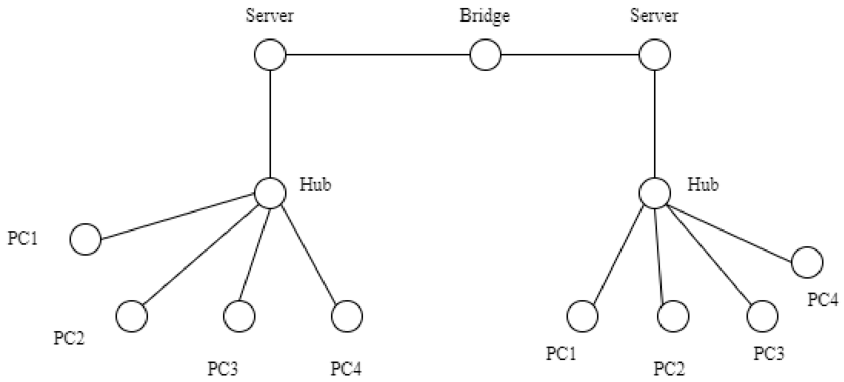


Figure 3

Consider a scenario where multiple servers are interconnected to form a network. These servers have the responsibility of overseeing critical services or enabling the transfer of data within an organization or for a service provider. The servers are interconnected to ensure redundancy and fairly spread the burden.

To ensure failure tolerance in this server network, split dominating sets must be identified. Imagine a situation in which there are two main server clusters, each with specific duties for critical tasks, and a few servers acting as bridges to facilitate data transfers and communication between these clusters.

In the event of a server failure or network, k disruption, isolating these critical bridge servers becomes crucial. By targeting these split dominating servers, the network can be intentionally partitioned to contain failures or disruptions within a specific cluster without affecting the overall system.

This strategy allows for more effective fault isolation and recovery. By disconnecting clusters, administrators can focus on restoring functionality to the affected cluster without worrying about affecting the operations of other clusters. This also helps avoid cascading errors that can occur if two clusters are highly interdependent.

Moreover, this approach aids in optimizing network resources and allows for more efficient resource allocation during maintenance or troubleshooting, as the impact is localized within a specific cluster.

In computer server networks, the concept of split domination can thus be employed to enhance fault tolerance, streamline maintenance, and ensure smoother operations even in the face of server failures or network disruptions.

Illustration:-

Let’s create a simple network representing a scenario with two clusters of connected by a few servers.

Let's represent servers as nodes and their connections as edges:

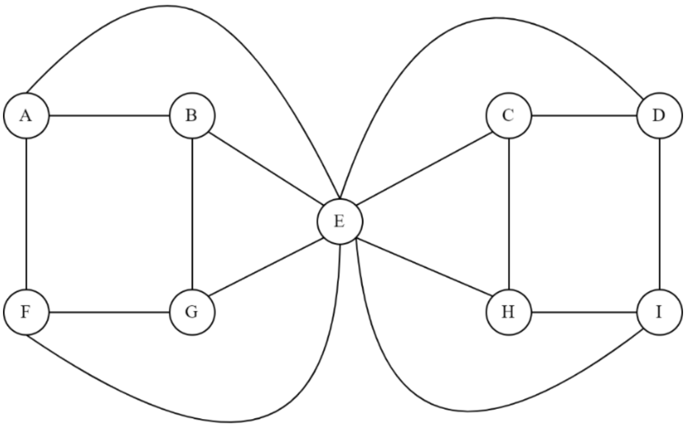


Figure 3

In this graph:

Servers A, B, F, G form one cluster.

Servers C, D, H, I form another cluster.

Servers E is a bridge between the two clusters.

In this scenario, servers E acts as a split dominating server, connecting both clusters. Removing server E would split the network into two separate clusters, A-B-F-G and C-D-H-I.

2) Public Health

Examine a social network analysis within the framework of illness transmission. Imagine you possess a network that depicts the connections between individuals within a community. Nodes symbolize individuals, while edges symbolize interactions or contacts among them.

When dealing with a communicable illness, the transmission of the infection can be represented by these exchanges. It is essential to identify split dominating sets in this network in order to develop effective measures for controlling the spread of the disease.

Consider a scenario where the network consists of two tightly interconnected subgroups, where individuals often engage within their own groups but occasionally have contacts with individuals from the other group. It is crucial to identify the smallest group of individuals who have links in both categories.

By targeting and isolating these individuals who act as bridges between the subgroups, health authorities can effectively contain the spread of the disease. Disrupting the connections between these split dominating individuals helps in preventing the transmission of the disease from one subgroup to another.

Strategies like targeted quarantines, increased monitoring, or specific interventions for these crucial individuals can significantly slow down the spread of the disease between the tightly connected subgroups, ultimately reducing the overall impact of the outbreak within the community.

This application demonstrates how the concept of split domination can be adapted to address challenges in public health and disease control.

Imagine this social network:

Nodes: Represent individuals in the community.

Edges: Represent interactions or contacts between individuals.

Subgroups: The network has two densely connected groups (red and blue), where individuals interact frequently within their groups.

Bridges: A few individuals (green) have connections in both groups, acting as bridges between the otherwise segregated communities.

Why are these bridge individuals crucial?

Disease Spread: In a contagious disease outbreak, the bridges become critical points for transmission. The infection can easily jump from one subgroup to another through their interactions.

Here's where split dominating sets come in:

A split dominating set is a minimal set of node, that dominates all the remaining nodes and whose removal disconnects the graph.

In this case, identifying the minimum set of bridge individuals who, if isolated or quarantined, would effectively cut off transmission between the subgroups, forms a split dominating set.

Benefits of targeting split dominating sets:

Reduced Transmission: By disrupting the connections of these key individuals, the disease spread between subgroups slows down significantly.

Targeted Interventions: Health authorities can focus their efforts (quarantines, monitoring, specific treatments) on a smaller group, optimizing resource allocation and response effectiveness.

Lower Overall Impact: By containing the outbreak within each subgroup, the overall number of infected individuals and the disease's burden on the community are reduced.

In conclusion, understanding and utilizing the concept of split dominating sets in social network analysis opens up exciting possibilities for tackling public health challenges and mitigating the spread of infectious diseases.

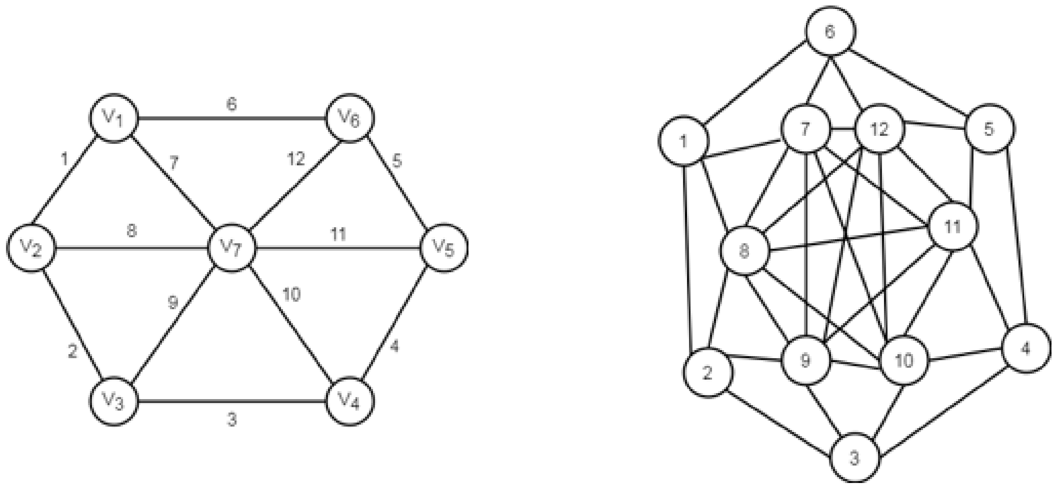
Illustration

Consider a working network of 6 peoples having connections as shown in graph W_7 figure 4(A), where V_7 represent the centre of connection say an office so all 6 are in same connections with interconnections as shown. Now observe the situation that V_2 is affected by any disease and it’s required to find out the removal of its neighbours without affecting the whole network.

It will be required to remove V_1 and V_3 along with V_2, so it's required to find the minimum set of connections whose removal removes the set $\{V_1,V_2,V_3\}$. That is it's required to find edge split dominating set or we can say split dominating set of line graph of this network with split dominating number $\gamma_s(L(W_7))=5$ for the removal of $\{V_1,V_2,V_3\}$. Split edge dominating set will be $\{6,7,8,9,3\}$ all other such possibility are listed in table can be easily detected by line graph figure 4(B) it is just an example. It will be a helpful in a bigger network.

Table for Graph in Figure 2

Minimum split dominating set of L(W7)	**Removed group of W7**	**Working Network**
{6,7,8,9 3}	$\{V_1,V_2,V_3\}$	$\{V_7,V_6,V_5,V_4\}$
{6,12,11,10,3}	$\{V_6,V_5,V_4\}$	$\{V_7,V_1,V_2,V_3\}$
{1,8,9,10,4}	$\{V_2,V_3,V_4\}$	$\{V_7,V_1,V_6,V_5\}$
{1,7,12,11,4}	$\{V_1,V_6,V_5\}$	$\{V_7,V_2,V_3,V_4\}$
{2,8,7,12,5}	$\{V_1,V_2,V_6\}$	$\{V_7,V_3,V_4,V_5\}$
{2,9,10,11,5}	$\{V_3,V_4,V_5\}$	$\{V_7,V_2,V_1,V_6\}$



(A) Wheel graph W_7 (B) Line Graph of Wheel Graph W_7

Figure 4

The situation may be possible for the network in Figure 4(B) itself from this figure, let's consider two sub group one is Outer one and second is inner one.

Where individuals frequently interact within their respective groups but occasionally have interactions across the groups. Identifying the minimum set of individuals who have connections in both groups becomes essential.

If we remove three individuals form inner subgroup and two individuals from outer sub group then it will reduced the spread of infectious diseases, in this situation also our finding will be helpful.

The generalized result for such network is derived in our previous finding as in split dominating number of line graph of W_n quoted in section 1.

III. Conclusion

Aiming to demonstrate the relevance of graph theory ideas such as Split dominance to various scientific and engineering domains is the primary objective of this study. The split domination of a line graph is employed when there is a need to divide a network based on its edges, whereas the split domination of a dual graph is used when the division of the network is based on the division of its regions. The objective is to facilitate researchers in comprehending the use of these concepts in their own investigations and endeavors.

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