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A Study on Intuitionistic Pre * Connected Spaces

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ABSTRACT:

The major goal of this work is to introduce and investigate the concepts of Intuitionistic Pre * Connected (In short 𝒥P*-Connected) Spaces using the concepts of intuitionistic pre * open (In short 𝒥P*O) sets. Also we give characterization for this connected space and discuss the relationship with other known intuitionistic connected spaces.

Keywords: 𝒥P*- connected, 𝒥P*- disconnected, 𝒥P – connected, 𝒥- connected.

AMS subject classification (2010): 54C05.

1. Introduction

D. Coker [1] introduced the idea of intuitionistic sets for the first time in 1996. In 2017, G. Sasikala and M. Navaneethakrishnan [5] give the definition of intuitionistic pre open sets in 𝒥TS. In 2021, G. Esther Rathinakani and M. Navaneethakrishnan [2,3,4] gives a new closure operator in intuitionistic topological spaces and define intuitionistic semi * open set. In 2023, we [6,7,8] introduced 𝒥P* open and 𝒥P* closed sets in 𝒥TS using the concepts of intuitionisic interior and intuitionistic generalized closure operators. Also we define 𝒥P*-continuous maps and 𝒥P*- open maps in 𝒥TS.

In this study, we define 𝒥P*- connected spaces using the concepts of 𝒥P*O sets. We also demonstrate that the 𝒥P*- connected space is intermediate between 𝒥P- connected space and 𝒥- connected space.

Definition 1.1 [1] Let $\grave{X}_J$ be a set that is not empty. An object with the form $M_J = <\grave{X}_J, M_{J1}, M_{J2}>$, where M_{J1} and M_{J2} are subsets of $\grave{X}_J$ satisfying $M_{J1} \cap M_{J2} = \phi$, is known as an intuitionistic set ($\mathcal{J}$S in short). The terms "set of members of M_J" and "set of non-members of M_J" refer to the sets M_{J1} and M_{J2} respectively.

Definition 1.2 [1] Assume that $\grave{X}_J$ is a non-empty set, that $M_J = <\grave{X}_J, M_{J1}, M_{J2}>$, that $N_J = <\grave{X}_J, N_{J1}, N_{J2}>$ is an $\mathcal{J}$S's and that $\{M_{Ji} : i \in J\}$ be arbitrary family of $\mathcal{J}$S's. Then,

a) $M_J \subseteq N_J$ iff $M_{J1} \subseteq N_{J1}$ and $M_{J2} \supseteq N_{J2}$.

b) $M_J = N_J$ iff $M_J \subseteq N_J$ and $M_J \supseteq N_J$.

c) The complement of M_J is defined as $M_J^c = <\grave{X}_J, M_{J2}, M_{J1}>$.

d) $\cup M_{Ji} = <\grave{X}_J, \cup M_{Ji1}, \cap M_{Ji2}>$ and $\cap M_{Ji} = <\grave{X}_J, \cap M_{Ji1}, \cup M_{Ji2}>$.

e) $M_J - N_J = M_J \cap N_J^c$.

f) $\ddot{\phi}_I = <\grave{X}_J, \phi, \grave{X}_J>$ and $\ddot{X}_I = <\grave{X}_J, \grave{X}_J, \phi>$.

Definition 1.3 [1] Assume that $\grave{X}_J$ is a non-empty set and τ_I is the set of $\mathcal{J}$S's of $\grave{X}_J$ then τ_I is known as an intuitionistic topology ($\mathcal{J}$T in short) on $\grave{X}_J$ if it meets the criteria listed below:

1) $\ddot{X}_I, \ddot{\phi}_I \in \tau_I$.

2) $M_J \cap N_J \in \tau_I$ for every $M_J, N_J \in \tau_I$.

3) $\cup M_{Ji} \in \tau_I$ for any arbitrary family $\{M_{Ji} : i \in J\} \subseteq \tau_I$.

The pair $(\grave{X}_J, \tau_I)$ is referred to as intuitionistic topological space ($\mathcal{J}$TS in short) and intuitionistic open set ($\mathcal{J}$OS in short) in $\grave{X}_J$ is referred to as $\mathcal{J}$S in τ_I. The intuitionistic closed set ($\mathcal{J}$CS in short) in $\grave{X}_J$ is regarded as the counterpart of $\mathcal{J}$OS in $\grave{X}_J$.

Definition 1.4 [1] If $(\grave{X}_J, \tau_I)$ is a $\mathcal{J}$TS and M_J is a $\mathcal{J}$S in $\grave{X}_J$ then the definition of the $\mathcal{J}$- interior operator of M_J and the $\mathcal{J}$- closure operator of M_J are as follows: (i) $\mathcal{J}$int$(M_J) = \cup \{N_J : N_J$ is $\mathcal{J}$OS in $\grave{X}_J$ & $M_J \supseteq N_J\}$. (ii) $\mathcal{J}$cl$(M_J) = \cap \{ N_J : N_J$ is $\mathcal{J}$CS in $\grave{X}_J$ & $M_J \subseteq N_J\}$.

Definition 1.5 [2] If $(\grave{X}_J, \tau_I)$ is an $\mathcal{J}$TS and a $\mathcal{J}$S M_J is known as the $\mathcal{J}$g- closed set if $\mathcal{J}$cl$(M_J) \subseteq U_J$ whenever $M_J \subseteq U_J$ and U_J is $\mathcal{J}$OS in $\grave{X}_J$. The $\mathcal{J}$g – open set in $\grave{X}_J$ is known as the $\mathcal{J}$g-closed set's counterpart.

Definition 1.6 [2] If $(\grave{X}_J, \tau_I)$ is an $\mathcal{J}$TS and M_J be a $\mathcal{J}$S in $\grave{X}_J$ then the definition of

a) $\mathcal{J}$g- closure of M_J is, $\mathcal{J}$cl*$(M_J) = \cap \{N_J : N_J$ is $\mathcal{J}$g- CS in $\grave{X}_J$ & $M_J \subseteq N_J\}$.

b) $\mathcal{J}$g- interior of M_J is, $\mathcal{J}$int*$(M_J) = \cup \{N_J : N_J$ is $\mathcal{J}$g- OS in $\grave{X}_J$ & $M_J \supseteq N_J\}$.

Definition 1.7 [5,7] If $(\grave{X}_J, \tau_I)$ is an $\mathcal{J}$TS and a $\mathcal{J}$S M_J in $\grave{X}_J$ is known as the

a) $\mathcal{J}$PO set if $M_J \subseteq \mathcal{J}$int($\mathcal{J}cl(M_J)$). The $\mathcal{J}$PC set in $\grave{X}_J$ is the $\mathcal{J}$PO set's counterpart.

b) $\mathcal{J}$P*O set if $M_J \subseteq \mathcal{J}$int($\mathcal{J}$cl*$(M_J)$). The $\mathcal{J}$P*C set in $\grave{X}_J$ is the $\mathcal{J}$P*O set's counterpart.

c) $\mathcal{J}\mathcal{R}$*O set if $M_J = \mathcal{J}$int($\mathcal{J}$cl*(M_J)). The $\mathcal{J}\mathcal{R}$*C set in $\grave{X}_J$ is the $\mathcal{J}\mathcal{R}$*O set's counterpart.

Definition 1.8 [5] If $(\grave{X}_J, \tau_I)$ is an $\mathcal{J}$TS and a $\mathcal{J}$S M_J in $\grave{X}_J$ is known as the $\mathcal{J}$P*- regular set if it is both $\mathcal{J}$P*O and $\mathcal{J}$P*C set.

Theorem 1.9 [5] If $(\grave{X}_J, \tau_{IT})$ is an $\mathcal{J}$TS then,

a) Every $\mathcal{J}$O set is $\mathcal{J}$P*O set.

b) Every $\mathcal{J}$C set is $\mathcal{J}$P*C set.

c) Every $\mathcal{J}$P*O set is $\mathcal{J}$PO set.

d) Every $\mathcal{J}$P*C set is $\mathcal{J}$PC set.

e) Every 𝒥ℛ*O set is 𝒥P*O set.
f) Every 𝒥ℛ*C set is 𝒥P*C set.
g) Arbitrary union of 𝒥P*O sets is 𝒥P*O set.
h) Intersection of 𝒥P*C sets is 𝒥P*C set.

Definition 1.10 Let $(\grave{X}_J,\tau_{IT})$ be an 𝒥TS. Then $(\grave{X}_J,\tau_{IT})$ is called the
a) 𝒥- connected space if $\ddot{X}_I$ cannot be expressed as the union of two disjoint nonempty 𝒥O sets in $\grave{X}_J$.
b) 𝒥P- connected space if $\ddot{X}_I$ cannot be expressed as the union of two disjoint nonempty 𝒥PO sets in $\grave{X}_J$.
c) 𝒥ℛ*- connected space if $\ddot{X}_I$ cannot be expressed as the union of two disjoint nonempty 𝒥ℛ*O sets in $\grave{X}_J$.

Definition 1.11 [6,8] Let $f_J : \grave{X}_J \rightarrow \grave{Y}_J$ is said to be
a) 𝒥P*- continuous map if $f_J^{-1}(V_J)$ is 𝒥P*O set in $\grave{X}_J$ for every 𝒥O set V_J in $\grave{Y}_J$.
b) 𝒥P*- irresolute map if $f_J^{-1}(V_J)$ is 𝒥P*O set in $\grave{X}_J$ for every 𝒥P*O set V_J in $\grave{Y}_J$.
c) Contra 𝒥P*- continuous map if $f_J^{-1}(V_J)$ is 𝒥P*C set in $\grave{X}_J$ for every 𝒥O set V_J in $\grave{Y}_J$.
d) Contra 𝒥P*- irresolute map if $f_J^{-1}(V_J)$ is 𝒥P*C set in $\grave{X}_J$ for every 𝒥P*O set V_J in $\grave{Y}_J$.
e) 𝒥P*- open map if $f_J(V_J)$ is 𝒥P*O set in $\grave{Y}_J$ for every 𝒥O set V_J in $\grave{X}_J$.
f) 𝒥P*- closed map if $f_J(V_J)$ is 𝒥P*C set in $\grave{Y}_J$ for every 𝒥C set V_J in $\grave{X}_J$.
g) Pre 𝒥P*- open map if $f_J(V_J)$ is 𝒥P*O set in $\grave{Y}_J$ for every 𝒥P*O set V_J in $\grave{X}_J$.
h) Pre 𝒥P*- closed map if $f_J(V_J)$ is 𝒥P*C set in $\grave{Y}_J$ for every 𝒥P*C set V_J in $\grave{X}_J$.

Theorem 1.12 [6] Let $f_J : \grave{X}_J \rightarrow \grave{Y}_J$ be a map then the followings are holds,
a) Every 𝒥P*- Irresolute map is 𝒥P*- continuous map.
b) Every Contra 𝒥P*- Irresolute map is Contra 𝒥P*- continuous map.

2. Intuitionistic Pre * Connected Spaces

Definition – 2.1. Let $(\grave{X}_J,\tau_{IT})$ be an 𝒥TS. Then $(\grave{X}_J,\tau_{IT})$ is called the 𝒥P*- disconnected if there exists an 𝒥P*O sets $M_J \neq \ddot{\phi}_I$ and $N_J \neq \ddot{\phi}_I$ such that $M_J \cup N_J = \ddot{X}_I$ and $M_J \cap N_J = \ddot{\phi}_I$.
Definition – 2.2. Let $(\grave{X}_J,\tau_{IT})$ be an 𝒥TS. Then $(\grave{X}_J,\tau_{IT})$ is called the 𝒥P*- connected if it is not an 𝒥P*- disconnected. (i.e), $\ddot{X}_I$ cannot be expressed as the union of two disjoint nonempty 𝒥P*O sets in $\grave{X}_J$ is called the 𝒥P*- connected.

Example – 2.3. Let $\grave{X}_J = \{a_{xj},b_{xj},c_{xj}\}$. Consider the 𝒥T, $\tau_{IT} = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\{a_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{b_{xj}\},\{c_{xj}\}>, <\grave{X}_J,\{a_{xj},b_{xj}\},\phi>, <\grave{X}_J,\phi,\{b_{xj},c_{xj}\}>\}$ then 𝒥P*O$(\grave{X}_J) = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\phi,\{b_{xj},c_{xj}\}>, <\grave{X}_J,\{a_{xj},b_{xj}\},\phi>, <\grave{X}_J,\{a_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{b_{xj}\},\{c_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}>, <\grave{X}_J,\{a_{xj},b_{xj}\},\{c_{xj}\}>\}$.
Clearly $(\grave{X}_J,\tau_{IT})$ is 𝒥P*- connected space.

Theorem – 2.4. Let $(\grave{X}_J,\tau_{IT})$ be an 𝒥TS then the followings are hold.
a) Every 𝒥P*- connected is 𝒥- connected.
b) Every 𝒥P*- connected is 𝒥ℛ*- connected.
c) Every 𝒥P- connected is 𝒥P*- connected.
Proof: (a) Let $\grave{X}_J$ be a 𝒥P*- connected. To prove, $\grave{X}_J$ is 𝒥- connected. Suppose $\grave{X}_J$ is 𝒥-disconnected then there exists nonempty disjoint 𝒥O sets M_J and N_J such that $\ddot{X}_I = M_J \cup N_J$. Since M_J and N_J are 𝒥O sets then M_J and N_J are 𝒥P*O sets. Therefore, $\grave{X}_J$ is 𝒥P*-disconnected. This is contradiction to our assumption. Hence $\grave{X}_J$ is 𝒥- connected.

(b) Let $\grave{X}_J$ be a $\mathscr{J}P^*$- connected. To prove, $\grave{X}_J$ is $\mathscr{J}\mathscr{R}^*$- connected. Suppose $\grave{X}_J$ is $\mathscr{J}\mathscr{R}^*$-disconnected then there exists nonempty disjoint $\mathscr{J}\mathscr{R}^*O$ sets M_J and N_J such that $\ddot{X}_I = M \cup N$. Since M_J and N_J are $\mathscr{J}\mathscr{R}^*O$ sets then M_J and N_J are $\mathscr{J}P^*O$ sets. Therefore, $\grave{X}_J$ is $\mathscr{J}P^*$-disconnected. This is contradiction to our assumption. Hence $\grave{X}_J$ is $\mathscr{J}\mathscr{R}^*$- connected.

(c) Let $\grave{X}_J$ be a $\mathscr{J}P$- connected. To prove, $\grave{X}_J$ is $\mathscr{J}P^*$- connected. Suppose $\grave{X}_J$ is $\mathscr{J}P^*$-disconnected then there exists nonempty disjoint $\mathscr{J}P^*O$ sets M_J and N_J such that $\ddot{X}_I = M_J \cup N_J$. Since M_J and N_J are $\mathscr{J}P^*O$ sets then M_J and N_J are $\mathscr{J}PO$ sets. Therefore, $\grave{X}_J$ is $\mathscr{J}P$-disconnected. This is contradiction to our assumption. Hence $\grave{X}_J$ is $\mathscr{J}P^*$- connected.

The converse of the above theorems need not be true as shows in the following example.

Example – 2.5. Let $\grave{X}_J = \{a_{xj},b_{xj}\}$. Consider the $\mathscr{J}T$, $\tau_{IT} = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\{a_{xj}\},\phi>, <\grave{X}_J,\{b_{xj}\},\phi>, <\grave{X}_J,\phi,\phi>\}$ then $\mathscr{J}P^*O(\grave{X}_J) = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\{a_{xj}\},\phi>, <\grave{X}_J,\{b_{xj}\},\phi>, <\grave{X}_J,\phi,\phi>, <\grave{X}_J,\{a_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{b_{xj}\},\{a_{xj}\}>\}$. Clearly (X,τ_{IT}) is $\mathscr{J}$- connected space. But $<\grave{X}_J,\{a_{xj}\},\{b_{xj}\}> \neq \phi_I \neq <\grave{X}_J,\{b_{xj}\},\{a_{xj}\}>$ and $<\grave{X}_J,\{a_{xj}\},\{b_{xj}\}> \cup <\grave{X}_J,\{b_{xj}\},\{a_{xj}\}> = \ddot{X}_I$. Also, $<\grave{X}_J,\{a_{xj}\},\{b_{xj}\}> \cap <\grave{X}_J,\{b_{xj}\},\{a_{xj}\}> = \ddot{\phi}_I$. Therefore, $\grave{X}_J$ is $\mathscr{J}P^*$- disconnected space.

Example – 2.6. In example – 2.5, $\mathscr{J}\mathscr{R}^*O(\grave{X}_J) = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\phi,\phi>\}$. Clearly, $\grave{X}_J$ is $\mathscr{J}\mathscr{R}^*$-connected space but not a $\mathscr{J}P^*$- connected space.

Example – 2.7. Let $\grave{X}_J = \{a_{xj},b_{xj},c_{xj}\}$. Consider the $\mathscr{J}T$, $\tau_{IT} = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\{a_{xj}\},\{c_{xj}\}>, <\grave{X}_J,\{c_{xj}\},\{a_{xj},b_{xj}\}>, <\grave{X}_J,\{a_{xj},c_{xj}\},\phi>\}$ then $\mathscr{J}P^*O(\grave{X}_J) = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\{a_{xj}\},\{c_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}>, <\grave{X}_J,\{c_{xj}\},\{a_{xj},b_{xj}\}>, <\grave{X}_J,\{a_{xj},c_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\phi>, <\grave{X}_J,\{a_{xj},c_{xj}\},\phi>\}$ and $\mathscr{J}PO(\grave{X}_J) = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\phi,\phi>, <\grave{X}_J,\phi,\{b_{xj}\}>, <\grave{X}_J,\phi,\{c_{xj}\}>, <\grave{X}_J,\{b_{xj}\},\{a_{xj}\}>, <\grave{X}_J,\{c_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\phi,\{a_{xj},b_{xj}\}>, <\grave{X}_J,\phi,\{b_{xj},c_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\{c_{xj}\}>, <\grave{X}_J,\{b_{xj},c_{xj}\},\{a_{xj}\}>, <\grave{X}_J,\{a_{xj}\},\phi>, <\grave{X}_J,\{b_{xj}\},\phi>, <\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}>, <\grave{X}_J,\{c_{xj}\},\phi>, <\grave{X}_J,\{c_{xj}\},\{a_{xj},b_{xj}\}>, <\grave{X}_J,\{a_{xj},c_{xj}\},\{b_{xj}\}>, <\grave{X}_J,\{a_{xj},b_{xj}\},\phi>, <\grave{X}_J,\{b_{xj},c_{xj}\},\phi>, <\grave{X}_J,\{a_{xj},c_{xj}\},\phi>\}$. Clearly, $\grave{X}_J$ is $\mathscr{J}P^*$- connected spaces. But, $<\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}> \neq \ddot{\phi}_I \neq <\grave{X}_J,\{b_{xj},c_{xj}\},\{a_{xj}\}>$ and $<\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}> \cup <\grave{X}_J,\{b_{xj},c_{xj}\},\{a_{xj}\}> = \ddot{X}_I$. Also, $<\grave{X}_J,\{a_{xj}\},\{b_{xj},c_{xj}\}> \cap <\grave{X}_J,\{b_{xj},c_{xj}\},\{a_{xj}\}> = \ddot{\phi}_I$. Therefore, $\grave{X}_J$ is $\mathscr{J}P$- disconnected space.

Theorem – 2.8. An $\mathscr{J}TS$ $(\grave{X}_J,\tau_{IT})$ has the only $\mathscr{J}P^*O$ and $\mathscr{J}P^*C$ sets are $\ddot{\phi}_I$ and $\ddot{X}_I$ itself then $(\grave{X}_J,\tau_{IT})$ is an $\mathscr{J}P^*$- connected.

Proof: Let $\ddot{\phi}_I$ and $\ddot{X}_I$ are both $\mathscr{J}P^*O$ and $\mathscr{J}P^*C$ sets in $\grave{X}_J$. To prove, $\grave{X}_J$ is $\mathscr{J}P^*$- connected. Suppose $\grave{X}_J$ is $\mathscr{J}P^*$- disconnected then there exists nonempty disjoint $\mathscr{J}P^*O$ sets M_J and N_J such that $\ddot{X}_I = M_J \cup N_J$. Therefore, $M_J = N_J^c$ is $\mathscr{J}P^*C$ set. Hence, M_J is both $\mathscr{J}P^*O$ and $\mathscr{J}P^*C$ set. This is contradiction to our assumption. Hence $\grave{X}_J$ is $\mathscr{J}P^*$- connected.

The converse of the above theorem need not be true as shows in the following example.

Example – 2.9. Let $\grave{X}_J = \{a_{xj},b_{xj}\}$. Consider the $\mathscr{J}T$, $\tau_{IT} = \{\ddot{X}_I, \ddot{\phi}_I, <\grave{X}_J,\phi,\{b_{xj}\}>, <\grave{X}_J,\{b_{xj}\},\phi>\}$ then $\mathscr{J}P^*O(\grave{X}_J) = \tau_{IT}$. Clearly $\grave{X}_J$ is $\mathscr{J}P^*$- connected space but $<\grave{X}_J,\phi,\{b_{xj}\}>$ is both $\mathscr{J}P^*O$ and $\mathscr{J}P^*C$ set in $\grave{X}_J$.

Theorem – 2.10. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two $\mathscr{J}TS$ and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be surjection $\mathscr{J}P^*$- Continuous Map then $\grave{Y}_J$ is $\mathscr{J}$- connected if $\grave{X}_J$ is $\mathscr{J}P^*$- connected.

Proof: Suppose $\grave{Y}_J$ is $\mathscr{J}$- disconnected then there exists nonempty disjoint $\mathscr{J}O$ sets M_J and N_J such that $\ddot{Y}_I = M_J \cup N_J$. Since f is $\mathscr{J}P^*$- continuous. Therefore, $f_J^{-1}(M_J)$ and $f_J^{-1}(N_J)$ are $\mathscr{J}P^*O$ sets in $\grave{X}_J$. Since, $M_J \neq \ddot{\phi}_I$ and $N_J \neq \ddot{\phi}_I$ then $f_J^{-1}(M_J) \neq \ddot{\phi}_I$ and $f^{-}{}_J{}^{1}(N_J) \neq \ddot{\phi}_I$. Since, $\ddot{Y}_I = M_J \cup N_J$. Therefore, $f_J^{-1}(\ddot{Y}_I) = \ddot{X}_I = f_J^{-1}(M_J) \cup f_J^{-1}(N_J)$. Also, $f_J^{-1}(M_J) \cap f_J^{-1}(N_J) = f_J^{-1}(M_J \cap N_J) = f_J^{-1}(\ddot{\phi}_I) = \ddot{\phi}_I$. Therefore, $\grave{X}_J$ is $\mathscr{J}P^*$- disconnected. This is contradiction to our assumption. Hence $\grave{Y}_J$ is $\mathscr{J}$- connected.

Corollary – 2.11. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be surjection 𝒥P*- Irresolute Map then $\grave{Y}_J$ is 𝒥- connected if $\grave{X}_J$ is 𝒥P*- connected.

Proof: Let $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be a surjection 𝒥P*- Irresolute Map and $\grave{X}_J$ be an 𝒥P*- connected space. We know that, every 𝒥P*- irresolute map is 𝒥P*- continuous map then by theorem – 2.10, $\grave{Y}_J$ is 𝒥- connected.

Theorem – 2.12. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be surjection 𝒥P*- Irresolute Map then $\grave{Y}_J$ is 𝒥P*- connected if $\grave{X}_J$ is 𝒥P*- connected.

Proof: Suppose $\grave{Y}_J$ is 𝒥P*- disconnected then there exists nonempty disjoint 𝒥P*O sets M_J and N_J such that $\ddot{Y}_I = M_J \cup N_J$. Therefore, $M_J = N_J^c$ is 𝒥P*- regular set in $\grave{Y}_J$. Since, f_J is 𝒥P*- irresolute map. Therefore, $f_J^{-1}(M_J)$ is 𝒥P*- regular set in $\grave{X}_J$. By theorem – 2.8, $\grave{X}_J$ is 𝒥P*- disconnected. This is contradiction to our assumption. Hence $\grave{Y}_J$ is 𝒥P*- connected.

Theorem – 2.13. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be injection Pre 𝒥P*- open and Pre 𝒥P*- closed Map then $\grave{X}_J$ is 𝒥P*- connected if $\grave{Y}_J$ is 𝒥P*- connected.

Proof: Suppose $\grave{X}_J$ is 𝒥P*- disconnected then there exists nonempty disjoint 𝒥P*O sets M_J and N_J such that $\ddot{X}_I = M_J \cup N_J$. Therefore, $M_J = N_J^c$ is 𝒥P*C set. Hence, M_J is both 𝒥P*O and 𝒥P*C set in $\grave{X}_J$. Since, f_J is both Pre 𝒥P*- open and Pre 𝒥P*- closed map. Therefore, $f_J(M_J)$ is 𝒥P*- regular set in $\grave{Y}_J$. By theorem – 2.8, $\grave{Y}_J$ is 𝒥P*- disconnected. This is contradiction to our assumption. Hence $\grave{X}_J$ is 𝒥P*- connected.

Theorem – 2.14. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be injection 𝒥P*- open and 𝒥P*- closed Map then $\grave{X}_J$ is 𝒥- connected if $\grave{Y}_J$ is 𝒥P*- connected.

Proof: Suppose $\grave{X}_J$ is 𝒥- disconnected then there exists nonempty disjoint 𝒥O sets M_J and N_J such that $\ddot{X}_I = M_J \cup N_J$. Therefore, $M_J = N_J^c$ is both 𝒥O and 𝒥C set in $\grave{X}_J$. Hence, M_J is both 𝒥P*O and 𝒥P*C set in $\grave{X}_J$. Since, f_J is both 𝒥P*- open and 𝒥P*- closed map. Therefore, $f_J(M_J)$ is 𝒥P*- regular set in $\grave{Y}_J$. By theorem – 2.8, $\grave{Y}_J$ is 𝒥P*- disconnected. This is contradiction to our assumption. Hence $\grave{X}_J$ is 𝒥- connected.

Theorem – 2.15. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be onto Contra 𝒥P*- continuous Map then $\grave{Y}_J$ is 𝒥- connected if $\grave{X}_J$ is 𝒥P*- connected.

Proof: Suppose $\grave{Y}_J$ is 𝒥- disconnected then there exists nonempty disjoint 𝒥O sets M_J and N_J such that $\ddot{Y}_I = M_J \cup N_J$. Therefore, $M_J = N_J^c$ is both 𝒥O and 𝒥C set in $\grave{Y}_J$. Since, f_J is Contra 𝒥P*- continuous map. Therefore, $f_J^{-1}(M_J)$ is 𝒥P*- regular set in $\grave{X}_J$. By theorem – 2.8, $\grave{X}_J$ is 𝒥P*- disconnected. This is contradiction to our assumption. Hence $\grave{Y}_J$ is 𝒥- connected.

Theorem – 2.16. Let $(\grave{X}_J,\tau_{IT})$ & $(\grave{Y}_J,\sigma_{IT})$ be two 𝒥TS and $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be onto Contra 𝒥P*- Irresolute Map then $\grave{Y}_J$ is 𝒥- connected if $\grave{X}_J$ is 𝒥P*- connected.

Proof: Let $f_J : (\grave{X}_J,\tau_{IT}) \rightarrow (\grave{Y}_J,\sigma_{IT})$ be a Contra 𝒥P*- Irresolute Map. Therefore, f_J is Contra 𝒥P*- continuous map. Since, $\grave{X}_J$ be a 𝒥P*- connected space then by theorem – 2.10, $\grave{Y}_J$ is 𝒥- connected.

3. Conclusion

We discussed the some of the properties of 𝒥P*– connected spaces in this paper. We intend to conduct research in the future 𝒥P* C_5 – connected space, 𝒥P*– connected sets, 𝒥P* – compact space, 𝒥P* – normal space and so on.

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