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Exploring Diverse Dimensions of Retrial Queueing in Computer Networks: A Comprehensive Review

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ABSTRACT:

This paper delves into the intricacies of retrial queuing within computer networks, synthesizing findings from various research papers. The exploration encompasses 45 distinct studies, each contributing unique insights to the understanding of retrial queuing dynamics. The reviewed papers span a range of topics, such as service time distribution, equilibrium pricing strategies, mathematical analysis of queues with phase service, breakdowns and repairs, strategic joining and pricing policies, priority, collisions, feedback customers, non-Markovian retrial systems, and adaptive permanent pooling. The insights derived from these studies contribute to the ongoing discourse on optimising the performance and efficiency of computer network systems.

Key Words: Bulk Arrival, Batch Service, Computer Networks, Matrix Analytic Method, Retrial Queue, Single Service, Supplementary Variable Technique.

1. Introduction

The word ‘queue’ derives from the Latin word cauda, which means tail, queueing theory is a branch of applied probability. Queueing Theory is a sophisticated mathematical modeling technique for estimating wait times. It dates back to the early 1900s, when studies were conducted to determine the most cost-effective way to build a telephone exchange's capacity. A. K. Erlang, a Danish engineer, noticed that the telephone exchange's demand is characterised by random phone calls, and the service is described by the random duration of each conversation. Retrial queue is a model where customers or jobs arrive and upon finding the server busy, they wait for some time before trying again. Waiting for a service is a typical occurrence in real life for example, in restaurants, telecommunication networks, banks etc. The natural unpredictability in the timing of demands and the length of time required to provide service causes mismatches and balances, thereby causing service delays. Because of the variability and interaction between the arrival and service processes, the dynamics of service systems are frequently more complicated, making congestion prediction challenging. As a result, queuing theory models must be used to predict the capacity required to attain certain levels of performance and higher service standards. Customers and servers are the two major components of a queueing system. The former is looking for a service that the server can deliver right away or later depending on the type of service and the quantity of consumers. A queue is formed when a client has to wait in line. Because clients arrive at random times and the system is unpredictable, the delays they experience are very variable and depend on the number of servers available and how quickly they can function. A queueing model can be used to convert arrival patterns and processing durations into significant system performance indicators like average client wait times. The paper is classified into the following sections: Performance Analysis and Optimization, Pricing and Monopoly Strategies, Reliability and Fault Tolerance, Queueing System Models, Call Centre and Network Applications, Priority and Feedback Mechanisms, Miscellaneous Topics, Conclusion and References

The following picture depicts the structure of a basic retrial queueing system and it explains the concept of retrials.

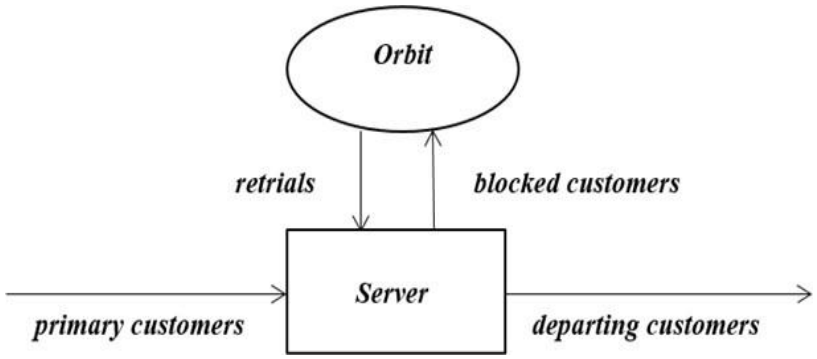


Figure 1: A Basic Flowchart of a Retrial Queue Model

Performance Analysis and Optimization

- **Performance Analysis and Optimization of a Retrial Queue with Working Vacations and Starting Failures**

Yang et al. investigated an M/M/1 retrial queue with working vacations, with the server subject to starting failures and the model follows a quasi-birth-and-death process. The arrivals follow

a Poisson process. It derived the stability condition and computes the stationary probability distribution of the orbit size using the matrix-geometric method. A cost model was devised to identify optimal service rates during normal and vacation periods using the canonical particle swarm optimization algorithm. Numerical results showcase how system parameters affect performance measures and optimal service rates. Potential applications of this model are in the healthcare sectors.

- **Tail Asymptotics of the Queue Size Distribution in the M/M/m Retrial Queue**

In an M/M/m retrial queue, the tail asymptotics for the joint distribution of queue size and the number of busy servers in the steady state were investigated. The customers were observed to arrive in a Poisson process. The stationary queue size distribution, with a fixed number of busy servers, was asymptotically represented by a geometric function multiplied by a power function. The decay rate of the geometric function remained constant and was determined by the offered load, irrespective of the number of busy servers. However, the exponent of the power function varies depending on the number of busy servers. The tail asymptotics of the queue size distribution was found, which stands as the main contribution of this model. A matrix differential equation for the vector probability-generating function of the stationary distribution of the censored Markov process was derived and numerical examples were provided to illustrate the result.

- **Retrial Queueing Models: A Survey on Theory and Applications**

Phung-Duc surveyed the theory and applications of retrial queues in various systems, including call centres, cellular networks, and local area networks. It covered theoretical research such as exact solutions, stability, and multidimensional models, as well as real-world applications. A survey of theoretical results for retrial queueing models, including those from real applications were provided. The objective was to serve as a basic reference for researchers entering the field.

- **Simulation of Finite-Source Retrial Queueing Systems with Collisions and Non-reliable Server**

Tóth et al. investigated a finite-source retrial queuing system with customer collisions and an unreliable server, employing a simulation program. The model assumed independent and generally distributed random variables. Notably, the study introduced sensitivity analysis using various distributions to evaluate key performance measures, showcasing the impact of different scenarios on customer numbers, response time, service duration, and orbit sojourn time. Assumptions include random breakdowns and repairs of the server based on its status (idle or busy). The possible applications of this retrial model are in quasi-random input models for cellular and local-area networks. It emphasized the practical significance of considering unreliable servers' impact on system reliability, aligning with recent literature on finite-source retrial queues. Using SimPack, a C/C++ simulation tool, the introduced finite-source retrial queue, adding a valuable dimension with sensitivity analysis was modelled. The outcomes indicated a normally distributed steady-state customer distribution in the service facility across all scenarios. The practical importance of the discussed model is used for understanding complex systems susceptible to breakdowns and collisions.

- **Optimization of the (MAP1, MAP2)/ (PH1, PH2)/*N* Retrial Queue Model of Wireless Cellular Networks with Channel Allocation**

Zhouo et al presented a queueing model with infinite waiting positions and N channels to assess wireless cellular networks that use channel allocation. The model accounted for different rates of Markov Arrival Process (MAP) arrivals for new calls and handoff calls. The behaviour of

the system was described by a quasi-birth-and-death process. Additionally, the channel holding time distributions are represented by distinct Phase-type (PH) distributions. By analysing the Markov chain underlying the queueing system and obtaining its ergodicity condition, the study derives key performance measures, including the blocking probability of new calls and the dropping probability of handoff calls, the idle probability of the target cell, and the mean queue lengths of two types of calls using matrix analytic techniques. Numerical results were presented to demonstrate how system parameters affect performance measures.

- **A Batch Arrival Retrial Queueing System with Two Phases of Service and Service Interruption**

Choudhury dealt with the steady-state behaviour of second phase of optional service and service interruption where breakdowns occur randomly at any instant while the server is serving the customers. Further, the concept of delay time was also introduced in the model. This model generalized both the classical M/G/1retrial queue with service interruption as well as the queueing with second optional service and service interruption. An extensive analysis of this model was carried out

- **Steady State Analysis of an *M*/*G*/1 Queue with Linear Retrial Policy and Two Phase Service under Bernoulli Vacation Schedule**

Choudhury considered a single server queueing system with two phases of heterogeneous service and Bernoulli vacation schedule operating under the linear retrial policy. This model extends both the classical M/G/1 retrial queue with linear retrial policy as well as the M/G/1 queue with two phases of service and Bernoulli vacation model. The rationale behind analyzing such a model is to understand its structure, which plays a role in many computer and communication networks and to delve deeper into the behaviour of the stationary system. The study involved recursive computation of limiting probabilities and the methodology was based on a Markov regenerative process.

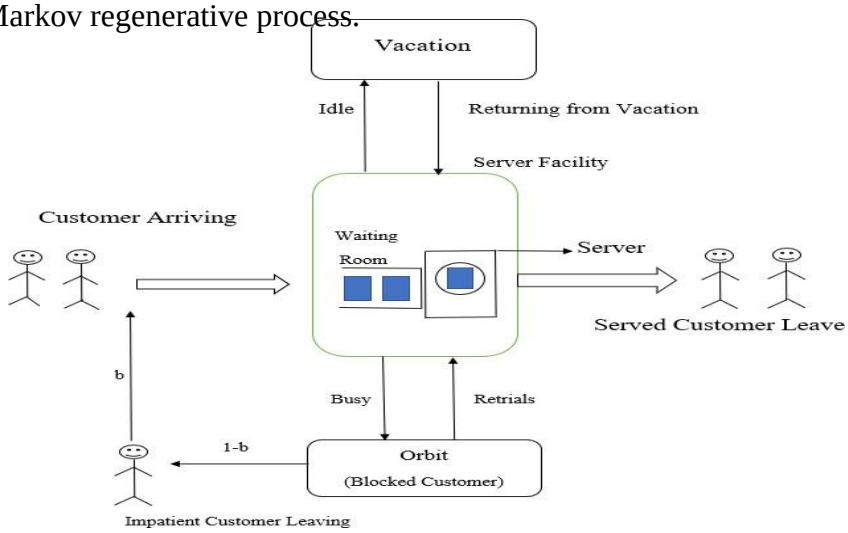


Figure 2: A Single Server Retrial Queue with Bernoulli Vacation

Pricing and Monopoly Strategies

- **Optimal Pricing Analysis of Computer Networks Based on a Queueing System Using Retrial Mechanism**

Zhang investigated a single server retrial queue with Poisson arrivals, exponential service times and unreliable server. It explored two ways of determining total retrial rate: one, based on the number of customers in the orbit and the other, known as constant retrial policy based on automatic mechanisms. The server charges a price for joining, affecting service demand via a

decreasing function. System characteristics and optimal pricing for profit maximization problems were studied using multiple demand models and illustrated using numerical examples. Service interruptions due to server breakdowns were considered. A comprehensive analysis compared to related works, focusing on steady-state status and general cases was provided.

- **Monopoly Pricing in a Retrial Queue with Delayed Vacations for Local Area Network Applications**

Wang et al. studied an M/M/1 retrial queue for LAN applications in which the server took delayed vacations after completing a service. An announced price was imposed on the customers who either decided to balk or enter the system based on the expected payoff and information available about the server's status. Furthermore, Wang et al. analysed how the social planner determined the price to charge each joining customer, where his main goal was to maximise the overall welfare of the server as well as the customers. The aim was to address two key questions, if the server aims to maximize revenue, what price should it set? And for the social planner, whose goal is to maximize the welfare of all customers and servers, what price should be chosen for net social benefit? The three main contributions were, one, it investigated a Stackelberg competition between the service provider and customers. Second, it discussed the optimal pricing strategy that would maximise the total welfare of the system with delayed vacation. Third, it contributed to the game-theoretic analysis of a particular queueing system with LAN applications.

- **Equilibrium Pricing Strategies in Retrial Queueing Systems with Complementary Services**

Zhang et al. presented a novel perspective on retrial queueing systems by exploring an unobservable model with complementary services based on real life instances like cloud computing and cognitive radio systems. The model involved two servers, one offering instantaneous service and the other providing delayed service within an M/M/1 retrial queue. The study derived optimal pricing strategies in 3 pricing schemes - fixed charge, time based and mixed - and analyzes which scheme is more profitable for the servers and social planners. It employed a Stackelberg game approach to derive optimal customer arrival rates and equilibrium prices set by servers. A notable gap in the study of decentralized customer decision-making within game-theoretic analyses of retrial queues was addressed. The unique insights into customer-service provider dynamics and equilibrium solutions enhance the understanding of retrial queueing systems.

Reliability and Fault Tolerance

- **A Retrial Queue for Modeling Fault-Tolerant Systems with Checkpointing and Rollback Recovery**

Dimitriou analysed a retrial queue that is used to model fault-tolerant systems with checkpointing and rollback recovery. It was assumed that service time of each job is broken into N heterogeneous modules and at the end of each of these modules, a checkpoint was established. The technique of checkpointing and rollback recovery consists of periodically saving the state at which the system is in on a secure device, so that upon system failure, recovery can be easier. Mean value analysis was applied to obtain performance metrics that are useful and the model used proved that stochastic decomposition property was satisfied. The aim was to provide an elucidation into the analysis of single server queues with checkpoint and rollback recovery policy in the case of failures. Applications of the model introduced can be

found in the power-saving capability of mobile devices and in long-running software applications.

- **Transient Analysis and Reliability Modeling of Fault-Tolerant System Operating Under Admission Control Policy with Double Retrial Features and Working Vacation**

Kumar et al. introduced a novel double retrial orbit queueing model for Fault-Tolerant Machining Systems (FTMS). Emphasising reliability and efficiency, it employed admission control via a threshold policy, working vacation, and redundancy. The study evaluates transient reliability, queueing indices, and performs sensitivity analysis on key parameters. Noteworthy applications include diverse domains where FTMS plays a crucial role. Retrial orbital queueing systems and optimal arrival control were introduced, effectively managing repair job overload. The threshold-based admission control proved to be beneficial for industrial decision-makers. The contribution spans both Markovian and non-Markovian models, addressing the practicality of retrial orbits for FTMS in finite populations. The inclusion of transient analysis added a time-critical perspective.

- **Homogeneous Finite-Source Retrial Queues with Server Subject to Breakdowns and Repairs**

Almási et al. discussed a complex model of a single server retrial queue with a finite number of homogeneous sources of calls and a single non reliable server. The server randomly breaks down depending on whether it is busy or idle, which affects the system's operations. The model construction assumes all random variables are exponentially distributed and independent. The study used the MOSEL tool to solve the problem and derived performance and reliability measures. The impact of the unreliability of the server on the mean response times of the calls was numerically calculated and graphed.

- **Reliability-Based Measure of a Retrial Machine Repair Problem with Working Breakdowns under the F-Policy**

Yen et al. delved into the reliability and sensitivity analysis of a retrial machine repair problem, specifically focusing on the F-policy, which regulates arrivals to a queue with a startup time for failed machines. A single-server queue with two priority classes was examined, where low-priority units make repeated attempts after being blocked. Server breakdowns only occur when failed machines are present, with failure and repair times modeled exponentially. Utilizing Laplace transform technique and matrix analytic method, the study analyzes system reliability and Mean Time to System Failure (MTTF). Through extensive numerical experimentation, impact of various parameter values on performance measures was explored. Using mathematical models the distribution observed from outside the system, the steady-state arrival unit distribution, and the influence of parameters on performance measures were investigated, highlighting its uniqueness in accommodating finite-source inputs and repeated attempts. From the results obtained, it can be observed that the effect of mean startup rate is ignorable and on increasing the probability of prohibit success; mean service rate enhanced the system reliability.

Queuing System Models

- **Retrial Multi-Server Queuing System with PHF Service Time Distribution as a Model of a Channel with Unreliable Transmission of Information**

Dudin et al. explored a retrial multi-server queuing system with an unreliable information transmission model. A Markovian arrival process, non-persistent customers, and server unreliability was incorporated. The total occupation time of a server follows a Phase-type with

Failures (PHF) distribution. The study uses a multi-dimensional level-dependent Markov chain to model system dynamics and introduces an efficient iterative algorithm to address computational challenges in finding the stationary distribution. Assumptions include Markovian arrival, PHF service time, and possible failures during service in a multi-server system. Dudina et al. contribute to retrial queue theory by considering more realistic scenarios, such as failures during service, and proposes a flexible PHF distribution. The ergodicity conditions was established, along with an introduction of a novel algorithm for computing the stationary distribution, offering valuable insights for real-world system dynamics and advancing queuing theory research.

- **Retrial Queues For Performance Modelling and Evaluation of Heterogeneous Networks**

Roszik et al. investigated a single retrial queue with heterogeneous sources of calls. The main goal was to give the steady-state performance measures of the system and to show how different parameters affected them. A tool named MOSEL (Modeling, Specification and Evaluation Language) developed at the University of Erlangen by Begain, Bolch and Herold (2001) was used. The steady-state performance metrics were graphically represented using IGL (Intermediate Graphical Language). This model can be considered as an extension of investigations for finite-source heterogeneous queueing systems. This model can be considered as a basis for performance modelling of bigger heterogeneous networks.

- **Stochastic Analysis of a Single Server Unreliable Queue with Balking and General Retrial Time**

An M/G/1 queue with general retrial times allowing balking and a server prone to breakdowns and repairs was considered. The methodology used is based on the general theory of stochastic orders. The insensitive limits for the stationary distribution of the embedded Markov chain in the considered system were found. A stochastic comparison analysis of the M/G/1 queue with server breakdowns and general as well as non-exponential service retrial time distributions was carried out by taking into account balking and defaulting behaviour of the customer was introduced. Retrial queues with general service times and non-exponential retrial time distributions have not been explored more because of the complexity of their results. For this reason, focus was given to the stochastic comparison method based on the theory of stochastic orders. In this technique, a complex system was bound to a new system which is simpler to solve. The goal was to provide qualitative estimates for performance metrics based on this simplified model. The model presented has applications in cognitive radio networks, manufacturing systems, etc.

- **Analysis of M/M/c Retrial Queue with Thresholds, PH Distribution of Retrial Times and Unreliable Servers**

Chakravarthy et al. dealt with a retrial queue with PH distribution of retrial times and a threshold-type policy where each server is prone to breakdowns and repairs. The matrix-analytic method was applied to carry out the steady-state analysis. A multi-server queue was analysed using the introduced threshold-based retrial times for customers in the orbit. With an objective to include PH-retrial times while being careful not to increase the complexity of the model, Chakravarthy(2020) proposed a new approach that involved threshold based PH-retrial times. This enabled the authors to study the model without worrying about exploding the state space. The queueing model described generalised threshold-type retrial queuing model discussed by author on unreliable servers. Upon server failure, the interrupted customer is allowed to handover his service to another server to complete its service.

- **A Retrial Queueing Model with Thresholds and Phase Type Retrial Times**

A retrial queueing model with threshold policy for orbiting customers in non-exponential retrial times was studied. The model was analysed using matrix-analytic methods and it was compared with traditional retrial queueing models through illustrative examples. The motivation for author was the need for including non-exponential retrial times while avoiding increasing the complexity of the retrial queuing model. The advantage of using threshold-type retrial times is that it can be used as further approximation to retrial queues with phase type retrial times. This model can be extended to include versatile arrival processes as well as robust services.

Call Center and Network Applications

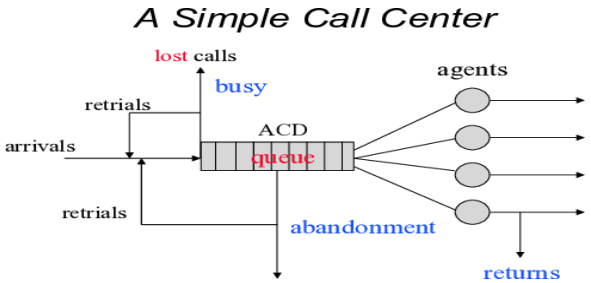


Figure 3: A Schematic Diagram Depicting a Call Centre Service

- **Strategic Joining and Pricing Policies in a Retrial Queue With Orbital Search and Its Application to Call Centers**

Gao et al. explored an M/M/1 retrial queue with orbital search, focusing on scenarios found in call centres. Using a blend of queueing theory and game theory, the study unveiled Nash equilibrium mixed joining strategies for individual customers. It extended the analysis to optimal joining probabilities and admission pricing, maximising administrator revenue and social profit. The methodology integrated queueing theory and game theory, with a groundbreaking consideration of server-orbit interaction. The main contribution is by introducing the server-orbit interaction schedule, reducing customer orbit times and enhancing system efficiency. Noteworthy was the thorough investigation of optimal joining strategies for individuals, administrators, and social planners. The derivation of Nash equilibrium probabilities and optimal pricing problems enriches the economic analysis of retrial queueing systems. The numerical examples offer practical insights, illustrating parameter impact on individual maximisation, administrator's maximisation, and social optimization.

- **Solution for a Retrial Queueing Problem in Cellular Networks with the Fractional Guard Channel Policy**

Author explored the intricacies of wireless cellular networks, introducing a retrial queueing problem aligned with the Fractional Guard Channel (FGC) admission control policy. A notable contribution was the investigation of retrial phenomena in FGC, an aspect previously overlooked in the literature. The author aimed to develop a comprehensive retrial queueing model, evaluating vital performance metrics affecting subscriber experience. In particular, Van do introduced an innovative algorithm for efficiently solving the retrial queue problem within the FGC policy framework. The significance of managing handover calls in wireless networks, with FGC policy as a key driver was emphasised. The proposed algorithm's efficacy was validated through a thorough comparison of analytical and simulation results, affirming its accuracy and rapid evaluation capabilities. Now, implications for optimising call admission policies in cellular networks, particularly in managing handover calls to enhance overall service quality have arised.

- **Analysis of a Semi-Open Queuing Network with a State Dependent Marked Markovian Arrival Process, Customers Retrials and Impatience**

A queueing network with single-server nodes and heterogeneous customers was considered. The service time of the nodes followed exponential distribution. The network dynamics were represented by a multidimensional Markov Chain with infinite states, varying behaviour and a unique infinitesimal generator structure. An explicit form for the generator was derived along with an efficient algorithm to compute the chain's stationary distribution. Expressions to calculate the network's performance metrics were provided. A few salient features include, more complex customers arrival process, it accounted for impatience of customers during their stay in the orbit and so on. In the model considered, the results of previous literature are broadened to the networks with state dependent arrival processes, account of impatience and non-persistence of customers in the orbit.

Priority and Feedback Mechanisms

- **A Study on M/G/1 Feedback Retrial Queue with Subject to Server Breakdown and Repair under Multiple Working Vacation Policy**

Rajadurai et al. explores a novel retrial queueing system, incorporating balking, feedback, and multiple working vacations, with a focus on real-world applications like network and mail services. The model assumes general retrial and service times, providing a nuanced understanding of system dynamics. Utilising the supplementary variable technique, the steady-state probability generating function for system size was derived. The expressions for average queue length of both the system as well as the orbit were obtained. Numerical examples validate analytical results, showcasing the model's versatility and practical applicability. Positioned as a generalised model, it offered insights for system managers and extended the applicability of existing queueing models.

- **On Multiserver Feedback Retrial Queue with Finite Buffer**

Kumar et al. discussed a queueing system with multiple servers, customer feedback, finite waiting positions and constant retrial rate. The system was studied as a Quasi-birth-and-death process and the necessary and sufficient condition for system stability were analysed. Important system performance measures are obtained using a matrix geometric method. The impact of various parameters on system performance measures and the algorithmic development of the full busy period were also discussed. Some applications of the system considered include, data transfer and telecommunication networks, and involve customers who leave and then return to initiate a demand. Additionally, the algorithmic development of the busy period of the system was presented.

- **Single Server Retrial Queues with Priority Calls**

Choi et al. presented a survey of retrial queues featuring two types of calls and introduced novel findings for various models. An $M_1,M_2/G/1$ retrial queue and its variants with varied service times, geometric loss, feedback, and many more, was studied. The model was classified into two categories based on the number of different calls, as single type of calls and several type of calls. Examples were provided for the two different types of models.

- **M/M/1 Retrial Queue with Collisions and Transmission Errors**

Lakaour et al. delved into the analysis of retrial queueing systems with a focus on collisions and transmission errors, particularly relevant to communication protocols like CSMA/CD and the DCF mode of IEEE 802.11. The study introduced a retrial policy with varying probabilities for a primary customer to access the server under different conditions, aiming to provide a

more comprehensive understanding of scenarios involving simultaneous collisions and transmission errors. Assumptions include a retrial queue with a classical retrial rate, collisions impacting both primary and in-service customers, and transmission errors affecting service completion. The proposed model, inspired by the IEEE 802.11 DCF mode, is suited for situations where stations cannot detect collisions or transmission errors while transmitting. The methodology involves presenting a mathematical model, establishing steady-state probabilities, and determining system stability. Performance metrics and theoretical results with numerical examples were presented to enhance understanding. The unique features, such as varying probabilities for joining the orbit based on server status, and the restriction of collisions to specific scenarios, add depth to the analysis.

- **Mathematical Analysis of Queue with Phase Service: an Overview**

Sharma adeptly distilled the extensive realm of phase service queueing models. These types of models serve as a crucial guide for congestion-related challenges, spotlighting the models' practical applications across diverse industries. From manufacturing systems to VoIP protocols, it showcases their relevance and versatility. The strength of the work lies in its application-oriented approach, emphasising the models' alignment with practical situations. Methodologies are demystified, making it accessible for scholars and practitioners. Performance measures and analyses enrich the content, providing valuable insights for evaluating production systems and communication networks. The objective is to provide information to industry experts and managers who want to use queueing theory to model congestion problems.

- **Analysis of a Retrial Queue with Two-Type Breakdowns and Delayed Repairs**

Gao et al. examined an M/G/1 retrial queue integrating both passive and active breakdowns, with a distinctive emphasis on delayed repairs for passive breakdowns—a novel contribution to the field. The model holds relevance in packet-switching networks, emphasising practical applications. Employing embedded Markov chain techniques and probability-generating functions, necessary and sufficient conditions for system stability was established. Through supplementary variable methods, the steady-state joint distribution of the orbit size and the server's state was derived. This lead to the computation of critical performance measures, offering insights into system behaviour. Author delved into reliability aspects, presenting indices such as steady-state availability, failure frequency, and mean time to the first failure of the server. These metrics contribute to the broader understanding of the system's robustness. Recognizing the importance of customer service quality, expressions for the Laplace transform of the sojourn time of an arbitrary customer were provided. The validation of Little's law in this context enhances the practical implications of the findings. Numerical examples offer a practical demonstration of the system's behaviour under various parameters.

Retrial Queue with Priority, Collisions and Feedback Customers

Nila et al. comprehensively examined a retrial queueing model featuring collisions, feedback, and priority customers. Originating from applications in packet switching networks, e-commerce, and production systems, the study employs the supplementary variable method to derive performance measures such as stable state probabilities and regular orbit size. Assumptions include customer arrivals following a Poisson process, with complex interactions involving collisions, pre-emptive recommence, orbital searches, and feedback. A detailed analysis of special cases and numerical results were provided, showcasing the model's applicability in diverse domains. Significance lies in the innovative approach to retrial

queueing systems, addressing real-world scenarios in telecommunication networks, call centres, and manufacturing**.**

- **A Non-Markovian Retrial Queueing System**

Atencia et al. investigated a discrete-time retrial queueing system where arriving customers choose between joining a retrial group and displacing the current server occupant. Idle servers initiate immediate service. The study employed a recursive algorithm, exploring customer waiting times and providing practical insights through numerical examples. Assumptions involve general retrial and service times, with only the retrial group's first customer granted direct server access. Additionally, a Markov chain was studied. The practical need for non-geometrical retrial times in computer and telecommunication networks was addressed. Discrete-time systems are crucial for handling simultaneous events. Significance lies in contributing to queueing theory, offering applicable models, and delving into nuanced scenarios involving task displacement. An inverse order discipline was adopted, considering negative customers and numerical examples were illustrated to showcase the effect of parameters on different performance characteristics.

- **Retrial Queue with Discipline of Adaptive Permanent Pooling**

Brugno et al. introduces an innovative approach to customer service in a single-server retrial queue, emphasising group-based service. The method allows admission during service and early termination of the admission period, addressing gaps in existing models. Employing a six-dimensional Markov chain, the study establishes ergodicity conditions and proposes a numerically stable algorithm. Formulas for calculating different performance measures of the system were provided and numerical experiments illustrate the dependency of the metrics on certain factors. Addressing retrial queue limitations, a more flexible and effective customer service discipline were offered, extending applicability to scenarios with early admission termination. The proposed approach finds practical utility in diverse systems, such as telecommunications, transportation, and e-commerce, allowing optimization of parameters like group sizes and service delays. Valuable tools for system managers were provided, aiding optimal decision-making regarding parameter values and showcasing potential improvements in system performance.

- **Cost Optimization and ANFIS Computing For Admission Control of M/M/1/K Queue with General Retrial Times and Discouragement**

Sanga et al. explored admission control policies in a single-server finite capacity retrial queueing model, emphasising discouraged customers and retrial dynamics. The introduced F-policy, restricting arrivals when the system is full, proves pivotal for congestion management. Furthermore, a cost function was established to calculate the expected cost of the system as well as to estimate optimal service rates. Notably, the study pioneers the use of artificial neuro fuzzy inference system (ANFIS) and genetic algorithm (GA) for performance analysis and cost optimization, a novel approach in queueing literature. The study's significance lies in its holistic consideration of finite capacity, retrial attempts, and discouragement. The integration of ANFIS and GA offers innovative tools for evaluating system performance and optimising control parameters, pushing the boundaries of traditional queueing analysis.

- **F-Policy for M/M/1/K Retrial Queueing Model with State-Dependent Rates: Performance and Safety Management**

Jain et al. investigated the F-policy in a single-server finite capacity Markovian retrial queueing model, addressing real-world congestion problems. The study incorporated state-dependent arrivals and service processes for enhanced realism. The birth–death process and Chapman–Kolmogorov equations were employed to analyse queueing characteristics. Jain et al. established steady-state queue size distributions, performance indices, and a cost function. Numerical examples, sensitivity analysis, and system descriptors are provided, to understand the system's behaviour. Retrial queueing models, particularly with finite capacity, have practical applications in diverse areas, exemplified by scenarios like call centres. The F-policy, restricting arrivals when system capacity is full, is crucial for maintaining service quality. The study builds on prior works in Markovian retrial queues and introduces state-dependent rates, addressing a gap in the literature. Significance of the study lies in its focus on admission control based on F-policy for a specific retrial queueing model, considering state-dependent rates, a novel contribution in the field. The use of a recursive method for steady-state analysis and the determination of optimal service rates and costs through heuristic approaches enhance its practical implications.

- **Cost Analysis of a Bulk Service Retrial Queue**

A bulk queueing system with server's choice of allowing reservice, multiple vacations and set-up times was studied by Haridass et al. The probability generating function of the queue size of the model was developed, including various performance measures and a cost model. The motivation for this model stems from a real-life situation observed in ESNs (Environmental Sensor Networks). The situation observed in ESNs can be modelled as a queueing system with a choice available to the server to admit re-service, multiple vacations and setup times. The model considered in is unique because of the server's choice of allowing re-service and set-up times introduced. The key improvement in the proposed model is the introduction of cost consideration based on the server's decision to accept or reject a service request. This model optimized the overall cost in the case where multiple service requests are queued up in a bulk. The results so arrived can be used to make a managerial decision whether to optimise the overall cost and find the best operating scheme in a waiting line system.

- **Performance Analysis of Retrial Queueing Systems Using Generalised Stochastic Petri Nets**

Gharbi et al. considered a retrial queuing system in which arriving customers, on finding the server busy, may repeat their call after a random period of time. The consideration of repeated calls posed great analytical difficulties. The method used for modelling and analysing retrial queueing systems was presented using Generalised Stochastic Petri ets (GSPNs) which benefited qualitative as well as quantitative analysis and provided the possibility of using results and software tools designed within that framework to obtain performance indices via analytic method or by other means. GSPNs also proved to be a powerful tool to analyse queueing systems that were difficult to be investigated by traditional methods. This procedure was also found to be useful to study non-markovian systems in which certain factors such as customer arrivals and service times are generally distributed.

- **Reliability Analysis of the Retrial Queue with Server Breakdowns and Repairs**

Wang et al. conducted a detailed analysis of the reliability of retrial queues was discussed. A single server system prone to breakdowns and repairs was studied. The supplementary variables method was used to obtain explicit expressions of some of the key reliability such as, availability, failure frequency and reliability function of the server. Some special queues such as the repairable M/G/1 queue and repairable retrial queue can be obtained from the results of the study. Furthermore, these results can be generalised to the repairable multi-server retrial queues.

- **Analysis of A Priority Retrial Queue with Dependent Vacation Scheme and Application to Power Saving in Wireless Communication Systems**

A priority retrial queue with heterogeneous repeated vacations was analysed. In this system, the server departs for vacation after completing a service. Additionally, the model derived has applications in power saving mechanisms in wireless communication systems. Author analysed a non-preemptive priority retrial queueing system with a dependent vacation policy in which the vacations depend on the number of previous vacations. The potential application that the proposed policy has is in power-saving mechanism in wireless communication networks that considered the important aspect of retrial customer behaviour. The energy gain using the dependent vacation policy was provided and compared it with other vacation schemes and it proposed metrics that produce efficiency of sleep mode operation. Furthermore, optimization problems were devised to find suitable values for critical parameters. These parameters are chosen to maximize energy efficiency achieved in power-saving mode, while ensuring that performance constraints are met.

- **PCS Networks with Correlated Arrival Process and Retrial Phenomenon**

The arrival of new calls and handoff calls to a PCS network was modelled with the help of Markovian Arrival Process. This allows correlation of the interarrival times among new calls, handoff calls and also between these two kinds of calls. Wei Li et al. developed an explicit expression of the infinitesimal generator matrix of the Markov chain governing the network in this model and found its complexity, under the general conditions. This serves as the significant achievement of the study because of the difficulty in developing this matrix. Methods to find the stationary probability were discussed and an efficient method from which the new call blocking probability and handoff call failure probability was introduced.

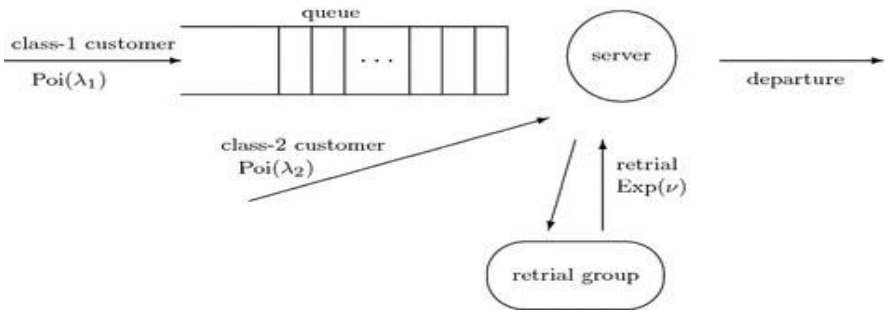
Also, the busy period of the orbit is introduced, which is from the network provider's perception. Its distribution and expectation were obtained. The results obtained can be used as guidelines to performance evaluation of a PCS network.

- **Analysis of a Single Server Batch Arrival Unreliable Queue with Balking and General Retrial Time**

A batch arrivals queue with general retrial time, breakdowns, repairs and reserved time were studied by Zirem et al. The customers are assumed to arrive in accordance with Poisson processes. This model used supplementary variables technique to conduct extensive analysis and to find the PGFs of the number of customers when the system is idle, busy, under repair or reserved. Then, important performance measures, stochastic decomposition property of the system size distribution, which is shown to be good and reliability indices are obtained. Additionally, some special cases were also discussed. Possible applications of this model are in fields such as cognitive radio network and manufacturing.

- **Energy Harvesting Cognitive Radio Networks with Strategic Users: A Two-Class Queueing Model with Retrials**

A single server retrial queueing model in CRNs was considered, and the server goes through an energy-harvesting period after each service. The study was done on how this time period impacts unlicensed users' equilibrium joining strategies. The authors also developed the equilibrium joining strategy for these self-interested users and prove the existence of its joining probability using Nash-equilibrium conditions. The welfare maximising joining strategy from the social planner's perspective and shows was explored, using numerical experiments, how charging an appropriate admission fees makes the strategic users adopt this strategy instead



- **On An Unreliable Retrial Queue with General Repeated Attempts and J Optional Vacations**

Yang et al. examined a single-server retrial queue with batch arrivals and a constant retrial rate, in which the server is allowed to take an additional vacation after the first one. Customers enter the system in accordance with a compound Poisson process. They discussed a variant vacation policy for an 1 retrial queue with server breakdowns and general retrial times.

The probability-generating functions of the system size distribution are derived using the supplementary variable technique along with its stochastic decomposition. System performance measures and reliability indices were also developed. To find the optimal value of J, a cost model was constructed. The impact of the value of on cost function and performance metrics of the system was investigated using numerical experiments.

- **An Energetic Retrial Queue with Vacations and It's Control**

Aissani presented findings about the non Markovian retrial queue featuring vacations, controlled by an arbitrary probability law, does not depend on the number of customers in the orbit. Serving a customer involves not only random time, but also a random energy requirement with an arbitrary probability distribution. When the server takes a vacation, the energy required for each vacation is arbitrarily distributed. A stochastically recursive relation suitable for discrete-event simulation was derived along with a formula for the generating function of the number of customers in the orbit in steady state. The model also demonstrated explicit forms of stochastic decomposition property. Performance measures and optimal control parameters for vacation and retrial policies were illustrated.

- **The M/G/1 Retrial Queue: New Descriptors of the Customer's Behaviour Amador**

Amadore et al. examined queuing systems with retrials, where customers make repeated attempts to enter service after some time instead of waiting in a queue. Specifically, concentration is given to the primary model of the M/G/1 type retrial system and analyzes the distribution of successful and blocked events among primary and retrial customers. The objectives were to study the

distribution of performance descriptors such as, successful retrials, successful arrivals and block retrials, of customer's behaviour. Methods to analyse performance measures computationally were studied and it was recommended to use the direct methods for computation than alternative ones.

2. Conclusion

This paper has surveyed the recent development of queueing theory in wireless communication networks, call centers, etc. The idea related to queueing theory and its applications have been presented in detail.in this paper with various models and their estimated waiting times and system designs. The paper tries to offer a thorough picture of each research paper in terms of its objectives, the assumptions it makes, the methodology utilised, and the paper's contribution to the field of retrial queueing research. Additionally, various numerical illustrations were presented in the surveyed works to validate their findings and demonstrate its application.

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