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Fixed Point Theorem in Random Fuzzy Metric Space

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Abstract
In the present paper a random fixed-point theorem is established in fuzzy metric space for rational expression. The theorem is based on the concept of Common E.A like properties and compatibility. The obtained result is generalization form of some well-known results in fuzzy metric spaces.
Keywords: Random Fixed point**,**Fuzzymetric space, Common Fixed point,Common E.A like property for random operator,weakly compatible mapping

1. Introduction

In 1965, L. A. Zedha [22] introduced a novel concept of fuzzy set, which is very useful to solve uncertainty problems. Whenever crisp set is unable to define the concept of fuzzy set used. In 1975,the concept of Fuzzy Metric Space was introduced by Kromosil and Michalek [10]. Letter on in 1994, the modified concept Continuous t-norm is introduced by Veeramani and George [6]. Many Authors and Researchers used many new concepts such as R-weakly Computing mapping, compatiblemapping, Weak Compatible mapping, CLR- Propertyetc and develop new results on Fuzzy Metric Space in different ways.The study of random fixed point is very useful in the iteration process. The literature from B.S. Choudhry [2-4], provide fruitful results in field of fixed-point theory named as random fixed point or fixed point theorems for random operator. The concept of Fuzzy-random-variable was introduced as an analogous notion to randomvariable in order to extend statistical analysis to situations when the outcomes of some random experiment are fuzzy sets. Volker [18] presented set theoretical concept of fuzzy-random-variables using the method of general topology and drawing on results from topological measure theory and the theory of analytic spaces. Some more results on this concept are given by Authors Shrivastava et.al[7]. Taking motivation from the said results the present paper deals with common random fixed-point theorems forrational expressionfor four mapping through Common E.A like property in random fuzzy metric space.Established results are generalization of well-known results in fuzzy metric spaces as Wadhwa et.al [20-21], Umashankar [17], Shrivastava,Shanghvi et.al [16] , Gupta et.al [8] ,and the researchers of [13-15, 19].

2. Preliminaries:

The basic definition of fuzzy metric spaces, t norm**,** compatible mapping, weakly compatible mapping, E.A. like property can be viewed in [5, 20-21]. The concept of random operator along with the fixed-point theory can be seen in [2-4]. Gupta, Dhagat, Shrivastava gave the modified fuzzy metric space as random fuzzy metric space. Taking the concept the modified definitions are given:

Throughout this paper, (Ω,Σ) denotes a measurable space. ξ: Ω → X is a measurable selector. X is any non-empty set. *is continuous t-norm, M is a fuzzy set in $X^2 \times [0,\infty)$.

Definition 2.1 (X, Ω, M,*) is said to be randomized fuzzy metric spaces (R.F.M.) if followings are true:

(1) $M(x(\xi), y(\xi), z(\xi)) = 0$

(2) $M(x(\xi), y(\xi), t) = 1\ for\ all\ t > 0 \quad iff\ \ x(\xi) = y(\xi)$.

(3) $M(x(\xi), y(\xi), t) = M(y(\xi), x(\xi), t)$

(4) $M(x(\xi), y(\xi), t_1) * M(y(\xi), z(\xi), t_2) \leq M(x(\xi), z(\xi), t_1 + t_2)$

(5) $M(x(\xi), y(\xi), *): [0, \infty] \to [0,1]$ is left continuous.

(6) $\lim_{t\to\infty} M(x(\xi), y(\xi), t) = 1$

$\forall x, y, z \in X\ , with\ measureble\ salector\ (\xi) and\ t_1, t_2, > 0$

Definition 2.2: Twoself-mappingP andQ on(X, Ω, M,*) are said to be compatible with measurable selector ξ if $\lim_{n\to\infty} M(PQ(\xi, x_n(\xi)), QP\ (\xi, x_n(\xi))\ , t) = 1\ for\ all\ t > 0$ whenever $\{x_n(\xi)\}$is a sequence in X such that

$\lim_{n\to\infty} P(\xi, x_n(\xi)) = \lim_{n\to\infty} Q(\ \xi, x_n(\xi)) = u(\xi)\ ,\ for\ all\ u(\xi) \in X$ with measurable selector ξ

Definition 2.3: Two self-mapping P and Q on (X, Ω, M,*) are said to be a weakly compatible with measurable selector ξif they commute at their coincidence point that is for $\forall\ u(\xi) \in X\ \ , P(\xi, u(\xi)) = Q(\xi, u(\xi)) implies\ that\ \ PQ(\xi, u(\xi)) = QP\ (\xi, u(\xi))\ for\ all\ t > 0$

Definition 2.4: Twoself-mappingP and Q on (X, Ω, M,*) are said to be Satisfy E.A property with measurable selector ξif their exist a sequence $\{x_n(\xi)\}$ in X such that

$$\lim_{n\to\infty} P(\xi, x_n(\xi)) = \lim_{n\to\infty} Q(\xi, x_n(\xi)) = u(\xi)\ ,\ for\ all\ u(\xi) \in X$$

Definition 2.5: Let P, Q,S and T be a self-mapping of (X, Ω, M,*) then (P, S) and (Q, T) said to satisfy Common E.A like property for measurable selector ξif their exist two sequence $\{x_n(\xi)\}\ and\ \{y_n(\xi)\}$ in X such that

$$\lim_{n\to\infty} P(\xi, x_n(\xi)) = \lim_{n\to\infty} S(\xi, x_n(\xi)) = \lim_{n\to\infty} Q(\ \xi, y_n(\xi)) = \lim_{n\to\infty} \mathrm{T}(\xi, y_n(\xi)) = z\,(\xi)$$

Where $z(\xi) \in S(X) \cap T(X)\ or\ z(\xi) \in P(X) \cap Q(X)$

Lemma 2.6: This lemma is motivated by [20-21]

(X, Ω, M,*) be a Random Fuzzy Metricspace then there exist$k \in (0,1)\ \ such\ that$
$M(x(\xi), y(\xi), kt) \geq M(x(\xi), y(\xi), t)\ \ then\ x(\xi) = y(\xi).$

$for\ all\ \ t > 0\ \ x, y \in X$ with measureble salector ξ

The implicit function is necessary to establish the main result.

Definition 2.7 Let ϕ be the set of all real continuous function $\phi: [0,1]^5 \to [0,1]$ non decreasing in each coordinate variable then

$$\phi(1,1,t,1,t) \geq t\ ,\quad \phi(1,t,1,t,t) \geq t\ ,\quad \phi(t,t,t,t,t) \geq t \qquad \forall t \in [0,1$$

3. Main Result

Theorem 3.1:Let $Q, U, N\ and\ J$ be a self-mapping of a Random Fuzzy Metric space (RFM) (X, Ω, M,*) satisfying the following conditions:

(i) $(Q, N) and\ (U, J)$Satisfying Common E.A like properties with random operator

(ii) $(Q, N) and\ (U, J)$are weakly compatible with random operator

(iii) For $\emptyset \in \phi$ then there exist $k \in (0,1)$ such that $\forall\, x(\xi), y(\xi), z(\xi) \in X$ with measureble selector ξ and $t > 0$

$M\big(Q(\xi, x(\xi)), U(\xi, y(\xi)), kt\big) \geq$

$$\phi\ \{M(N\,(\xi, x(\xi)), J(\xi, y(\xi), t), M\big(N(\xi, x(\xi)), U(\xi, y(\xi)), t\big), M\big(Q(\xi, x(\xi)), J(\xi, y(\xi), t\big),$$
$$\frac{M(N\,(\xi, x(\xi)), U(\xi, y(\xi)), t) + M(N(\,\xi, x(\xi), J(\xi, y(\xi)), t)}{1 + M(U(\xi, y(\xi)), J(\xi, y(\xi)), t)} * M(U(\xi, y(\xi)), J(\xi, y(\xi)), t)\ ,$$
$$\frac{M(Q(\xi, x(\xi), N(\xi, x(\xi)), t) + M(N(\xi, x(\xi)), U(\xi, y(\xi)), t)}{1 + M(Q(\xi, x(\xi)), U(\xi, y(\xi)), t)} * M(Q(\xi, x(\xi)), U(\xi, y(\xi)), t)\ \}$$

Then Q, U, N and J have common random fixed point.

Proof: Since (Q, N) and (U, J) satisfying common E.A like properties then there exist two sequences $\{x_n(\xi)\}$ and $\{y_n(\xi)\}$ in X:

$$\lim_{n\to\infty} Qx_n(\xi) = \lim_{n\to\infty} Nx_n(\xi) = \lim_{n\to\infty} Uy_n(\xi) = \lim_{n\to\infty} Jy_n(\xi) = z(\,\xi)$$

Where , $z(\xi) \in N(X) \cap J(X)$ or $z(\xi) \in P(X) \cap U(X)$

Let $z(\xi) \in N(X) \cap J(X)$ and $\lim_{n\to\infty} Qx_n(\xi) = z(\xi) \in N(X)$ then $z(\xi) = Nu(\xi)$

To Prove, $Qu(\xi) = Nu(\xi)$

Put $x(\xi) = u(\xi)$ and $y(\xi) = y_n\,(\xi)$ in(iii)

$M(Q(\xi, u(\xi)), U(\xi, y_n(\xi)), kt)$

$$\geq \phi\{M(N(\xi, u(\xi)), J(\xi, y_n(\xi)), t), M(N(\xi, u(\xi)), U(\xi, y_n(\xi)), t), M(Q(\xi), u(\xi), J(\xi, y_n(\xi)), t),$$
$$\frac{M(N(\xi, u(\xi)), U(\xi, y_n(\xi)), t) + M(N(\xi, u(\xi)), J(\xi, y_n(\xi)), t)}{1 + M\big(U\big(\xi, y_n(\xi), J(\xi, y_n(\xi))\big)\big), t\big)} * M(U(\xi, y_n(\xi)), J(\xi, y_n(\xi), t)\ ,$$
$$\frac{M(Q(\xi, u(\xi)), N(\xi, u(\xi)), t) + M(N(\xi, u(\xi), U(\xi, y_n(\xi)), t)}{1 + M(Pu, Uy_n, t)}$$
$$* M(Q(\xi, u(\xi)), U(\xi, y_n(\xi)), t)\ \}$$

$M(Q(\xi, u(\xi)), z(\xi), kt)$

$$\geq \phi\{M(z(\xi), z(\xi), t), M(z(\xi), z(\xi), t), M(Q(\xi, u(\xi)), z(\xi), t),$$
$$\frac{M(z(\xi), z(\xi), t) + M(z(\xi), z(\xi), t)}{1 + M(z(\xi), z(\xi), t)} * M(\,z(\xi), z(\xi), t)\ ,$$
$$\frac{M(Q(\xi, u(\xi), z(\xi), t) + M(z(\xi), z(\xi), t)}{1 + M(Q(\xi, u(\xi), z(\xi), t)} * M(Q(\xi, u(\xi), z(\xi)), t)\ \}$$

$$M(Q(\xi, u(\xi), z(\xi)), kt) \geq \phi\{\,1\,, 1\,, M(Q(\xi, u(\xi), z(\xi), t), 1, M(Q(\xi, u(\xi), z(\xi)), t)\ \}$$

$M(Q(\xi, u(\xi), z(\xi), kt) \geq\ M(Q(\xi, u(\xi), z(\xi)), , t)$

$\Rightarrow Q\big(\xi, u(\xi)\big) = z(\xi)$

$\Rightarrow Q\big(\xi, u(\xi)\big) = z(\xi)\ = N(Q(\xi, u(\xi)) = z(\xi)$

Since (Q, N) *is* random weakly compatible then

$$Q\big(\xi, z(\xi)\big) = QN(\xi, u(\xi)) = NQ(\xi, u(\xi)) = N\big(\xi, z(\xi)\big)$$

Again, $\lim_{n\to\infty} U\big(\xi, y_n(\xi)\big) = z(\xi) \in J(X)$ then $z(\xi) = J(\xi, v(\xi))$

Now toProve, $J(\xi, v(\xi)) = U(\xi, v(\xi))$

Now from (iii)

$M\big(Q\big(\xi, x_n(\xi)\big), U\big(\xi, v(\xi)\big), kt\big) \geq$

$$\phi\{M(N(\,\xi, x_n(\xi), J(\xi, v(\xi)), t), M(N(\xi, x_n(\xi)), U(\xi(, v(\xi)), t), M(Q(\xi, x_n(\xi)), J(\xi, v(\xi)), t),$$

$$\frac{M(N(\xi,x_n(\xi)),U(\xi,v\,(\xi)),t)+M(N(\xi,x_n(\xi)),J(\xi,v(\xi)),t)}{1+M(U(\xi,v(\xi)),J(\xi,v(\xi),t)} * M(U(\xi,v(\xi),J(\xi,v(\xi),t)\,,$$
$$\frac{M(Q(\xi,x_n(\xi)),N(\xi,x_n(\xi)),t)+M(N(\,\xi,x_n(\xi)),U(\xi,v(\xi)),t)}{1+M(Px_nx,Uv,t)}$$
$$* M(Q(\xi,x_n(\xi)),U(\xi,v(\xi)),t)\,\}$$
$$M(z(\xi),U(\xi,v(\xi)),kt)$$
$$\geq \phi\{M(z(\xi),z(\xi),t),M(z(\xi),U(\xi,v(\xi),t),M(z(\xi),z(\xi),t),$$
$$\frac{M(z(\xi),U(\xi,v(\xi)),t)+M(z(\xi),z(\xi),t)}{1+M(U(\xi,v(\xi),z(\xi),t)} * M(U(\xi,v(\xi),z(\xi)),t)\,,$$
$$\frac{M(z(\xi),z(\xi),t)+M(z(\xi),U(\xi,v(\xi)),t)}{1+M(z,Uv,t)} * M(z,U(\xi,v(\xi)),t)\,\}$$
$$M(z(\xi),U(\xi,v(\xi)),kt)$$
$$\geq \phi\{1,M(z(\xi),U(\xi,v(\xi)),t),1,M(U(\xi,v(\xi),z(\xi)),t)\,,M(z(\xi),U(\xi,v(\xi),t)\,\}$$
$$M(z(\xi),U(\xi,v(\xi)),kt)\geq M(z(\xi),U(\xi,v(\xi)),t)$$
$$\Rightarrow z(\xi)=U(\xi,v(\xi))\quad\Rightarrow U(\xi,v(\xi))=z(\xi)=J(\xi,v(\xi))$$

Since (U,J) is random weakly compatible then

$$U\big(\xi,z(\xi)\big)=UJ\big(\xi,v(\xi)\big)=JU(\xi,v(V))=J(\xi,z(\xi))$$

Now to Prove $Q(\xi,z(\xi))=z(\xi)$

From (iii), $M\big(Q\big(\xi,z(\xi)\big),U\big(\xi,y_n(\xi)\big),kt\big)\geq$

$$\phi\{M(N(\xi,z(\xi)),J(\xi,y_n(\xi)),t),M(N(\,\xi,z(\xi)),U(\xi,y_n(\xi)),t),M(Q(\xi,z(\xi)),J(\xi,y_n(\xi)),t),$$
$$\frac{M(N(\xi,z(\xi)),U(\xi,y_n(\xi)),t)+M(N(\xi,z(\xi)),J(\xi,y_n(\xi)),t)}{1+M(Uy_n,Iy_n,t)}$$
$$* M(U(\xi,y_n(\xi)),J(\xi,y_n(\xi)),t)\,,$$
$$\frac{M(Q(\xi,z(\xi)),N(\,\xi,z(\xi)),t)+M(N(\xi,z(\xi)),U(\xi,y_n(\xi)),t)}{1+M(Pz,Uy_n,t)}$$
$$* M(Q(\xi,z(\xi)),U(\xi,y_n(\xi)),t)\,\}$$
$$M(Q(\xi,z(\xi),z(\xi),kt))$$
$$\geq \phi\{M(Q(\xi,z(\xi),z(\xi),t),M(Q(\xi,z(\xi),z(\xi),t),M(Q(z(\xi),z(\xi))),t),$$

$$\frac{M(Q(\xi,z(\xi),z(\xi)),t)+M(Q(\xi,z(\xi),z(\xi),z(\xi)),t)}{1+M(z(\xi),z(\xi),t)} * M(z(\xi),z(\xi),t)\,,$$

$$\frac{M(Q(\xi,z(\xi),z(\xi)),t)+M(Q(\xi,z(\xi),z(\xi)),t)}{1+M(Q(\xi,z(\xi),z(\xi)),t)} * M(Q(\xi,z(\xi),z(\xi)),t)\}$$
$$M\big(Q\big(\xi,z(\xi),z(\xi)\big),t\big)$$
$$\geq \phi\{\;M\big(Q\big(\xi,z(\xi),z(\xi)\big),t\big),M\big(Q\big(\xi,z(\xi),z(\xi)\big),t\big),M\big(Q\big(\xi,z(\xi),z(\xi)\big),t\big),$$
$$M(Q(\xi,z(\xi),z(\xi)),t),MM(Q(\xi,z(\xi),z(\xi)),t)\}$$

$$M(Q(\xi,z(\xi),z(\xi)),t)\geq\; M(Q(\xi,z(\xi),z(\xi)),t)$$
$$\Rightarrow\quad M(Q(\xi,z(\xi),t)=z\,(\,\xi)$$
$$\Rightarrow.\,M(Q(\xi,z(\xi),t)=z\,(\,\xi)\;=N(\,\xi,\;z(\,\xi))\_$$

Now to Prove $U(\xi,z(\xi))=z(\xi)$

Again from (iii)

$$M\big(Q\big(\xi,x_n(\xi)\big),U\big(\xi,z(\xi)\big),kt\big)$$

$$\geq \phi\{M(N(\xi,x_n(\xi)),J(\xi,z(\xi)),t),M(N(\xi,x_n(\xi)),U(\xi,z(\xi)),t),M(Q(\xi,x_n(\xi)),J(\xi,z(\xi)),t),$$

$$\frac{M(N(\xi,x_n(\xi)),U(\xi,z(\xi)),t)+M(N(\xi,x_n(\xi)),J(\xi,z(\xi)),t)}{1+M(U(\xi,v(\xi)),J(\xi,v(\xi)),t)} * M(U(\xi,v(\xi)),J(\xi,v(\xi)),t),$$

$$\frac{M(Q(\xi,x_n(\xi)),N(\xi,x_n(\xi)),t)+M(N(\xi,x_n(\xi)),U(\xi,z(\xi)),t)}{1+M(Q(\xi,x_n(\xi)),U(\xi,z(\xi)),t)} * M(Q(\xi,x_n(\xi)),U(\xi,z(\xi)),t)\}$$

$$M(z(\xi),U(\xi,z(\xi)),kt)$$
$$\geq \phi\{M(z(\xi),U(\xi,z(\xi)),t),M(z(\xi),U(\xi,z(\xi)),t),M(z(\xi),U(\xi,z(\xi)),t),$$
$$\frac{M(z(\xi),U(\xi,z(\xi)),t)+M(z(\xi),U(\xi,z(\xi)),t)}{1+M(U(\xi,v(\xi)),U(\xi,z(\xi)),t)} * M(U(\xi,v(\xi)),U(\xi,z(\xi)),t),$$
$$\frac{M(z(\xi),z(\xi),t)+M(z(\xi),U(\xi,z(\xi)),t)}{1+M(z(\xi),U(\xi,z(\xi)),t)} * M(z(\xi),U(\xi,z(\xi)),t)\}$$
$$M\big(z(\xi),U\big(\xi,v(\xi)\big),kt\big) \geq \phi\{M\big(z(\xi),U\big(\xi,z(\xi)\big),t\big),M\big(z(\xi),U\big(\xi,z(\xi)\big),t\big),$$

$$M(z(\xi),U(\xi,z(\xi)),t),M(z(\xi),U(\xi,z(\xi)),t),M(z(\xi),U(\xi,z(\xi)),t)\}$$

$$M(z(\xi),U(\xi,z(\xi)),kt) \geq M(z(\xi),U(\xi,z(\xi)),t)$$

⇒we get $U\big(\xi,z(\xi)\big)=z(\xi)$

$$\Rightarrow U(\xi,z(\xi))=z(\xi)=J(\xi,z(\xi))$$

Hence, $Q(\xi,z(\xi))= \quad U(\xi,z(\xi))=N(\xi,z(\xi))= J(\xi,z(\xi))=z(\xi)$

HenceQ,U,N *and* J *have a Common random Fixed Point.*

Uniqueness: Suppose z and s are two common random fixed point ofQ,U,N *and* J with $z \neq$ s then from (3.1)

$$M(Q(\xi,z(\xi),U(\xi,s(\xi)),kt) \geq \phi\{M\big(N\big(\xi,z(\xi)\big),J\big(\xi,s(\xi)\big),t\big),$$
$$M\big(N\big(\xi,s(\xi)\big),U(\xi,s(\xi),t\big),M(Q(\xi,z(\xi)),J(\xi,s(\xi)),t),$$
$$\frac{M(N(\xi,z(\xi)),U(\xi,s(\xi)),t)+M(N(\xi,z(\xi)),J(\xi,s(\xi)),t)}{1+M\big(U\big(\xi,s(\xi)\big),J\big(\xi,s(\xi)\big),t\big)} * M\big(U\big(\xi,s(\xi)\big),J\big(\xi,s(\xi)\big),t\big),$$
$$\frac{M\big(Q\big(\xi,z(\xi)\big),N(\xi,s(\xi)),t\big)+M\big(N\big(\xi,z(\xi)\big),U(\xi,s(\xi)),t\big)}{1+M\big(Q\big(\xi,z(\xi)\big),U(\xi,s(\xi)),t\big)} * M\big(Q\big(\xi,z(\xi)\big),U(\xi,s(\xi)),t\big)\}$$
$$M(z(\xi),s(\xi),kt) \geq \phi\{M(z(\xi),s(\xi),t),M(z(\xi),s(\xi),t),M(z(\xi),s(\xi),t),$$

$$\frac{M(z(\xi),s(\xi),t)+M(z(\xi),s(\xi),t)}{1+M(z(\xi),s(\xi),t)} * M(s(\xi),s(\xi),t),$$
$$\frac{M(z(\xi),z(\xi),t)+M(s(\xi),s(\xi),t)}{1+M(z(\xi),s(\xi),t)} * M(z(\xi),s(\xi),t)\}$$
$$M(z(\xi),s(\xi),kt) \geq \phi\{M(z(\xi),s(\xi),t),M(z(\xi),s(\xi),kt),M(z(\xi),s(\xi),t)$$
$$M(z(\xi),s(\xi),t),M(z(\xi),s(\xi),kt)\}$$
$$M(z(\xi),s(\xi),kt) \geq \phi\{M(z(\xi),s(\xi),t)\}$$

$\Rightarrow z(\xi) = s(\xi)$. Hence Random fixed point is unique.

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