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Common Fixed-Point Theorems for Rational Expression in Fuzzy Metric Space using E.A like Property

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Abstract: In this Paper, we are proving common fixed point for rational expression using by common E.A like property and weakly compatible mapping in fuzzy metric space and also we define new class of function ψ ,which can be helpful for the researcher and accelerate sustainable development for the existence and uniqueness of common fixed point in which the pair of the maps are satisfying E.A like property, which is generalized the result of K. Wadhwa et.al.[13].

Keywords: *Fuzzy metric space, Common Fixed point, Common E.A like property, weakly compatible mapping*

Introduction

The notion of Fuzzy Metric Space was first presented by Kromosil and Michalek in 1975 [1]. Veeramani and George introduced the modified Continuous t-norm approach in 1994 [16]. Numerous authors and researchers developed new findings on fuzzy metric space in various ways by utilizing several novel notions, such as compatible mapping, weak compatible mapping, R-weakly Computing mapping, and CLR-property, among others. R. Vasuki[8] extended the outcome for 2-Metric space in 1999. Sushil Sharma [19][20] defined the property E.A. for the first time in 2002, establishing certain common fixed point theorems and defining the property as a novel concept in fuzzy metric spaces under rigorous contractive conditions. 2013 saw the introduction of the novel idea of the E.A. like property in fuzzy metric space by K. Wadhawa et al. [13]. This quality is crucial in ensuring that one does not need the completeness of the entire space, the continuity of mappings, or the proximity of range subspaces. Fuzzy Metrics are applied in many fields, including communication, control theory, neural network theory, image processing, medical sciences, stability theory, and applied sciences.
This work presents the proof of a few common fixed point theorems for new rational expressions for four mappings through weakly compatible and common E.A.-like properties in a fuzzy metric space. More work on fixed point related to fuzzy metric space with specified properties for various conditions can be seen in [23-31] . Established results are very useful in uncertainty and decision-making problems.

1. Preliminaries:

Definition 2.1[16] A binary operation $*: [0,1] \times [0,1] \rightarrow [0,1]$ is a continuous t-norm if * satisfying the condition:

(1) * is commutative and associative.
(2) * is continuous.
(3) $a * 1 = a \;,\; \forall a \in [0,1]$
(4) $a * b \leq c * d \;, whenever\; a \leq c \; and \; b \leq d \; for \; \forall \; a, b, c, d \in [0,1]$

Definition 2.2[16] A 3-tuple $(\Xi, \mathfrak{M}, *)$ is said to be Fuzzy Metric space if Ξ is an arbitrary set, * is a continuous t-norm and $\mathfrak{M}$is a Fuzzy set on $\Xi^2 \times [0, \infty)$ satisfying the following conditions:$\forall \; x, y, z \in \Xi, s, t > 0$

(1) $\mathfrak{M}(x, y, z) > 0$
(2) $\mathfrak{M}(x, y, t) = 1 \; for \; all \; t > 0 \quad iff \;\; x = y.$
(3) $\mathfrak{M}(x, y, t) = \mathfrak{M}(z, x, t)$
(4) $\mathfrak{M}(x, y, t_1) * \mathfrak{M}(z, z, t_2) \leq \mathfrak{M}(x, z, t_1 + t_2)$

$$\forall x, y, z \in \Xi \; and \; t_1, t_2, > 0$$

(5) $\mathfrak{M}(x, y, *): [0, \infty] \rightarrow [0,1]$ is left continuous.
(6) $\lim_{t\to\infty} \mathfrak{M}(x, y, t) = 1.$

Definition 2.3[4] Two self mapping A and B on a fuzzy mapping $(\Xi, \mathfrak{M}, *)$ are said to be compatible if $\lim_{n\to\infty} \mathfrak{M}(ABx_n, BAx_n, t) = 1 \; for\ all\ t > 0$ whenever $\{x_n\}$is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u\,,\ for\ all\ u \in \Xi$

Definition 2.4[13] Two self mapping A and B on a fuzzy mapping $(\Xi, \mathfrak{M}, *)$ are said to be a weakly compatible if they commute at their coincidence point that is for $\forall\, u \in \Xi\ , Au = Bu\ implies\ that\ \ ABu = BAu\ for\ all\ t > 0$

Definition 2.5[19] Two self mapping A and B on a fuzzy mapping $(\Xi, \mathfrak{M}, *)$ are said to be Satisfy E.A property if their exist a sequence $\{x_n\}$in Ξsuch that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = u\,,\ for\ all\ u \in \Xi$$

Definition 2.6[13] Let A, B,S and T be a self mapping of a Fuzzy Metric space $(\Xi, \mathfrak{M}, *)$ then (A,S) and (B,T) said to satisfy Common E.A like property if their exist two sequence $\{x_n\}\ and\ \{y_n\}$ in Ξ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} \mathrm{T}\, y_n = z$, Where $z \in S(\Xi) \cap T(\Xi)\ or\ z \in A(\Xi) \cap B(\Xi)$

Lemma 2.7[8] $\mathfrak{M}(x, y, .)$ is non decreasing function $for\ all\ x, y, z \in \Xi$

The following definition and result are define Mishra[14].

Lemma 2.8[14]Let$(\Xi, \mathfrak{M}, *)$ be a Fuzzy Metricspace if their exist$k \in (0,1)$ $such\ that$

$$\mathfrak{M}(x, y, kt) \geq \mathfrak{M}(x, y, t)\ \ then\ x = y\ and\ , k \in (0,1), t\ > 0\ \ for\ all\ x, y \in \Xi$$

In this paper, we will define the following implicit function which will be support in the main result.

Definition 2.9: Letϕ be the set of all real continuous function $\phi: [0,1]^5 \to [0,1]$ non decreasing in each coordinate variable and such that

(i) $\phi(1,1,t,t,1) \geq t$;
(ii) $\phi(1,t,1,1,1) \geq t$;
(iii) $\phi(t,t,t,1,1) \geq t$; $\forall t \in [0,1]$

Main Result

Theorem 3.1: Let $P, B, N\ and\ T$ be a self mapping of a Fuzzy Metric space $(\Xi, \mathfrak{M}, *)$ satisfying the following conditions:

(i) $(P, N) and (B, T)$ Satisfying common E.A like properties.
(ii) $(P, N) and (B, T)$ are weakly compatible.
(iii) For $\emptyset \in \phi$ then there exist $k \in (0,1)$ such that $\forall x, y, z \in \Xi$ and $t > 0$.

$$\mathfrak{M}(Px, By, kt) \geq \phi \begin{Bmatrix} \mathfrak{M}(Nx, Ty, t), \mathfrak{M}(Nx, By, t), \mathfrak{M}(Px, Ty, t), \\ \mathfrak{M}(Nx, Px, t), \dfrac{\mathfrak{M}(Px, By, t). \mathfrak{M}(Nx, Ty, t)}{\mathfrak{M}(Nx, By, t). \mathfrak{M}(Px, Ty, t)} \end{Bmatrix}$$

Then P, B, N and T have a common fixed point.

Proof: Since $(P,N)$and (B,T) satisfying common E.A like properties then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Ξ such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z.$

Where , $z \in N(\Xi) \cap T(\Xi)$ or $z \in P(\Xi) \cap B(\Xi)$.

Let $z \in N(\Xi) \cap T(\Xi)$ and $\lim_{n\to\infty} Px_n = z \in N(\Xi)$ $then$ $z = Nu$, where $x \in \Xi$.

To Prove, $Pu = Nu$

Put $x = u$ and $y = y_n$ in inequality (iii)

$$\mathfrak{M}(Pu, By_n, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Nu, Ty_n, t), \mathfrak{M}(Nu, By_n, t), \mathfrak{M}(Pu, Ty_n, t), \\ \mathfrak{M}(Nu, Pu, t), \dfrac{\mathfrak{M}(Pu, By_n, t). \mathfrak{M}(Nu, Ty_n, t)}{\mathfrak{M}(Nu, By_n, t). \mathfrak{M}(Pu, Ty_n t)} \end{matrix} \right\}$$

$$\mathfrak{M}(Pu, z, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(z, z, t), \mathfrak{M}(z, z, t), \mathfrak{M}(Pu, z, t), \\ \mathfrak{M}(z, Pu, t), \dfrac{\mathfrak{M}(Pz, z, t). \mathfrak{M}(Nz, z, t)}{\mathfrak{M}(Nz, z, t). \mathfrak{M}(Pz, z, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(Pu, z, kt) \geq \phi\{1 , 1 , \mathfrak{M}(Pu, z, t), \mathfrak{M}(z, Pu, t), 1 \}$$

$$\mathfrak{M}(Pu, z, kt) \geq \mathfrak{M}(Pu, z, , t)$$

By lemma 2.8, we get $Pu = z$ $i.e$ $Pu = z = Nu$

Since (P,N) is weakly compatible then $Pz = PNu = NPu = Nz$

Again, $\lim_{n\to\infty} By_n = z \in T(X)$ $then$ $z = Tv$

Now to Prove, $Tv = Bv$

Put $x = x_n$ and $y = v$ in (iii)

$$\mathfrak{M}(Px_n, Bv, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Nx_n, Tv, t), \mathfrak{M}(Nx_n, Bv, t), \mathfrak{M}(Px_n, Tv, t), \\ \mathfrak{M}(Nx_n, Px_n, t), \dfrac{\mathfrak{M}(Px_n, Bv, t). \mathfrak{M}(Nx_n, Tv, t)}{\mathfrak{M}(Nx_n, Bv, t). \mathfrak{M}(Px_n, Tv, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z, Bv, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(z, z, t), \mathfrak{M}(z, Bv, t), \mathfrak{M}(z, z, t), \\ \mathfrak{M}(z, z, t), \dfrac{\mathfrak{M}(z, Bv, t). \mathfrak{M}(z, z, t)}{\mathfrak{M}(z, Bv, t). \mathfrak{M}(z, z, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z, Bv, kt) \geq \phi\{1, \mathfrak{M}(z, Bv, t), 1,1,1 \}$$
$$\mathfrak{M}(z, Bv, kt) \geq \mathfrak{M}(z, Uv, t)$$

By lemma 2.8, we get $z = Bv$ $i.e$ $Bv = z = Tv$

Since (U,T) is weakly compatible then $Bz = BTv = TBv = Tz$

Now to Prove $Pz = z$ then

Put $x = z \;\; and \; y = y_n$ in (iii)

$$\mathfrak{M}(Pz, By_n, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Nz, Ty_n, t), \mathfrak{M}(Nz, By_n, t), \mathfrak{M}(Pz, Ty_n, t), \\ \mathfrak{M}(Nz, Pz, t), \dfrac{\mathfrak{M}(Pz, By_n, t). \mathfrak{M}(Nz, Ty_n, t)}{\mathfrak{M}(Nz, By_n, t). \mathfrak{M}(Pz, Ty_n, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(Pz, z, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), \\ \mathfrak{M}(Pz, Pz, t), \dfrac{\mathfrak{M}(Pz, z, t). \mathfrak{M}(Pz, z, t)}{\mathfrak{M}(Pz, z, t). \mathfrak{M}(Pz, z, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(Pz, z, kt) \geq \phi\{ \mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), 1,1\}$$

$$\mathfrak{M}(Pz, z, kt) \geq \mathfrak{M}(Pz, z, t)$$

By lemma 2.8, we get $Pz = z$ i.e $Pz = z = Nz$

Now to Prove $Bz = z$

Put $x = x_n \;\; and \; y = z$ in (iii)

$$\mathfrak{M}(Px_n, Bz, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Nx_n, Tz, t), \mathfrak{M}(Nx_n, Bz, t), \mathfrak{M}(Px_n, Tz, t), \\ \mathfrak{M}(Nx_n, Px_n, \mathrm{t}), \dfrac{\mathfrak{M}(Px_n, Bz, t). \mathfrak{M}(Nx_n, Tz, t)}{\mathfrak{M}(Nx_n, Bz, t). \mathfrak{M}(Px_n, Iv, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z, Bz, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, t), \\ \mathfrak{M}(z, z, t), \dfrac{\mathfrak{M}(z, Bz, t). \mathfrak{M}(z, Bz, t)}{\mathfrak{M}(z, Bz, t). \mathfrak{M}(z, Bz, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z, Bv, kt) \geq \phi\{\mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, t) , \mathfrak{M}(z, Bz, t), 1,1 \}$$
$$\mathfrak{M}(z, Bz, kt) \geq \mathfrak{M}(z, Bz, t)$$

By lemma 2.8, we get $Bz = z \;\; i.e \;\; Bz = z = Tz$

Hence, $Pz = Bz = Nz = Tz = z$

Hence P, B, N and T have a Common Fixed Point.

Uniqueness: Suppose z_1 and z_2 are two common fixed point of $P, B, N \; and \; T$ with $z_1 \neq z_2$ then from (3.1)

$$\mathfrak{M}(Pz_1, Bz_2, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(Nz_1, Tz_2, t), \mathfrak{M}(Nz_1, Bz_2, t), \mathfrak{M}(Pz_1, Tz_2, t), \\ \mathfrak{M}(Nz_1, Pz_1, t), \dfrac{\mathfrak{M}(Pz_1, Bz_2, t). \mathfrak{M}(Nz_1, Tz_2, t)}{\mathfrak{M}(Nz_1, Bz_2, t). \mathfrak{M}(Pz_1, Tz_2, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z_1, z_2, kt) \geq \phi \left\{ \begin{matrix} \mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), \\ \mathfrak{M}(z_1, z_1, t), \dfrac{\mathfrak{M}(z_1, z_2, t). \mathfrak{M}(z_1, z_2, t)}{\mathfrak{M}(z_1, z_2, t). \mathfrak{M}(z_1, z_2, t)} \end{matrix} \right\}$$

$$\mathfrak{M}(z_1, z_2, kt) \geq \phi\{\mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), 1,1\}$$

By definition of Implicit function, we get, $\mathfrak{M}(z_1, z_2, kt) \geq \phi\{\mathfrak{M}(z_1, z_2, t)\}$
By lemma 2.8, we get. $z_1 = z_2$.

Remark * *Now we define new class of* ψ *as follows.*

Let ψ *be the class of all mapping* $\psi : [0,1] \to [0,1]$ *such that*
(a) ψ *is non-decreasing and* $\lim_{n\to\infty} \psi^{n}(p) = 1, \forall p \in (0,1]$;
(b) $\psi(p) > p, \forall p \in (0,1)$;
(c) $\psi(1) = 1$;

Example: *Define* $\psi: [0,1] \to [0,1]$ *by* $\psi(p) = \frac{2p}{p+1}, \forall p \in [0,1]$,

$$\psi^2(p) = \frac{4p}{3p+1},\ \psi^3(p) = \frac{8p}{7p+1} \ldots\ldots, \psi^n(p) = \frac{2^n p}{(2^n-1)p+1}, \forall p \in [0,1].$$

$$\lim_{n\to\infty} \psi^n(p) = \frac{2^n p}{(2^n-1)p+1} = 1, \forall p \in [0,1]$$

Clearly, $\psi(p) > p$ *and* $\psi(1) = 1, \forall p \in [0,1]$

Theorem 3.2: Let P, B, N *and* T be a self mapping of a Fuzzy Metric space $(\Xi, \mathfrak{M}, *)$ with $a * b = \min(a, b)$, satisfying the following conditions:

(i) (P, N) *and* (B, T) Satisfying Common E.A like properties.
(ii) (P, N) *and* (B, T) are weakly compatible.
(iii) $\mathfrak{M}(Px, By, t) \geq$

$$\psi\left\{min\begin{bmatrix}\mathfrak{M}(Nx, Ty, t), \mathfrak{M}(Nx, By, 2t), \mathfrak{M}(Px, Ty, t), \mathfrak{M}(Nx, Px, t), \\ \frac{\mathfrak{M}(Px,By,t).\mathfrak{M}(Nx,Ty,t)}{\mathfrak{M}(Nx,By,t).\mathfrak{M}(Px,Ty,t)}\end{bmatrix}\right\}$$

$\forall x, y \in \Xi$ *and* $t > 0$
then P, B, N and T have a Common Fixed Point.

Proof: Since (P, N) *and* (B, T) Satisfying common E.A like properties then there exist two sequences $\{x_n\}$ *and* $\{y_n\}$ in Ξ such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$.

Where , $z \in N(\Xi) \cap T(\Xi)$ *or* $z \in P(\Xi) \cap B(\Xi)$.

Let $z \in N(\Xi) \cap T(\Xi)$ and $\lim_{n\to\infty} Px_n = z \in N(\Xi)$ *then* $z = Nu$, where $x \in \Xi$.

To Prove, $Pu = Nu$

Put $x = u$ *and* $y = y_n$ in (iii)

$$\mathfrak{M}(Pu, By_n, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(Nu, Ty_n, t), \mathfrak{M}(Nu, By_n, 2t), \mathfrak{M}(Pu, Ty_n, t), \\ \mathfrak{M}(Nu, Pu, t), \frac{\mathfrak{M}(Pu, By_n, t). \mathfrak{M}(Nu, Ty_n, t)}{\mathfrak{M}(Nu, By_n, t). \mathfrak{M}(Pu, Ty_n t)}\end{bmatrix}\right\}$$

$$\mathfrak{M}(Pu, z, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(z, z, t), \mathfrak{M}(z, z, 2t), \mathfrak{M}(Pu, z, t), \mathfrak{M}(z, z, t), \\ \frac{\mathfrak{M}(Pz, z, t). \mathfrak{M}(Nz, z, t)}{\mathfrak{M}(Nz, z, t). \mathfrak{M}(Pz, zt)}\end{bmatrix}\right\}$$

$\mathfrak{M}(Pu,z,t) \geq \psi\{\ min[1\ ,1\ ,\mathfrak{M}M(Pu,z,t),\mathfrak{M}(z,Pu,t),1]\}$
$\mathfrak{M}(Pu,z,t) \geq \psi\ \{\mathfrak{M}(Pu,z,t)\}$
Since $\psi(p) \geq p$ for all $p \in (0,1]$, it is only possible when $\mathfrak{M}(Pu,z,t) = 1$
That is $Pu = z$

$$i.e\ Pu = z = Nu$$

Since $(P,N)\ is$ weakly compatible then $Pz = PNu = NPu = Nz$

Again, $\lim_{n\to\infty} By_n = z \in I(\Xi)\ \ then\ \ \ z = Tv$

Now to Prove, $Tv = Bv$

Put $x = x_n\ \ \ and\ y = v\ \ in\ (iii)$

$$\mathfrak{M}(Px_n,Bv,t) \geq \psi\left\{min\left[\begin{matrix}\mathfrak{M}(Nx_n,Tv,t),\mathfrak{M}(Nx_n,Bv,2t),\mathfrak{M}(Px_n,Tv,t),\\ \mathfrak{M}(Nx_n,Px_n,t),\dfrac{\mathfrak{M}(Px_n,Bv,t).\mathfrak{M}(Nx_n,Tv,t)}{\mathfrak{M}(Nx_n,Bv,t).\mathfrak{M}(Px_n,Tv,t)}\end{matrix}\right]\right\}$$

$$\Xi(z,Bv,t) \geq \psi\left\{min\left[\begin{matrix}\Xi(z,z,t),\mathfrak{M}(z,Bv,2t),\mathfrak{M}(z,z,t),\\ \mathfrak{M}(z,z,t),\dfrac{\mathfrak{M}(z,Bv,t).\mathfrak{M}(z,z,t)}{\mathfrak{M}(z,Bv,t).\mathfrak{M}(z,z,t)}\end{matrix}\right]\right\}$$

$\mathfrak{M}(z,Bv,t) \geq \psi\{min[1,\mathfrak{M}(z,Bv,2t),1,1,1]\}$
$\mathfrak{M}(z,Bv,t) \geq \psi\{[\mathfrak{M}(z,Bv,t)]\}$
Since $\psi(p) \geq p$ for all $p \in (0,1]$, it is only possible when $\mathfrak{M}(z,Bv,t) = 1$
That is $Bv = z$

$$i.e\ \ Bv = z = Tv$$

Since (U,I) is weakly compatible then $Bz = BTv = TBv = Tz$

Now to Prove $Pz = z$ then

Put $x = z\ \ and\ y = y_n$ in (iii)

$$\mathfrak{M}(Pz,By_n,t) \geq \psi\left\{min\left[\begin{matrix}\mathfrak{M}(Nz,Ty_n,t),\mathfrak{M}(Nz,By_n,2t),\mathfrak{M}(Pz,Ty_n,t),\\ \mathfrak{M}(Nz,Pz,t),\dfrac{\mathfrak{M}(Pz,By_n,t).\mathfrak{M}(Nz,Ty_n,t)}{\mathfrak{M}(Nz,By_n,t).\mathfrak{M}(Pz,Ty_n,t)}\end{matrix}\right]\right\}$$

$$\mathfrak{M}(Pz,z,t) \geq \psi\left\{min\left[\begin{matrix}\mathfrak{M}(Pz,z,t),\mathfrak{M}(Pz,z,2t),\mathfrak{M}(Pz,z,t),\\ \mathfrak{M}(Pz,Pz,t),\dfrac{\mathfrak{M}(Pz,z,t).\mathfrak{M}(Pz,z,t)}{\mathfrak{M}(Pz,z,t).\mathfrak{M}(Pz,z,t)}\end{matrix}\right]\right\}$$

$\mathfrak{M}(Pz,z,kt) \geq \psi\{min[\mathfrak{M}(Pz,z,t),\mathfrak{M}(Pz,z,2t),\mathfrak{M}(Pz,z,t),1,1]\}$
$M(Pz,z,t) \geq \psi\{\mathfrak{M}(Pz,z,t)\}$
Since $\psi(p) \geq p$ for all $p \in (0,1]$, it is only possible when $\mathfrak{M}(Pz,z,t) = 1$
That is $Pz = z\ \ \ i.e\ \ Bv = z = Nz$
Now to Prove $Bz = z$

Put $x = x_n\ \ and\ y = z$ in (iii)

$$\mathfrak{M}(Px_n, Bz, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(Nx_n, Tz, t), \mathfrak{M}(Nx_n, Bz, 2t), \mathfrak{M}(Px_n, Tz, t), \\ \mathfrak{M}(Nx_n, Px_n, \mathrm{t}), \dfrac{\mathfrak{M}(Px_n, Bz, t).\mathfrak{M}(Nx_n, Tz, t)}{\mathfrak{M}(Nx_n, Bz, t).\mathfrak{M}(Px_n, Tv, t)}\end{bmatrix}\right\}$$

$$\mathfrak{M}(z, Bz, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, 2t), \mathfrak{M}(z, Bz, t), \\ \mathfrak{M}(z, z, \mathrm{t}), \dfrac{\mathfrak{M}(z, Bz, t).\mathfrak{M}(z, Bz, t)}{\mathfrak{M}(z, Bz, t).\mathfrak{M}(z, Bz, t)}\end{bmatrix}\right\}$$

$$\mathfrak{M}(z, Bv, t) \geq \psi\{min[\mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, 2t), \mathfrak{M}(z, Bz, t), 1,1]\}$$

$$M(z, Bz, t) \geq \psi\{\mathfrak{M}(z, Bz, t)\}$$

Since $\psi(p) \geq p$ for all $p \in (0,1]$, it is only possible when $\mathfrak{M}(z, Bz, t) = 1$

That is $Bz = z$ $i.e$ $Bz = z = Tz$

Hence, $Pz = Bz = Nz = Tz = z$

HenceP, B, N and T $have$ a $Common$ $Fixed$ $Point.$

Uniqueness: Suppose z_1 and z_2 are two common fixed point of P, B, N and T with $z_1 \neq z_2$ then from (3.2)

$$\mathfrak{M}(Pz_1, Bz_2, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(Nz_1, Tz_2, t), \mathfrak{M}(Nz_1, Bz_2, 2t), \mathfrak{M}(Pz_1, Tz_2, t), \\ \mathfrak{M}(Nz_1, Pz_1, t), \dfrac{\mathfrak{M}(Pz_1, Bz_2, t).\mathfrak{M}(Nz_1, Tz_2, t)}{\mathfrak{M}(Nz_1, Bz_2, t).\mathfrak{M}(Pz_1, Tz_2, t)}\end{bmatrix}\right\}$$

$$\mathfrak{M}(z_1, z_2, t) \geq \psi\left\{min\begin{bmatrix}\mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, 2t), \mathfrak{M}(z_1, z_2, t), \\ \mathfrak{M}(z_1, z_1, t), \dfrac{\mathfrak{M}(z_1, z_2, t).\mathfrak{M}(z_1, z_2, t)}{\mathfrak{M}(z_1, z_2, t).\mathfrak{M}(z_1, z_2, t)}\end{bmatrix}\right\}$$

$$\mathfrak{M}(z_1, z_2, t) \geq \psi\{min[\mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, 2t), \mathfrak{M}(z_1, z_2, t), 1,1]\}$$

$$\mathfrak{M}(z_1, z_2, t) \geq \psi\{M(z_1, z_2, t)\}$$

Since $\psi(p) \geq p$ for all $p \in (0,1]$, it is only possible when $\mathfrak{M}(z_1, z_2, t) = 1$

That is $z_1 = z_2$

Hence proof the theorem.

Theorem 3.3: Let P, B, N and T be a self mapping of a Fuzzy Metric space $(\Xi, \mathfrak{M}, *)$ with $a * b = \min(a, b)$, satisfying the following conditions:

(i) $(P, N) and$ (B, T) Satisfying common E.A like properties.

(ii) $(P, N) and$ (B, T) are weakly compatible.

(iii) $$\mathfrak{M}(Px, By, t) \geq \psi\left\{\begin{matrix}\varpi\big(\mathfrak{M}(Nx, Ty, t)\big), \varpi\big(\mathfrak{M}(Nx, By, 2t)\big), \varpi\big(\mathfrak{M}(Px, Ty, t)\big), \\ \varpi\big(\mathfrak{M}(Nx, Px, t)\big), \varpi\left(\frac{\mathfrak{M}(Px, By, t).\mathfrak{M}(Nx, Ty, t)}{\mathfrak{M}(Nx, By, t).\mathfrak{M}(Px, Ty, t)}\right)\end{matrix}\right\}$$

$\forall\, x, y \in \Xi$ and $t > 0$

$then$ P, B, N and T $have$ a $Common$ $Fixed$ $Point.$

***Proof**:* Since $(P, N) and$ (B, T)Satisfying common E.A like properties then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in Ξ such that $\lim_{n\to\infty} Px_n = \lim_{n\to\infty} Nx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$

Where , $z \in N(\Xi) \cap T(\Xi)$ or $z \in P(\Xi) \cap B(\Xi)$

Let $z \in N(\Xi) \cap T(\Xi)$ and $\lim_{n\to\infty} Px_n = z \in N(\Xi)$ $then$ $z = Nu$, where $x \in \Xi$

To Prove, $Pu = Nu$

Put $x = u$ and $y = y_n$ in (iii)

$$\mathfrak{M}(Pu, By_n, kt) \geq \phi\left\{\begin{matrix}\varpi\big(\mathfrak{M}(Nu, Ty_n, t)\big), \varpi\big(\mathfrak{M}(Nu, By_n, t)\big), \varpi\big(\mathfrak{M}(Pu, Ty_n, t)\big), \\ \varpi\big(\mathfrak{M}(Nu, Pu, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Pu, By_n, t).\,\mathfrak{M}(Nu, Ty_n, t)}{\mathfrak{M}(Nu, By_n, t).\,\mathfrak{M}(Pu, Ty_n t)}\right)\end{matrix}\right\}$$

$$\mathfrak{M}(Pu, z, kt) \geq \phi\left\{\begin{matrix}\varpi\big(\mathfrak{M}(z, z, t)\big), \varpi\big(\mathfrak{M}(z, z, t)\big), \varpi\big(\mathfrak{M}(Pu, z, t)\big), \\ \varpi\big(\mathfrak{M}(z, Pu, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Pz, z, t).\,\mathfrak{M}(Nz, z, t)}{\mathfrak{M}(Nz, z, t).\,\mathfrak{M}(Pz, zt)}\right)\end{matrix}\right\}$$

$$\mathfrak{M}(Pu, z, kt) \geq \phi\left\{\begin{matrix}\varpi(1), \varpi(1), \varpi\big(\mathfrak{M}(Pu, z, t)\big), \\ \varpi\big(\mathfrak{M}(z, Pu, t)\big), \varpi(1)\end{matrix}\right\}$$

$by\ \ using\ remark *$ Properties then we get

$$\mathfrak{M}(Pu, z, kt) \geq \phi\{\,1\,, 1\,, \mathfrak{M}(Pu, z, t), \mathfrak{M}(z, Pu, t), 1\,\}$$

$$\mathfrak{M}(Pu, z, kt) \geq\ \mathfrak{M}(Pu, z,, t)$$

By lemma 2.8, we get $Pu = z$ $i.e\ Pu = z = Nu$

Since (P, N) is weakly compatible then $Pz = PNu = NPu = Nz$

Again, $\lim_{n\to\infty} By_n = z \in T(X)$ $then$ $z = Tv$

Now to Prove, $Tv = Bv$

Put $x = x_n$ $and\ y = v\ \ in\ (iii)$

$$\mathfrak{M}(Px_n, Bv, kt) \geq \phi\left\{\begin{matrix}\varpi\big(\mathfrak{M}(Nx_n, Tv, t)\big), \varpi\big(\mathfrak{M}(Nx_n, Tv, t)\big), \varpi\big(\mathfrak{M}(Nx_n, Bv, t)\big) \\ , \varpi\big(\mathfrak{M}(Nx_n, Px_n, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Px_n, Bv, t).\,\mathfrak{M}(Nx_n, Tv, t)}{\mathfrak{M}(Nx_n, Bv, t).\,\mathfrak{M}(Px_n, Tv, t)}\right)\end{matrix}\right\}$$

$$\mathfrak{M}(z, Bv, kt) \geq \phi\left\{\begin{matrix}\varpi\big(\mathfrak{M}(z, z, t)\big), \varpi\big(\mathfrak{M}(z, Bv, t)\big), \varpi\big(\mathfrak{M}(z, z, t)\big), \\ \varpi\big(\mathfrak{M}(z, z, t)\big), \varpi\left(\dfrac{\mathfrak{M}(z, Bv, t).\,\mathfrak{M}(z, z, t)}{\mathfrak{M}(z, Bv, t).\,\mathfrak{M}(z, z, t)}\right)\end{matrix}\right\}$$

$$\mathfrak{M}(z, Bvkt) \geq \phi\left\{\begin{matrix}\varpi(1), \varpi\big(\mathfrak{M}(z, Bv, t)\big), \varpi(1), \\ \varpi(1), \varpi(1)\end{matrix}\right\}$$

$by\ \ using\ remark *$ Properties then we get

$$\mathfrak{M}(z, Bv, kt) \geq \phi\{1, \mathfrak{M}(z, Bv, t), 1,1,1\,\}$$

$$\mathfrak{M}(z, Bv, kt) \geq \mathfrak{M}(z, Uv, t)$$

By lemma 2.8, we get $z = Bv$ $i.e$ $Bv = z = Tv$

Since (U,T) is weakly compatible then $Bz = BTv = TBv = Tz$

Now to Prove $Pz = z$ then

Put $x = z\ and\ y = y_n$ in (iii)

$$\mathfrak{M}(Pz, By_n, kt) \geq \phi \begin{Bmatrix} \varpi\big(\mathfrak{M}(Nz, Ty_n, t)\big), \varpi\big(\mathfrak{M}(Nz, By_n, t)\big), \varpi\big(\mathfrak{M}(Pz, Ty_n, t)\big), \\ \varpi\big(\mathfrak{M}(Nz, Pz, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Pz, By_n, t).\mathfrak{M}(Nz, Ty_n, t)}{\mathfrak{M}(Nz, By_n, t).\mathfrak{M}(Pz, Ty_n, t)}\right) \end{Bmatrix}$$

$$\mathfrak{M}(Pz, z, kt) \geq \phi \begin{Bmatrix} \varpi\big(\mathfrak{M}(Pz, z, t)\big), \varpi\big(\mathfrak{M}(Pz, z, t)\big), \varpi\big(\mathfrak{M}(Pz, z, t)\big), \\ \varpi\big(\mathfrak{M}(Pz, Pz, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Pz, z, t).\mathfrak{M}(Pz, z, t)}{\mathfrak{M}(Pz, z, t).\mathfrak{M}(Pz, z, t)}\right) \end{Bmatrix}$$

$$\mathfrak{M}(Pz, z, kt) \geq \phi\{\varpi\big(\mathfrak{M}(Pz, z, t)\big), \varpi\big(\mathfrak{M}(Pz, z, t)\big), \varpi\big(\mathfrak{M}(Pz, z, t)\big), \varpi(1), \varpi(1)\}$$

$by\ \ using\ remark *$ Properties then we get

$$\mathfrak{M}(Pz, z, kt) \geq \phi\{\mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), \mathfrak{M}(Pz, z, t), 1,1\}$$

$$\mathfrak{M}(Pz, z, kt) \geq \mathfrak{M}(Pz, z, t)$$

By lemma 2.8, we get $Pz = z$ i.e $Pz = z = Nz$

Now to Prove $Bz = z$

Put $x = x_n\ and\ y = z$ in (iii)

$$\mathfrak{M}(Px_n, Bz, kt) \geq \phi \begin{Bmatrix} \varpi\big(\mathfrak{M}(Nx_n, Tz, t)\big), \varpi\big(\mathfrak{M}(Nx_n, Bz, t)\big), \varpi\big(\mathfrak{M}(Px_n, Tz, t)\big), \\ \varpi\big(\mathfrak{M}(Nx_n, Px_n, \mathrm{t})\big), \varpi\left(\dfrac{\mathfrak{M}(Px_n, Bz, t).\mathfrak{M}(Nx_n, Tz, t)}{\mathfrak{M}(Nx_n, Bz, t).\mathfrak{M}(Px_n, Iv, t)}\right) \end{Bmatrix}$$

$$\mathfrak{M}(z, Bz, kt) \geq \phi \begin{Bmatrix} \varpi\big(\mathfrak{M}(z, Bz, t)\big), \varpi\big(\mathfrak{M}(z, Bz, t)\big), \varpi\big(\mathfrak{M}(z, Bz, t)\big), \\ \varpi\big(\mathfrak{M}(z, z, t)\big), \varpi\left(\dfrac{\mathfrak{M}(z, Bz, t).\mathfrak{M}(zBz, t)}{\mathfrak{M}(z, Bz, t).\mathfrak{M}(z, Bz, t)}\right) \end{Bmatrix}$$

$$\mathfrak{M}(z, Bz, kt) \geq \phi\{\varpi\big(\mathfrak{M}(z, Bz, t)\big), \varpi\big(\mathfrak{M}(z, Bz, t)\big), \varpi\big(\mathfrak{M}(z, Bz, t)\big), \varpi(1), \varpi(1)\}$$

$by\ \ using\ remark *$ Properties then we get

$$\mathfrak{M}(z, Bv, kt) \geq \phi\{\mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, t), \mathfrak{M}(z, Bz, t), 1,1\}$$

$$\mathfrak{M}(z, Bz, kt) \geq \mathfrak{M}(z, Bz, t)$$

By lemma 2.8, we get $Bz = z\ \ i.e\ \ Bz = z = Tz.$

Hence, $Pz = Bz = Nz = Tz = z.$

Hence $P, B, N\ and\ T\ have\ a\ Common\ Fixed\ Point.$

Uniqueness: Suppose z_1 and z_2 are two common fixed point of $P, B, N\ and\ T$ with $z_1 \neq z_2$ then from (3.3)

$$\mathfrak{M}(Pz_1, Bz_2, kt) \geq \phi \left\{ \begin{matrix} \varpi\big(\mathfrak{M}(Nz_1, Tz_2, t)\big), \varpi\big(\mathfrak{M}(Nz_1, Bz_2, t)\big), \varpi\big(\mathfrak{M}(Pz_1, Tz_2, t)\big), \\ \varpi\big(\mathfrak{M}(Nz_1, Pz_1, t)\big), \varpi\left(\dfrac{\mathfrak{M}(Pz_1, Bz_2, t). \mathfrak{M}(Nz_1, Tz_2, t)}{\mathfrak{M}(Nz_1, Bz_2, t). \mathfrak{M}(Pz_1, Tz_2, t)}\right) \end{matrix} \right\}$$

$$\mathfrak{M}(z_1, z_2, kt) \geq \phi \left\{ \begin{matrix} \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \\ \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi\left(\dfrac{\mathfrak{M}(z_1, z_2, t). \mathfrak{M}(z_1, z_2, t)}{\mathfrak{M}(z_1, z_2, t). \mathfrak{M}(z_1, z_2, t)}\right) \end{matrix} \right\}$$

$$\mathfrak{M}(z_1, z_2, kt) \geq \phi\{ \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi\big(\mathfrak{M}(z_1, z_2, t)\big), \varpi(1), \varpi(1)\}$$

$$\mathfrak{M}(z_1, z_2, kt) \geq \phi\{\mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), \mathfrak{M}(z_1, z_2, t), 1,1\}$$

By definition of Implicit function, we get, $\mathfrak{M}(z_1, z_2, kt) \geq \phi\{\mathfrak{M}(z_1, z_2, t)\}$
By lemma 2.8, we get. $z_1 = z_2$.

Conclusion:

In this paper our result has been remove continuity of mapping and containment of ranges which improve the result of K. Wadhwa and H.dubey By using new class function, which can be helpful for the researcher and accelerate sustainable development for the existence and uniqueness of common fixed point. This result can be extended in fuzzy 2-metric space, fuzzy 3-metric space and Interval Valued fuzzy metric spaces.

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